BMS particles

based on joint work with

Yannick **HERFRAY** & Xavier **BEKAERT** (IDP Tours)

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Bethe Colloquium
Bethe Center for Theoretical Physics
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Infrared effects in QED and gravity call for an **extended** notion of particle.

Mass and spin as particle labels

Electron	Photon	Graviton
mass m_e spin ½	mass 0 helicity ±1	mass 0 helicity ±2

The familiar notions of **mass** and **spin**/helicity arise as labels from the **representation theory** of **spacetime symmetries**.

What is a particle?

Wigner (1939)

A particle = a unitary irreducible representation (UIR) of the spacetime symmetry group

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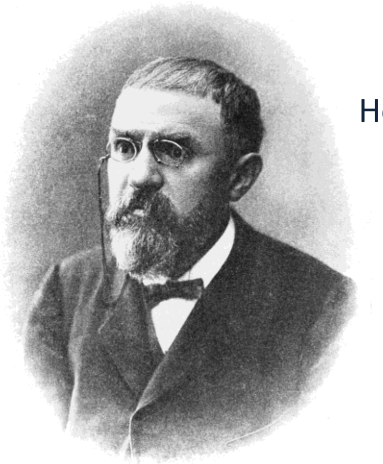
SYMMETRY

Poincaré group



Henri Poincaré

Poincaré transformations preserve the Minkowski geometry

$$\eta_{\mu\nu}\, dx^\mu dx^\nu = -dt^2 + d\mathbf{x^2}$$

Symmetry group of special relativity

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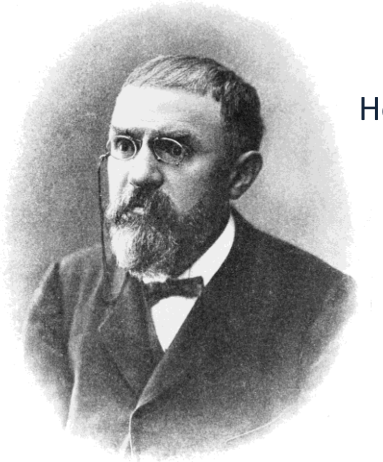

Henri Poincaré

$$x^\mu \mapsto \Lambda^\mu{}_\nu \, x^\nu + a^\mu$$

$$ISO(3,1) \simeq \underbrace{\boxed{SO(3,1)}}_{\text{6 Lorentz transformations}} \ltimes \underbrace{\boxed{\mathbb{R}^{3,1}}}_{\text{4 spacetime translations}}$$

6 Lorentz transformations
(3 rotations + 3 boosts)

4 spacetime translations

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$$(\Lambda, a) \longmapsto U(\Lambda, a)$$

QUANTUM STATE

$$|\Psi\rangle$$

Representation

Irreducible

Unitary

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Irreducible

one particle species

Unitary

probabilities are preserved

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Wigner's classification:

Poincaré UIRs are labelled by mass and spin.

Gravity changes the story

Special relativity

Poincaré group

translations

momentum p_μ

Particles = UIRs of Poincaré

General relativity

BMS group

supertranslations

supermomentum $\mathcal{P}$

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UIR of BMS group = “BMS particle”

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1 **BMS** group

supertranslations

supermomentum $\mathcal{P}$

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Goals of this talk

Extending the notion of "particle"

Deeply connected to the **infrared problem**

Amplitudes between conventional Fock states suffer from infrared **divergences**

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In QED and gravity, scattering inevitably produces arbitrarily soft photons or gravitons.

— asymptotic states in the *interacting* theory ≠ states of the *free* theory —

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by looking at “inclusive cross sections” $\supset$ sum over all photons with energy smaller than the detector resolution

→ Infrared-safe observables ✓ [Bloch, Nordsieck ‘37][Yennie et al. ‘61][Weinberg ‘65]

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a well-defined S-matrix calls for more: a proper identification of asymptotic states

→ Note: not a new idea! [Chung '65][Kibble '68] [Dollard '71][Kulish, Fadeev '70]

BUT new insights: representation theory of the asymptotic symmetries


UNIVERSITÄT BONN
bctp
Bethe Center for
Theoretical Physics
Bethe Forum
Bootstrap Approaches to Scattering Amplitudes
September, 14 - 18, 2026
Bonn, Germany
Speakers include
Elisabetta Armanini
Paolo Benincasa
Justin Berman
Laura Donnay
Giulia Fardelli
Maddalena Ferragatta
Andrea Guerrieri
Yifei He
Gregory Korchemsky
Elli Pomoni
Francesco Riva
Michele Santagata
Qinglin Yang
Alexander Zhiboedov
Organizers:
Agnese Bissi
Claude Duhr
Florian Loebbert
Bethe Center for Theoretical Physics
Physikalisches Institut
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53115 Bonn
Phone: (+49) 228 / 73 3770
e-mail: theory@physik.uni-bonn.de
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Additional information and application form:
https://indico.hiskp.uni-bonn.de/event/1433/

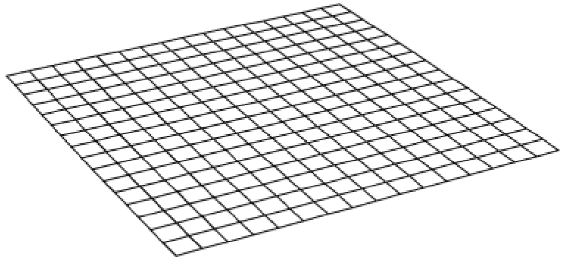
Outline

- BMS symmetries
- BMS particles
  - – hard representations
  - – generic representations
- Infrared physics from representation theory
- Outlook

BMS symmetries

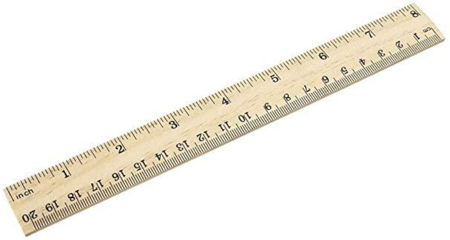
A surprise in asymptotically flat spacetimes

Minkowski metric (flat spacetime) in 4D



The geometry is described by the metric

$$ds^2_{\text{flat}} = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$



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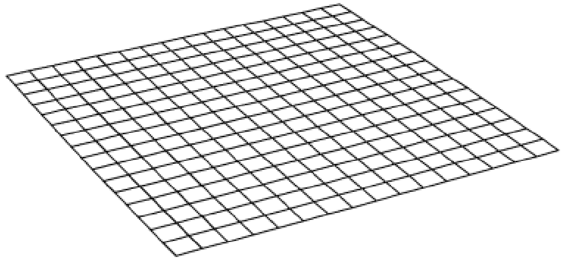

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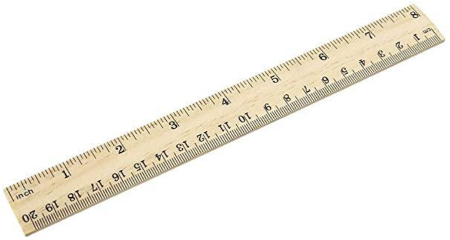
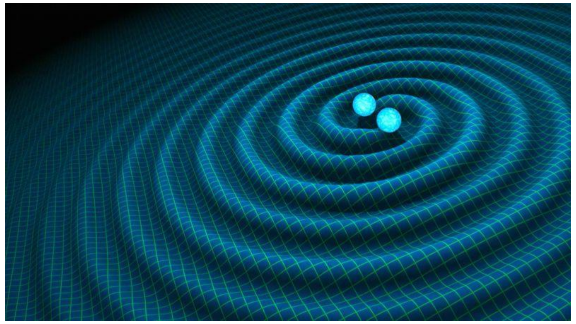


***Asymptotically* flat spacetime** (as $r \to \infty$)



Curved spacetime that looks flat seen from a far distance. The deviation from flatness is dictated by the **large *r* behavior** of the metric.

$$ds^2 = ds^2_{\text{flat}} + \mathcal{O}(\tfrac{1}{r})$$

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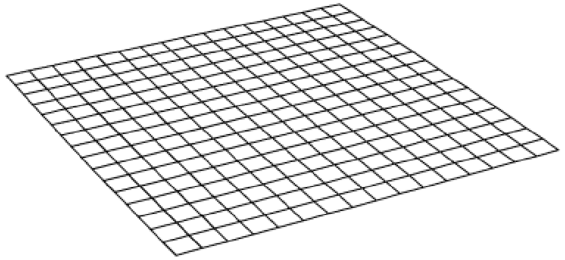

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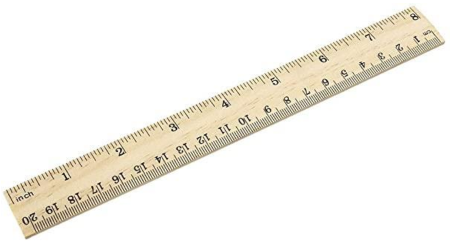

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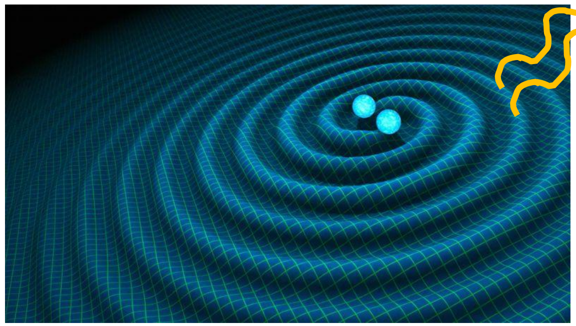

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These terms allow to include a huge set of interesting systems.

<u>Ex</u>: Kerr black holes, **gravitational wave** emission from black hole mergers, …

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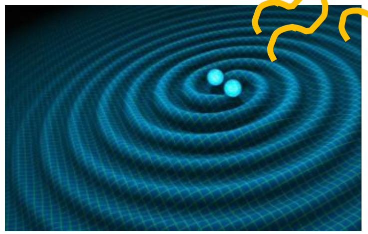

Back in the 1960s: debate about the **existence of gravitational waves**!

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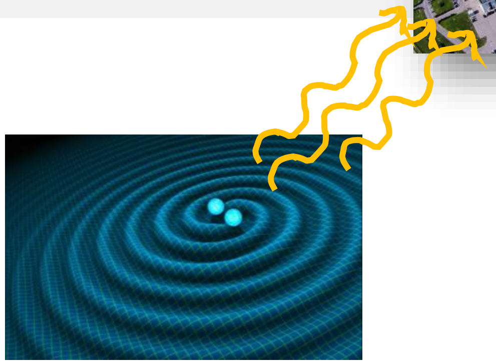

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4 Poincaré translations → ∞ number of **BMS ``supertranslations''**

$$u \to u + \mathcal{T}(\theta, \phi)$$

'retarded' time

$$u = t - r$$

arbitrary function

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$$\mathcal{T}(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} T_{\ell m} Y_{\ell m}(\theta, \phi).$$

$$\underbrace{\ell = 0, 1}_{\text{4 ordinary translations}} \quad \subset \quad \underbrace{\ell = 0, 1, 2, 3, \ldots}_{\text{infinitely many supertranslations}}$$

BMS group

= symmetry group of 4D asymptotically flat spacetimes

$$\mathrm{BMS}_4 \simeq \mathrm{SO}(3,1) \ltimes \mathcal{E}[1](S^2)$$

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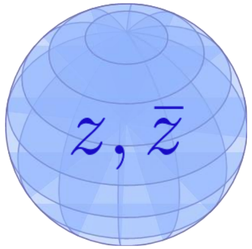
Lorentz acts on the celestial sphere



$$S^2 \cong \mathbb{C} \cup \{\infty\}$$

as $z \to z' = \dfrac{az+b}{cz+d}$ $\quad SL(2,\mathbb{C})$

$$ad - bc = 1$$

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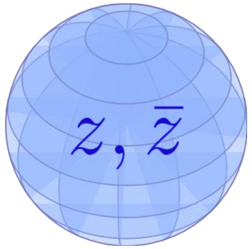

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= weight-1 conformal densities

In CFT notation, it means

$$(h,\bar{h}) = \left(-\tfrac{1}{2}, -\tfrac{1}{2}\right)$$

$$\phi(z,\bar{z}) \longmapsto \left(\frac{\partial z'}{\partial z}\right)^{h} \left(\frac{\partial \bar{z}'}{\partial \bar{z}}\right)^{\bar{h}} \phi(z',\bar{z}')$$

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supertranslations $\mathcal{T}(z,\bar{z})$

- The Poincaré group is included inside BMS:

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Usual translations are recovered from supertranslations via

$$q_\mu \left| \begin{array}{ccc} \mathbb{R}^{3,1} & \hookrightarrow & \mathcal{E}[1] \\ T^\mu & \mapsto & \mathcal{T}(z,\bar{z}) = T^\mu q_\mu(z,\bar{z}) \end{array} \right.$$

$$q^\mu(z,\bar{z}) = (1+|z|^2, z+\bar{z}, -i(z-\bar{z}), 1-|z|^2)$$

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BMS symmetries were originally disregarded…

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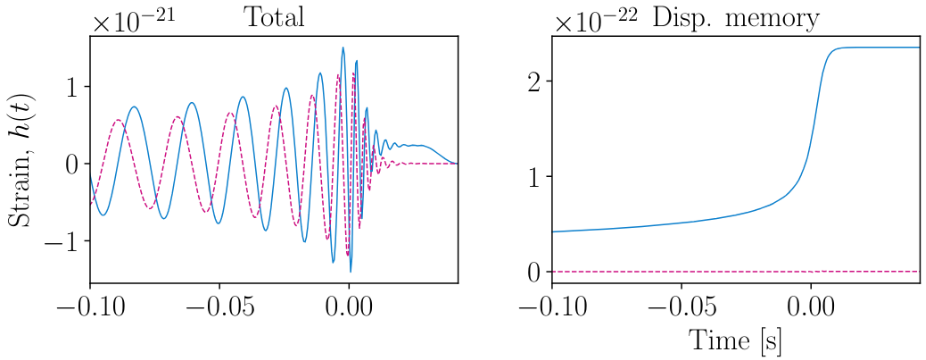
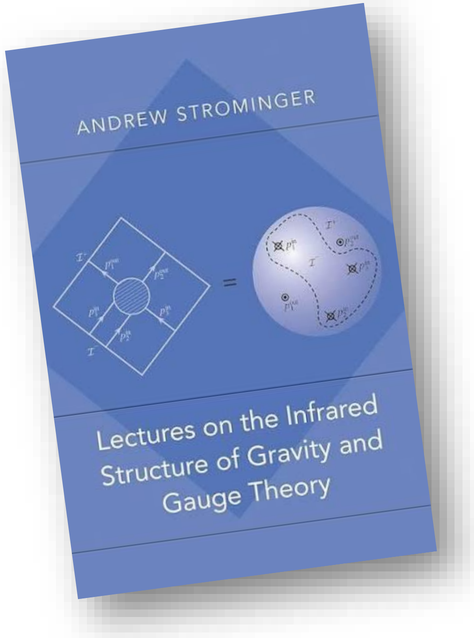
…but 50 years later physicists realized that they

- have associated low-energy **observables** (gravitational wave memory effects)

[Strominger–Zhiboedov '14]



[Goncharov, LD, Harms '23]



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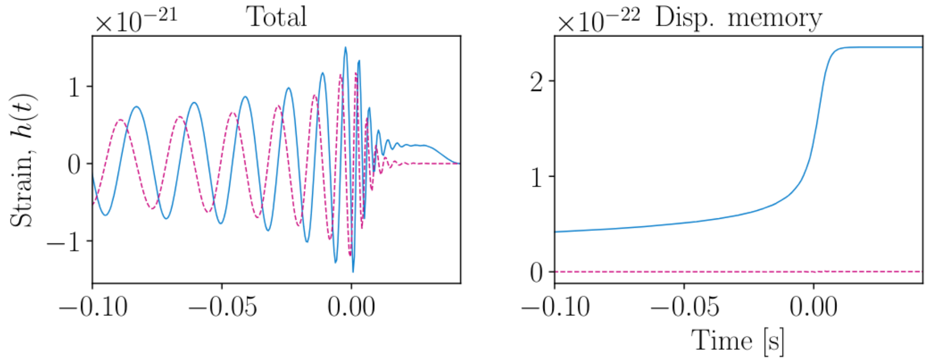

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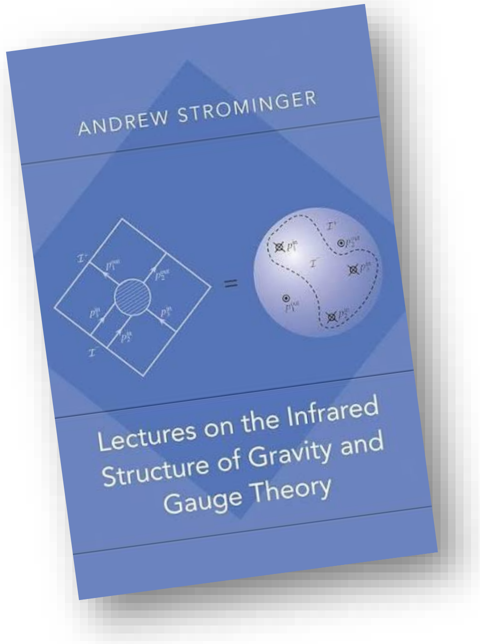
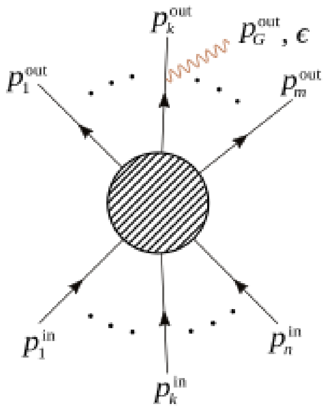


- are related to universal results in **Quantum Field Theory**



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Poincaré representations

Induced representations : a systematic way to construct all one-particle Poincaré UIRs from

1) Lorentz **orbit** $\longrightarrow$ **mass**
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Lorentz orbit (mass-shell)

Choose a reference momentum k_μ.
Its Lorentz orbit is

$$\mathcal{O}_k = \{\Lambda k \mid \Lambda \in SO(3,1)\} \simeq \frac{SO(3,1)}{\ell_k}$$

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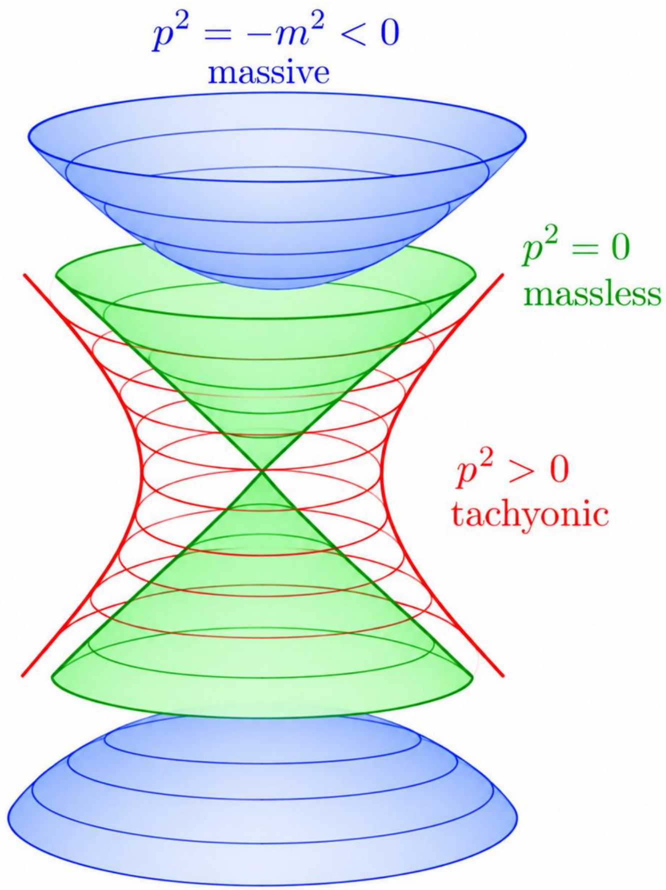
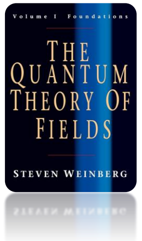
Wigner's classification

little group	ℓ_k	$SU(2)$	$ISO(2)$	$SL(2,\mathbb{R})$
mass-shell	$\mathcal{O}_k$	H^3	$\mathbb{R}\times S^2$	dS_3

e.g $k_\mu = (m,0,0,0) \longrightarrow$ preserved by $\ell_k = SU(2)$





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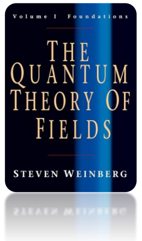
$$\left(a^\mu \,,\, \Lambda^\mu{}_\nu \right) \in \mathbb{R}^{3,1} \rtimes SO(3,1)$$

BMS

$$\left(\mathcal{T}(z,\bar{z}) \,,\, \Lambda^\mu{}_\nu \right) \in \quad \mathcal{E}[1](S^2) \rtimes SO(3,1)$$



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1. Define momenta as dual to translations

$$a^\mu \leftrightarrow p_\mu \qquad \langle p, a\rangle := p_\mu a^\mu$$

BMS

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Define **supermomenta** as dual to supertranslations

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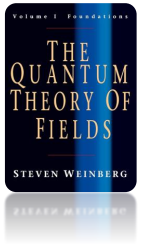

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1. Define momenta as dual to translations

$$a^\mu \leftrightarrow p_\mu \qquad \langle p, a \rangle := p_\mu a^\mu$$

2. Classify the **orbits** of the Lorentz group $\mathcal{O}_k$

In practice, fix a reference momentum k_μ
and find its stabilizer.

$$\ell_k = \{ \Lambda \in SO(3,1) \mid \Lambda k = k \}$$

ℓ_k	$SU(2)$	$ISO(2)$	$SL(2,\mathbb{R})$
$\mathcal{O}_k$	H^3	$\mathbb{R} \times S^2$	dS_3

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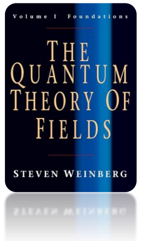

Poincaré

$$\left(a^\mu\,,\,\Lambda^\mu{}_\nu\right) \in \mathbb{R}^{3,1} \rtimes SO(3,1)$$

1. Define momenta as dual to translations

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2. Classify the **orbits** of the Lorentz group $\mathcal{O}_k$

In practice, fix a reference momentum k_μ
and find its stabilizer.

$$\ell_k = \{\Lambda \in SO(3,1) \mid \Lambda k = k\}$$

ℓ_k	$SU(2)$	$ISO(2)$	$SL(2,\mathbb{R})$
$\mathcal{O}_k$	H^3	$\mathbb{R}\times S^2$	dS_3

BMS

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Classify the **orbits** of the Lorentz group $\mathcal{O}_\mathcal{K}$

In practice, fix a reference supermomentum $\mathcal{K}(z,\bar z)$
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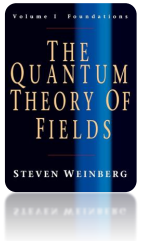
Task: find them all



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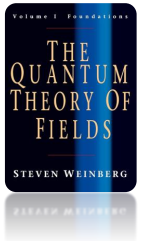
[McCarthy '70s] : there are **11** possible **little groups**

$\ell_{\mathcal{K}}$ with mass-shell of dimensions

$$Dim\left(\mathcal{O}_{\mathcal{K}}\right) \in \{0, 3, 4, 5, 6\}$$



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3. Poincaré particle
= wavefunction with support on $\mathcal{O}_k$

+ spin (UIR of little group)

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BMS particle
= wavefunction with support on $\mathcal{O}_\mathcal{K}$

+ BMS spin (UIR of BMS little group)

	Poincaré little group ℓ_p	**Orbit** (mass shell) $\frac{SL(2,\mathbb{C})}{\ell_p}$	**BMS little group** $\ell_{\mathcal{P}}$	**Extra d.o.f.** (Fibre of BMS shell) $\frac{\ell_p}{\ell_{\mathcal{P}}}$	**BMS shell dim.**
Massless momentum	$ISO(2)$	$\mathbb{R}^+ \times S^2$	$ISO(2) = \ell_p$	$\{e\}$ = trivial	3
			$\mathbb{R}^2$	S^1	4
			$\mathbb{R}$	$S^1 \times \mathbb{R}$	5
			$U(1)$	$\mathbb{R}^2$	5
			$\{e\}$ = trivial	$ISO(2) = S^1 \times \mathbb{R}^2$	6
Massive momentum	$SU(2)$	$\mathbb{H}^3$	$SU(2) = \ell_p$	$\{e\}$ = trivial	3
			$U(1)$	S^2	5
			$\{e\}$ = trivial	$SU(2) = S^3$	6

[LD, Herfray '26]

Questions we would like to answer:

What does a generic BMS particle look like?

- How does it differ from an ordinary Poincaré particle?
- What are the additional infrared degrees of freedom?

Link with infrared physics?

[LD, Herfray ‘26] [Bekaert, Herfray ‘25] [Bekaert, LD, Herfray ‘24]

hard BMS UIR

Hard BMS UIRs

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$$\Phi(X) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[\hat{b}(\vec{p}) e^{ip\cdot X} + \hat{b}(\vec{p})^\dagger e^{-ip\cdot X}\right] \qquad \left[\hat{b}(\vec{p}), \hat{b}(\vec{p}\,')^\dagger\right] = (2\pi)^3\, 2p^0\, \delta^{(3)}(\vec{p} - \vec{p}\,')$$

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Poincaré generators:

$$\hat{P}^\mu = \int \frac{d^3p}{(2\pi)^3\, 2p^0}\, p^\mu \hat{b}^\dagger(\vec{p})\hat{b}(\vec{p})$$

$$\hat{J}^{\mu\nu} = i\int \frac{d^3p}{(2\pi)^3\, 2p^0}\left(p^\mu \frac{\partial}{\partial p_\nu} - p^\nu \frac{\partial}{\partial p_\mu}\right)\hat{b}^\dagger(\vec{p})\hat{b}(\vec{p})$$

Poincaré algebra:

$$\begin{aligned}
\left[\hat{P}^\mu, \hat{P}^\nu\right] &= 0\\
\left[\hat{J}^{\mu\nu}, \hat{P}^\rho\right] &= i\eta^{\nu\rho}\hat{P}^\mu - i\eta^{\mu\rho}\hat{P}^\nu\\
\left[\hat{J}^{\mu\nu}, \hat{J}^{\rho\sigma}\right] &= i\left(\eta^{\sigma\mu}\hat{J}^{\nu\rho} - \eta^{\rho\mu}\hat{J}^{\nu\sigma} + \eta^{\rho\nu}\hat{J}^{\mu\sigma} - \eta^{\sigma\nu}\hat{J}^{\mu\rho}\right)
\end{aligned}$$

Hard BMS UIRs

Bekaert – LD – Herfray

Each momentum can be associated to a hard supermomentum via

$$p^\mu = \int d^2z\, P(z,\bar z) q^\mu(z,\bar z) \qquad q^\mu(z,\bar z) = (1+|z|^2, z+\bar z, -i(z-\bar z), 1-|z|^2)$$

$$P(z,\bar z) = \begin{cases} \omega\,\delta^{(2)}(z-\zeta) & \text{if } p^2 = 0,\ p_\mu = \omega q_\mu(\zeta,\bar\zeta), \\ -\dfrac{m^4}{2\pi\,(q(z,\bar z)\cdot p)^3} & \text{if } p^2 < 0\,. \end{cases}$$

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closes (with Lorentz) the algebra:

$$\left[\hat P(z,\bar z), \hat P(w,\bar w)\right] = 0$$

$$\left[\hat J^{\mu\nu}, \hat P(z,\bar z)\right] = i\left(\mathcal Y^z_{\mu\nu}\partial_z + \mathcal Y^{\bar z}_{\mu\nu}\partial_{\bar z} + \frac{3}{2}\left(\partial_z \mathcal Y^z_{\mu\nu} + \partial_{\bar z}\mathcal Y^{\bar z}_{\mu\nu}\right)\right)\hat P(z,\bar z)$$

$$\left[\hat J^{\mu\nu}, \hat J^{\rho\sigma}\right] = i\left(\eta^{\sigma\mu}\hat J^{\nu\rho} - \eta^{\rho\mu}\hat J^{\nu\sigma} + \eta^{\rho\nu}\hat J^{\mu\sigma} - \eta^{\sigma\nu}\hat J^{\mu\rho}\right)$$

BMS algebra

summary:

each familiar Poincaré particle can be lifted to a hard BMS UIR of supermomentum

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However:

hard particles are **not enough**

Supermomentum conservation

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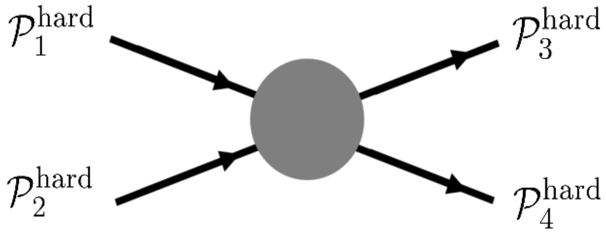

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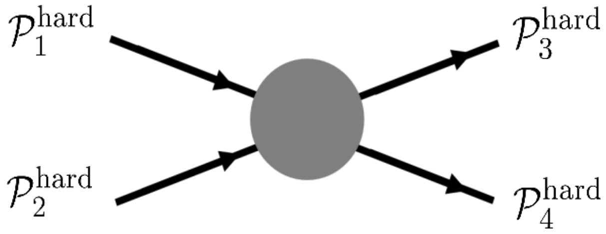

$$\boxed{\mathcal{S}(z,\bar{z}) = \frac{1}{2\pi}\sum_i (p_i \cdot q(z,\bar{z})) \ln |p_i \cdot q(z,\bar{z})|}$$

obstruction to supermomentum conservation

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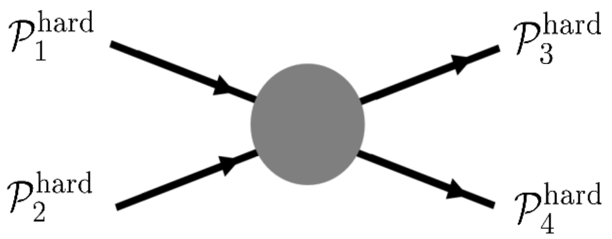
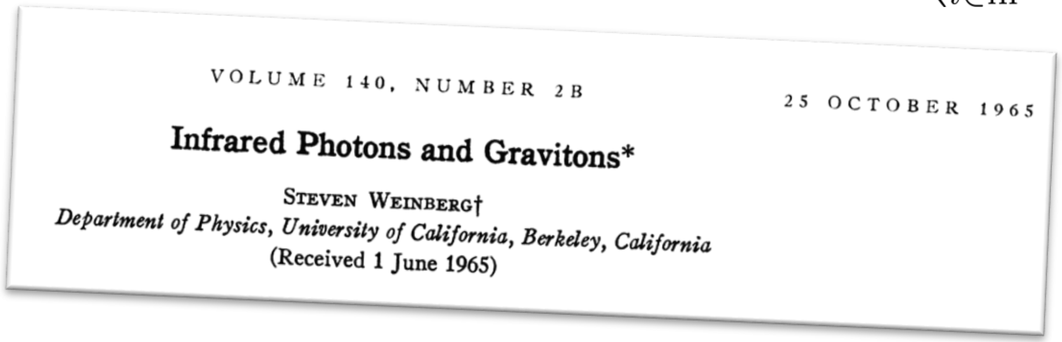


- This is the content of Weinberg's soft graviton theorem ! [Weinberg '65]

$$\lim_{\omega \to 0} \omega \, \langle \text{out} | \hat a_+^{\text{out}}(\omega, z, \bar z) \hat S | \text{in} \rangle = \kappa \left(\sum_{i \in \text{in}} \frac{(\varepsilon^+ \cdot p_i)^2}{q \cdot p_i} - \sum_{i \in \text{out}} \frac{(\varepsilon^+ \cdot p_i)^2}{q \cdot p_i} \right) \langle \text{out} | \hat S | \text{in} \rangle$$


VOLUME 140, NUMBER 2B 25 OCTOBER 1965

Infrared Photons and Gravitons*

STEVEN WEINBERG†

Department of Physics, University of California, Berkeley, California

(Received 1 June 1965)

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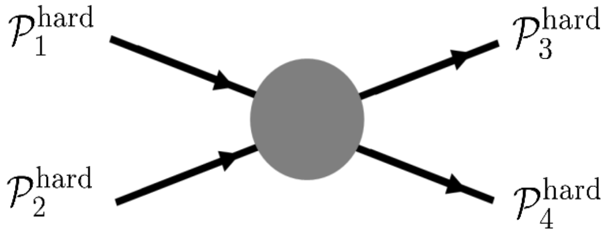

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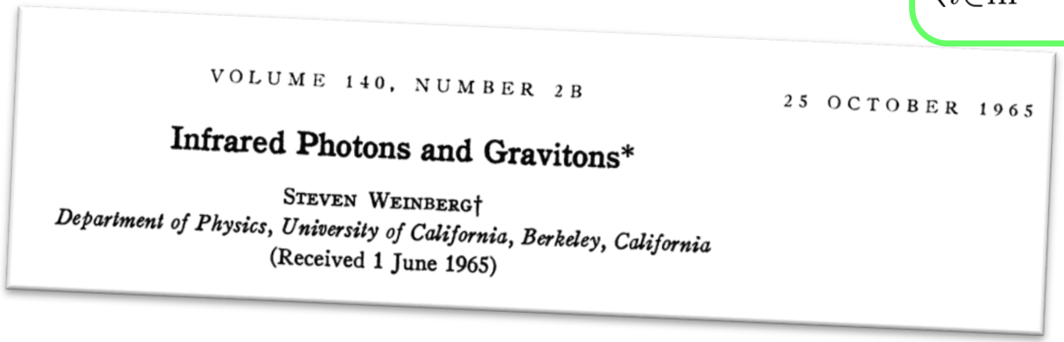
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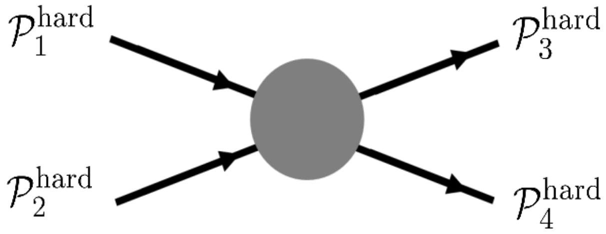

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⟶ we need to include generic UIRs in the Hilbert space

(generic) BMS particles

BMS wavefunctions

[Bekaert, LD, Herfray '24]

- Generic representations will have a generic supermomentum $\mathcal{P}(z, \bar{z})$

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Usual wavefunctions

$$|\Psi\rangle = \int_{\mathcal{O}_p} d\mu(p)\Psi(p)|p\rangle$$

BMS wavefunctions

$$|\Psi\rangle = \int_{\mathcal{O}_\mathcal{P}} d\mu(\mathcal{P})\,\Psi(\mathcal{P})|\mathcal{P}\rangle$$

eigenstate of the supermomentum operator $\hat{\mathcal{P}}|\mathcal{P}\rangle = \mathcal{P}|\mathcal{P}\rangle$

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- Key result

$$\mathcal{P}(z,\bar{z}) = \underbrace{\boxed{\partial_z^2\partial_{\bar{z}}^2\mathcal{N}}}_{\text{Soft part}} + \underbrace{\boxed{P(z,\bar{z})}}_{\text{Hard part}}$$

extra IR degrees of freedom

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extra IR degrees of freedom

This decomposition is unique and Lorentz invariant.

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$$\mathcal{P}(z,\bar{z}) = \partial_z^2\partial_{\bar{z}}^2\mathcal{N} \;+ P(z,\bar{z}) \quad \rightarrow$$

BMS state:

$$|\mathcal{P}\rangle = |p, \partial^2\mathcal{N}\rangle$$

extra label

In QED

[LD, Herfray '26]

The same representation-theoretic structure appears in **QED**:

$$\mathrm{SO}(3,1) \ltimes \left(\mathbb{R}^{3,1} \times \mathcal{C}^{\infty}(S^2)\right)$$

$$\mathcal{T} := \left(T^{\mu}, \varepsilon(z,\bar{z})\right)$$

infinite-dimensional U(1) transformations

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- **QED supermomentum**

$$\mathcal{P}(z,\bar{z}) = \left(p_{\mu}, \mathcal{Q}(z,\bar{z})\right)$$

$$\in (\mathbb{R}^{3,1})^* \times \mathcal{E}[-2]$$

pairing: $\langle \mathcal{P}, \mathcal{T} \rangle := p_{\mu} T^{\mu} + \int d^2 z\, \mathcal{Q}(z,\bar{z}) \varepsilon(z,\bar{z})$

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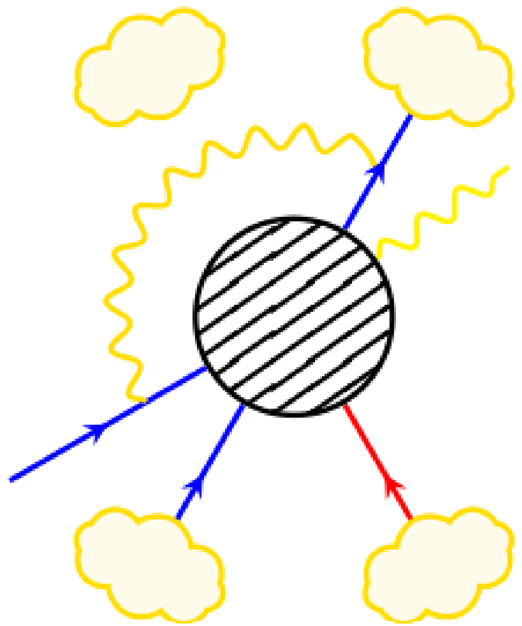
- Lorentz-invariant decomposition

$$\mathcal{P}(z,\bar{z}) = \underbrace{\boxed{\left(p_\mu, Q(z,\bar{z})\right)}}_{\text{hard part}} + \underbrace{\boxed{\left(0_\mu, \partial_z\partial_{\bar{z}}\mathcal{N}(z,\bar{z})\right)}}_{\text{soft part}} \qquad \mathcal{N}(z,\bar{z}) \in \mathcal{E}[0]$$

Infrared physics from representation theory

A new light on infrared divergences (in QED and gravity)



A new light on infrared divergences (in QED and gravity)

- **IR divergences = failure** of **conventional** states to conserve supermomentum

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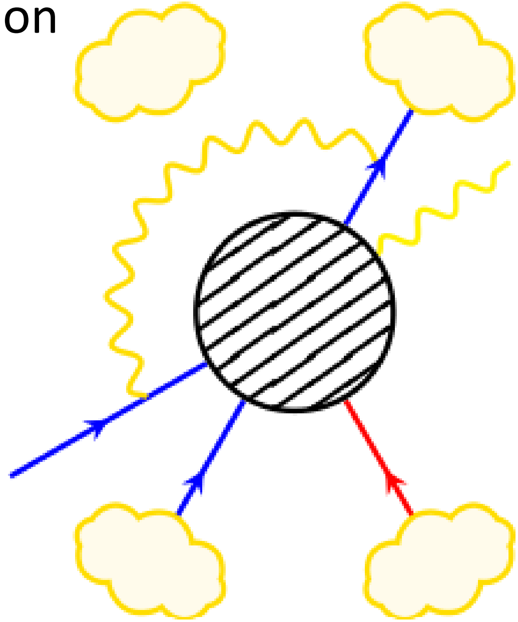

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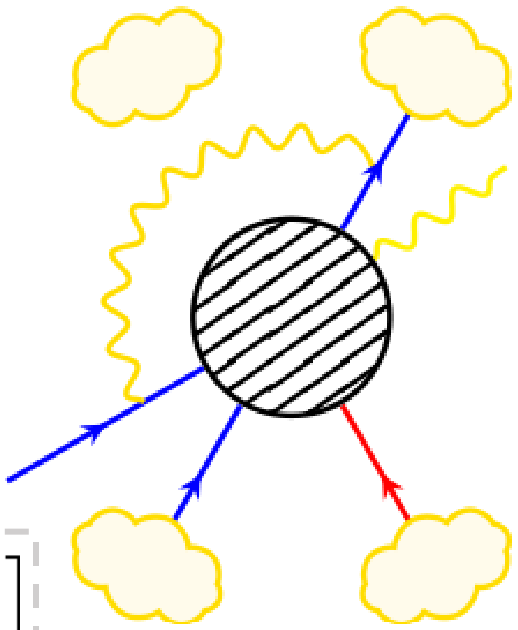

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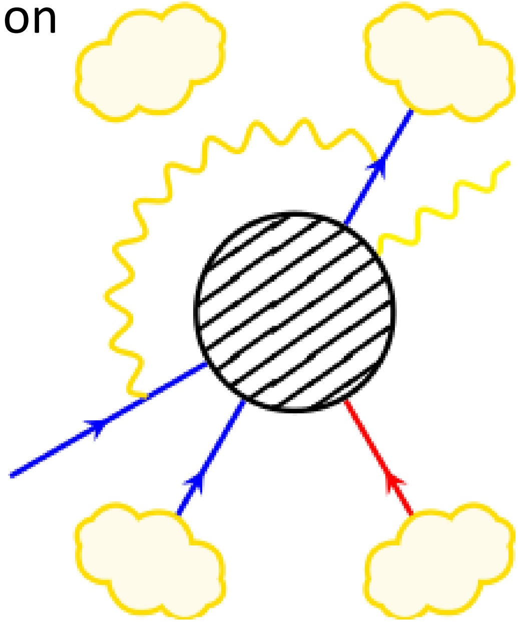

- **Dressed states** (``coherent states'', ``attaching soft clouds'', etc.)
  [Chung '65][Kibble '68] [Dollard '71][Kulish, Fadeev '70]

dressed state construction $\approx$ **shoehorn BMS states** into the conventional QFT language.

→ see [LD, Herfray '26] for more detail

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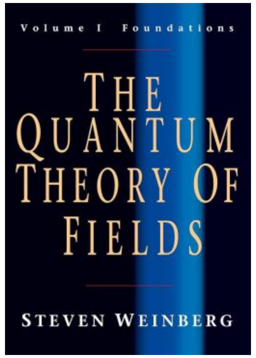
Towards a fully BMS-invariant (infrared safe) extension of QFT?



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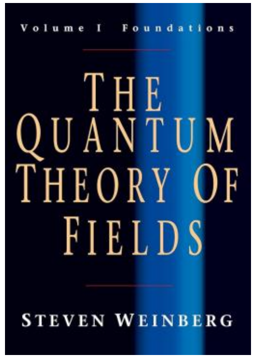

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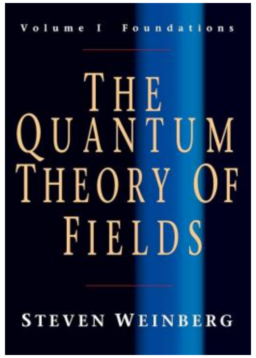

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