# Multi-trace operators in $\mathcal{N}$=4 Super Yang-Mills

Giulia Fardelli

Bootstrap Approaches to Scattering Amplitudes

Based on WIP with Fitzpatrick, Li, Mei, Novikova
and 2508.20158, 2508.20160, 2403.07079 with Fitzpatrick, Li

# Table of Contents

# Table of Contents

# Overview and motivations

| $\mathrm{CFT}_d$ | | $\mathrm{AdS}_{d+1}$ |
|---|---|---|
| Multi-trace(twist) operators $\mathcal{O}\partial_{\mu_1}\cdots\partial_{\mu_i}\mathcal{O}\partial_{\mu_j}\mathcal{O}\cdots\mathcal{O}$ | $\longleftrightarrow$ | Multi-particle states |

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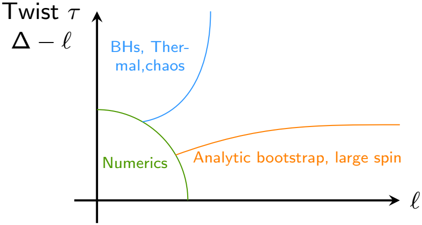
- Study multi-particle dynamics through CFT correlation functions
- They control certain kinematic limits of *higher-point* correlators
- Chart the spectrum of a CFT



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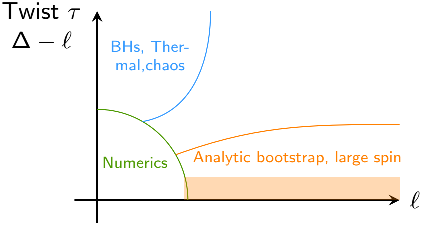
CFT$_d$ AdS$_{d+1}$

Multi-trace(twist) operators $\mathcal{O}\partial_{\mu_1}\cdots\partial_{\mu_i}\mathcal{O}\partial_{\mu_j}\mathcal{O}\cdots\mathcal{O}$ $\longleftrightarrow$ Multi-particle states

- Study multi-particle dynamics through CFT correlation functions [Ma, Zhou '22]
- They control certain kinematic limits of *higher-point* correlators [Antunes,Bercini,Costa,Goncalves,Harris,Kaviraj,Vieira,Vilas Boas,Mann,Quintavalle,Schomerus]
- Chart the spectrum of generic CFTs



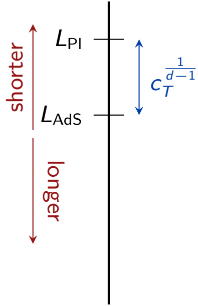

Multi-trace ops belong to the low-E part of the spectrum

In certain regimes (i.e. at large spin $\ell\to\infty$) they become semi-classical [Alday, Maldacena '07; Fitzpatrick, Kaplan, Poland, Simmons-Duffin '12; Komargodski, Zhiboedov '12; Alday '16]

# Large spin EFT in AdS

[Fitzpatrick, Kaplan, Poland, Simmons-Duffin '12; Fitzpatrick, Kaplan, Walters '14; GF, Fitzpatrick, Li '24,'25]

How can we describe CFTs holographically at *large spin*?



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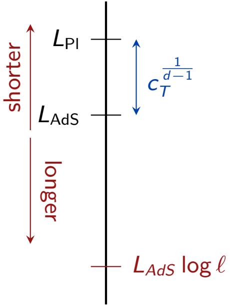
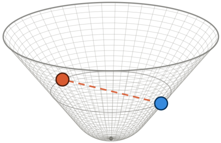



At large spin there is a new *IR scale* where gravity is weakly coupled, so that we can build an EFT



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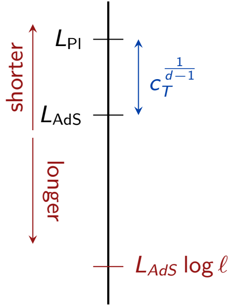


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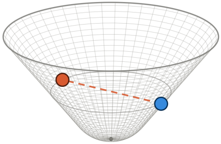
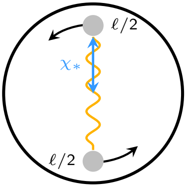


Particles interact *weakly* due to large separation




Rest frame geodesic separation $\chi = \log\left(\frac{2\ell}{\Delta}\right)$

Interactions $\sim e^{-\tau_{\text{ex}}\chi} = \frac{1}{\ell^{\tau_{\text{ex}}}}$

# Large spin EFT in AdS

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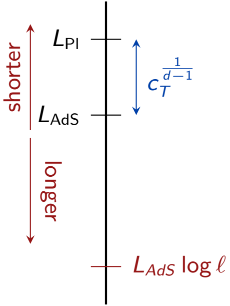



At large spin there is a new *IR scale* where gravity is weakly coupled, so that we can build an **EFT**

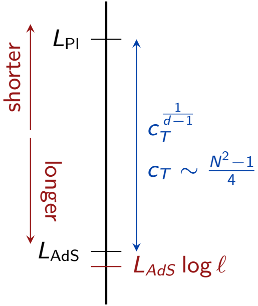
1. Choose a twist cutoff $\Lambda_\tau$
2. *Long-distance effects*: keep long-range potential from exchanges below $\Lambda_\tau$

$$e^{\tau\chi} \sim \frac{1}{\ell^\tau}$$

3. *Short distance effects*: add local counterterms to correct for discrepancy at small $\ell$

# $\mathcal{N} = 4$ Super Yang-Mills

$\mathcal{N} = 4$ SYM at large $N$ is a great playground!




$\mathcal{N} = 4$ SYM with SU($N$) gauge group at **large N** and $\boldsymbol{\lambda} \to \infty$

$\updownarrow$

Type IIB SUGRA on $\text{AdS}_5 \times \text{S}^5$

Single-trace operators $\mathcal{O}_p \sim \text{tr}\,(\phi^p)$ correspond to single-particle states in AdS and composite operators made of $Q$ $\mathcal{O}_p$s to $Q$-particle states

Multi-trace operators can be studied looking at CFT *correlation functions*

- $\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2 \rangle$ at *higher loops* (higher $1/N$ corrections) [Bissi, GF, Georgoudis '20, Drummond, Paul '22]
- 4-pt fns w/ *external* multi-trace operators, e.g. $\mathcal{O}_2^n$ or $\frac{1}{4}$-BPS [Bissi, GF, Manenti '21, '24; Aprile, Giusto, Russo '24,'25 + Vilas Boas '26] [Bissi, GF, Puerta-Ramisa (WIP)]
- *higher-point* functions [Alday, Goncalves, Nocchi, Pereira, Vilas Boas, Zhou]

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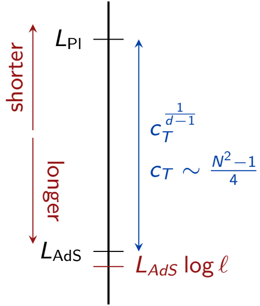


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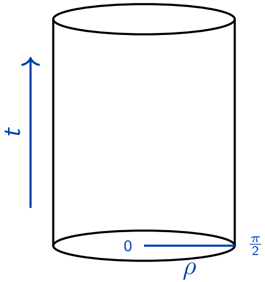
Multi-trace operators can be studied looking at CFT *correlation functions*

*Main challenge* of correlator-based method is the high-degeneracy of operators at fixed spin $\ell$. For ex. $\#$ leading-twist 3-particle primaries $\xrightarrow{\ell\to\infty} \ell/6$

# Table of Contents

We will take an alternative approach to determine the spectrum of multi-particle operators $\Rightarrow$ construct a **Hamiltonian** in AdS




**AdS in global coordinates**

AdS time translation $\leftrightarrow$ CFT dilatation

$$E_{\mathrm{AdS}} \leftrightarrow \Delta_{\mathrm{CFT}}$$

**Free scalar** $m^2 = \Delta_\phi(\Delta_\phi - d)$

$$\Phi(t,\rho,\Omega) = \sum \phi_{n\ell L} a_{n\ell L} + \phi^*_{n\ell L} b^\dagger_{n\ell L}$$

$$\Phi(0)|\mathrm{vac}\rangle \leftrightarrow b^\dagger_{000}|\mathrm{vac}\rangle\,, \qquad \partial_{\mu_1}\cdots\partial_{\mu_\ell}\Phi(0) \leftrightarrow |\ell\rangle \equiv b^\dagger_{0\ell\ell}|\mathrm{vac}\rangle$$

[Fitzpatrick, Shih '11; GF, Fitzpatrick, Li '24,'25]

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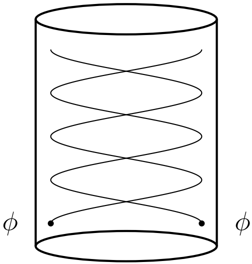


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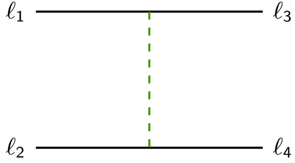
$$\Phi(t, \rho, \Omega) = \sum \phi_{n\ell L} a_{n\ell L} + \phi^*_{n\ell L} b^\dagger_{n\ell L}$$

Double-trace operator $$[\phi, \phi]_{0,\ell} = \sum_{\ell_1+\ell_2=\ell} \alpha_{\ell_1\ell_2} |\ell_1 \ell_2\rangle$$

$$E = \langle [\phi, \phi]_{0,\ell} | H_{\mathrm{GFF}} | [\phi, \phi]_\ell \rangle = 2\Delta_\phi + \ell$$

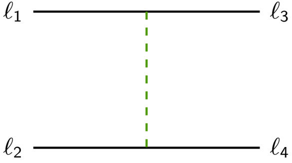
[Fitzpatrick, Shih '11; GF, Fitzpatrick, Li '24,'25]




$$\gamma_\ell = \langle[\phi,\phi]_\ell|H_2|[\phi,\phi]_\ell\rangle$$
$$= \sum \alpha_{\ell_1\ell_2}\alpha_{\ell_3\ell_4}\int d\rho d\Omega\langle\ell_1\ell_2|V_{\text{eff}}|\ell_3\ell_4\rangle$$
$$= \sum \alpha_{\ell_1\ell_2}\alpha_{\ell_3\ell_4} V_{\ell_1\ldots\ell_4} b^\dagger_{\ell_1} b^\dagger_{\ell_2} a_{\ell_3} a_{\ell_4}$$

# AdS Hamiltonian approach II

[Fitzpatrick, Shih '11; GF, Fitzpatrick, Li '24,'25]

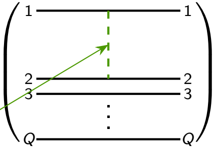



$$
\begin{aligned}
\gamma_\ell &= \langle [\phi,\phi]_\ell | H_2 | [\phi,\phi]_\ell \rangle \\
&= \sum \alpha_{\ell_1\ell_2}\alpha_{\ell_3\ell_4} \int d\rho d\Omega \langle \ell_1\ell_2 | V_{\text{eff}} | \ell_3\ell_4 \rangle \\
&= \sum \alpha_{\ell_1\ell_2}\alpha_{\ell_3\ell_4} V_{\ell_1\ldots\ell_4} b^\dagger_{\ell_1} b^\dagger_{\ell_2} a_{\ell_3} a_{\ell_4}
\end{aligned}
$$

$$\boxed{H = H_{\text{GFF}} + H_2}$$

$Q$-particle state $\qquad \Delta = Q\Delta_\phi + \ell + \gamma_\ell$

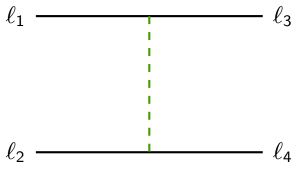
Eig




$= \gamma_\ell$

Only 2-body interactions @ $1/N^2$

[Fitzpatrick, Shih '11; GF, Fitzpatrick, Li '24,'25]




$$\begin{aligned}\gamma_\ell &= \langle [\phi,\phi]_\ell | H_2 | [\phi,\phi]_\ell \rangle \\ &= \sum \alpha_{\ell_1\ell_2}\alpha_{\ell_3\ell_4} \int d\rho d\Omega \langle \ell_1\ell_2 | V_{\rm eff} | \ell_3\ell_4 \rangle \\ &= \sum \alpha_{\ell_1\ell_2}\alpha_{\ell_3\ell_4} V_{\ell_1\ldots\ell_4} b^\dagger_{\ell_1} b^\dagger_{\ell_2} a_{\ell_3} a_{\ell_4}\end{aligned}$$

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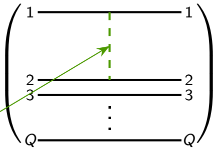
Eig  $= \gamma_\ell$


Only 2-body interactions @ $1/N^2$

To build $H$

- basis of states
- 2-body inter

# Table of Contents

# Basis of states in $\mathcal{N} = 4$ SYM

Supergraviton $\mathcal{O}_2$

$$\Delta = 2 \quad [0, 2, 0]_R$$

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Supergraviton $\mathcal{O}_2 \xrightarrow{Q} \ldots \xrightarrow{Q} \ldots \xrightarrow{Q} \ldots \xrightarrow{Q} \mathcal{L}_2$

$\Delta = p \quad [0, p, 0]_R$

KK modes $\mathcal{O}_p \xrightarrow{Q} \ldots \xrightarrow{Q} \ldots \xrightarrow{Q} \ldots \xrightarrow{Q} \mathcal{L}_p$

$\Delta = p + 2$
$[0, p-2, 0]_R$
$\mathrm{U}(1)_Y$ charge $+4$

[Intriligator '98; Intriligator, Skiba '99]

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We want to build multi-particle states out of $\mathcal{L}$s, in the **singlet** of $\mathrm{SU}(4)_R$

$$O = [\mathcal{L}_p \cdots \mathcal{L}_p]_{n,\ell} = \sum \partial_{\mu_1} \cdots \partial_{\mu_{k_1}} \mathcal{L}_p \Box^i \mathcal{L}_p \cdots \Box^j \partial_{\mu_{k_2}} \cdots \partial_{\mu_\ell} \mathcal{L}_p$$

- We only need to impose that $O$ is a primary $[K^\mu, O] = 0$
- Bonus $\mathrm{U}(1)_Y$ symmetry $\Longleftrightarrow$ No mixing between states with different number of particles
- In this basis, "hidden $10d$ symmetry" is manifest

# Extracting 2-body interactions from correlators

Hamiltonian matrix elements can be obtained by the SUGRA action, but much more convenient to start from **correlators**

$$\langle \mathcal{L}_p \mathcal{L}_p \bar{\mathcal{L}}_q \bar{\mathcal{L}}_q \rangle \quad = \quad \Delta^{(8)} \quad \langle \mathcal{O}_p \mathcal{O}_p \mathcal{O}_q \mathcal{O}_q \rangle$$

[Drummond, Gallot, Sokatchev '06; Goncalves '14; Korchemsky, Sokatchev '15; Aprile, Drummond, Heslop, Paul '18]

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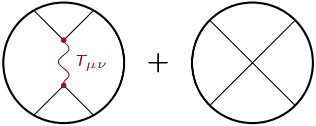
Mellin tfm $\downarrow$    $\downarrow$ difference operator    $\downarrow$ Reduced Mellin ampl [Rastelli, Zhou '17]

$$\mathcal{M}^{\mathcal{L}}_{ppqq}(s,t) \quad = \quad \mathbb{D}(s,t) \qquad \widetilde{\mathcal{M}}_{ppqq} = \sum \frac{a_{ijk}\sigma^i \tau^j}{(s-s_k)(t-t_j)(\tilde{u}-\tilde{u}_i)}$$

$\Delta^{(8)} \sim Q^8 \bar{Q}^8$ kills all protected contributions and isolates the interactions producing the anomalous dimensions

$$\mathcal{M}^{\mathcal{L}}_{2222} = \frac{24}{c}\left(\frac{2\left(3s^2 - 42s + 148\right)}{t-2} + \frac{8(s-6)^2}{t-4} + \frac{(s-4)(s-6)}{t-6} + t \leftrightarrow u\right)$$

$=$ 

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1. Use $\mathcal{M}^{\mathcal{L}}$ to classify interactions and extract the couplings
2. Transform back to position space and use

Correlator $$\left\langle \mathcal{L}_p(x_1)\mathcal{L}_p(x_2)\bar{\mathcal{L}}_q(x_3)\bar{\mathcal{L}}_q(x_4) \int dt d\rho d\Omega \frac{\sin^{d-1}\rho}{\cos^{d+1}\rho} V_{\rm int}(x) \right\rangle$$

$H$ − matrix $$\langle [\mathcal{L}_p, \mathcal{L}_p] | \int d\rho d\Omega \frac{\sin^{d-1}\rho}{\cos^{d+1}\rho} V_{\rm int}(\rho,\Omega) | [\bar{\mathcal{L}}_q, \bar{\mathcal{L}}_q] \rangle$$

# Table of Contents

# Double-trace operators

[Alday, Bissi '17; Aprile, Drummond, Heslop, Paul '17, '18; Caron-Huot, Trinh '18]

- **Leading twist** $\tau = 8$: $[\mathcal{L}_2\mathcal{L}_2]_{0,\ell}$

$$\boxed{\tau_{[\mathcal{L}_p\mathcal{L}_p]_{n,\ell}} = 2(p+2) + 2n}$$

$$\gamma_{0,\ell} = -\frac{24}{c(\ell+1)(\ell+6)}$$

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$$\begin{array}{ccc} & [\mathcal{L}_2\mathcal{L}_2]_{1,\ell} & [\mathcal{L}_3\mathcal{L}_3]_{0,\ell} \\ \begin{array}{c}[\bar{\mathcal{L}}_2\bar{\mathcal{L}}_2]_{1,\ell}\\[3em] [\bar{\mathcal{L}}_3\bar{\mathcal{L}}_3]_{0,\ell}\end{array} & \multicolumn{2}{c}{\begin{pmatrix} -\frac{120}{c(\ell+1)(\ell+8)} & -\frac{720}{c(\ell+1)\sqrt{(\ell+2)(\ell+7)}(\ell+8)} \\[2em] -\frac{720}{c(\ell+1)\sqrt{(\ell+2)(\ell+7)}(\ell+8)} & -\frac{120\left(\ell^2+9\ell+44\right)}{c(\ell+1)(\ell+2)(\ell+7)(\ell+8)} \end{pmatrix}} \end{array}$$

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- **SubSubLeading twist** $\tau = 12$:

$$\begin{array}{c c}
 & \begin{array}{ccc} \quad [\mathcal{L}_2\mathcal{L}_2]_{2,\ell} & \qquad\qquad [\mathcal{L}_3\mathcal{L}_3]_{1,\ell} & \qquad\qquad\qquad [\mathcal{L}_4\mathcal{L}_4]_{0,\ell} \end{array} \\
\begin{array}{c} [\bar{\mathcal{L}}_2\bar{\mathcal{L}}_2]_{2,\ell} \\[4ex] [\bar{\mathcal{L}}_3\bar{\mathcal{L}}_3]_{1,\ell} \\[4ex] [\bar{\mathcal{L}}_4\bar{\mathcal{L}}_4]_{0,\ell} \end{array} &
\begin{pmatrix}
-\frac{360}{c(\ell+1)(\ell+10)} & -\frac{720\sqrt{21}}{c(\ell+1)\sqrt{\ell+3}\sqrt{\ell+8}(\ell+10)} & -\frac{2880\sqrt{35}}{c(\ell+1)\sqrt{\ell+2}\sqrt{\ell+3}\sqrt{\ell+8}\sqrt{\ell+9}(\ell+10)} \\
* & -\frac{360\left(\ell^2+11\ell+94\right)}{c(\ell+1)(\ell+3)(\ell+8)(\ell+10)} & -\frac{720\sqrt{15}\left(\ell^2+11\ell+66\right)}{c(\ell+1)\sqrt{\ell+2}(\ell+3)(\ell+8)\sqrt{\ell+9}(\ell+10)} \\
* & * & -\frac{360\left(\ell^4+22\ell^3+219\ell^2+1078\ell+3120\right)}{c(\ell+1)(\ell+2)(\ell+3)(\ell+8)(\ell+9)(\ell+10)}
\end{pmatrix}
\end{array}$$

# Double-trace operators

[Alday, Bissi '17; Aprile, Drummond, Heslop, Paul '17, '18; Caron-Huot, Trinh '18]

$$\boxed{\tau_{[\mathcal{L}_p\mathcal{L}_p]_{n,\ell}} = 2(p+2)+2n}$$

- **Leading twist** $\tau = 8$: $[\mathcal{L}_2\mathcal{L}_2]_{0,\ell}$

$$\gamma_{0,\ell} = -\frac{24}{c(\ell+1)(\ell+6)}$$

- **SubLeading twist** $\tau = 10$:

$$\gamma_{1,\ell} = \left\{-\frac{120}{c(\ell+1)(\ell+2)}, -\frac{120}{c(\ell+7)(\ell+8)}\right\}$$

- **SubSubLeading twist** $\tau = 12$:

$$\gamma_{2,\ell} = \left\{-\frac{360(\ell+7)}{c(\ell+1)(\ell+2)(\ell+3)}, -\frac{360}{c(\ell+3)(\ell+8)}, -\frac{360(\ell+4)}{c(\ell+8)(\ell+9)(\ell+10)}\right\}$$

# $10d$ Hidden symmetry

[Caron-Huot, Trinh '18; Aprile, Drummond, Paul, Santagata '20; Abl,Heslop, Lipstein '20]

$[\mathcal{L}_p\mathcal{L}_p]_{n-p+2,\ell}$ degenerate under SO(4,2)$\times$ SO(6) at $N \to \infty$

$$\mathrm{SO}(10,2) \to \mathrm{SO}(4,2) \times \mathrm{SO}(6)$$

$$X^{\mathfrak{a}} \quad = ( \quad P^A \quad , \quad y^m \; )$$

One $10d$ free massless scalar $\Delta = 4$

$$\Phi(X) = \sum y_{m_1} \cdots y_{m_{p-2}} \mathcal{L}_p^{m_1 \cdots m_{p-2}}(P)$$

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$$[\Phi\Phi]_L\sim\Phi\partial_{\mathfrak{a}_1}\cdots\partial_{\mathfrak{a}_L}\Phi\,,\qquad \Delta^{(10d)}=L+8 \text{ and } \ell\leq L\leq 2n+\ell\,,$$
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When restricted to SO(4,2)$\times$SO(6), acting on $[\mathcal{L}_p\mathcal{L}_p]_{n-p+2,\ell}$

$$\begin{array}{c}
\begin{array}{ccccc} [\mathcal{L}_2\mathcal{L}_2]_n & [\mathcal{L}_3\mathcal{L}_3]_{n-1} & & & [\mathcal{L}_{n+2}\mathcal{L}_{n+2}]_0 \end{array} \\
\mathcal{C}^{(10,2)} = \begin{pmatrix} c_{2,2} & c_{2,3} & 0 & \cdots & 0 \\ c_{2,3} & c_{3,3} & c_{3,4} & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & c_{n+1,n+2} & c_{n+2,n+2} \end{pmatrix}
\end{array}$$

$$\boxed{\mathrm{Eig}(\mathcal{C}^{(10,2)}) = \mathrm{Eig}(H_2)}$$

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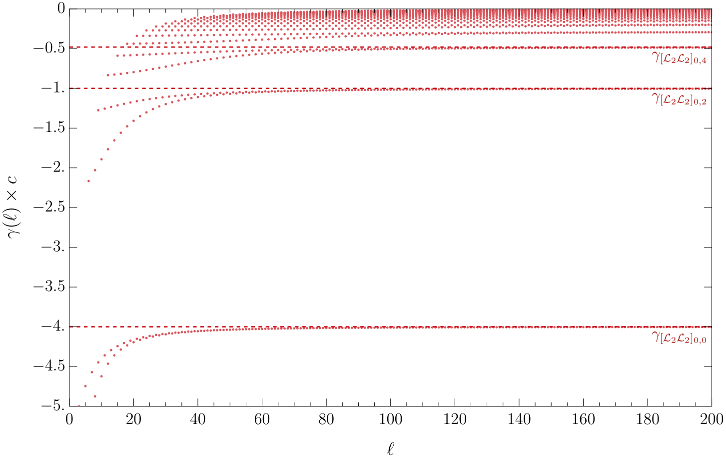
$$R_\ell: \quad [\mathcal{L}_3\mathcal{L}_3]_{0,\ell} - \sqrt{\frac{\ell+2}{\ell+7}}[\mathcal{L}_2\mathcal{L}_2]_{1,\ell}$$

$$R_{\ell+2}: \quad [\mathcal{L}_3\mathcal{L}_3]_{0,\ell} + \sqrt{\frac{\ell+7}{\ell+2}}[\mathcal{L}_2\mathcal{L}_2]_{1,\ell}$$

# Leading triple-trace operators $[\mathcal{L}_2\mathcal{L}_2\mathcal{L}_2]_{0,\ell}$



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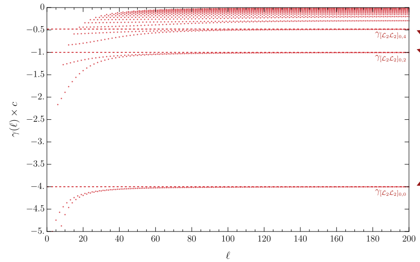
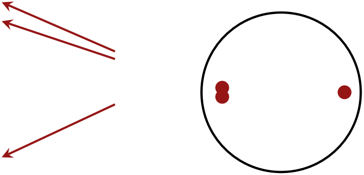


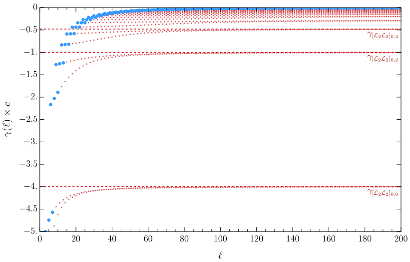



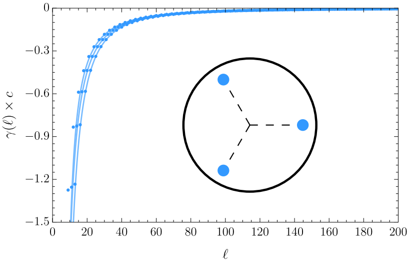
$[\mathcal{L}_2[\mathcal{L}_2\mathcal{L}_2]_k]_{\ell-k}$

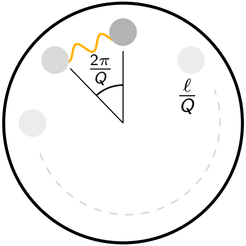
$$\gamma_\ell^{(Q=3)} = \gamma_{[\mathcal{L}_2\mathcal{L}_2]_{0,k}} + \frac{\#}{\ell^2}$$

$$\gamma_\ell^{(Q)} = \min_{\ell^*}\left(\gamma_{\ell^*}^{(Q-1)}\right) + \frac{\#}{\ell^{\tau_{\rm ex}}}$$

# Leading triple-trace operators $[\mathcal{L}_2\mathcal{L}_2\mathcal{L}_2]_{0,\ell}$




$$\gamma^{(Q)}(\ell) \sim \frac{Q}{2}\sum_{m=1}^{Q-1} \gamma^{(Q=2)}\left(\frac{2\ell}{Q}\left|\sin\left(\frac{\pi m}{Q}\right)\right|\right)$$

$$\gamma^{(Q=3)}(\ell) \xrightarrow{\ell\to\infty} -\frac{216}{\ell^2}$$

[See also Kravchuk, Mann '24 for an alternative interpretation]

# Table of Contents

# $\mathcal{N} = 4$ is not analytically solvable at $N^{-2}$

WIP w/ Fitzpatrick, Novikova

At fixed spin $\ell$

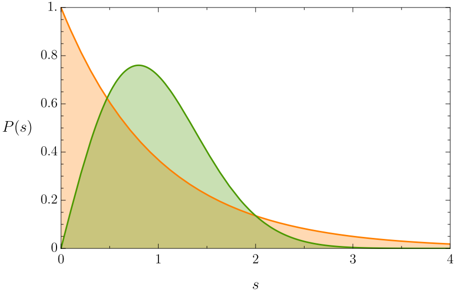
$$s_n = E_{n+1} - E_n\,, \quad \tilde{s}_n = \frac{s_n}{\langle s_n \rangle}\,, \qquad n = 1, \ldots, N_\ell - 1$$

Integrable: levels are **uncorrelated** and spacing distribution

$$P(s) = e^{-s} \qquad \text{Poisson}$$

Chaotic: levels **repel** like eigenvalues of a random matrix

$$P(s) = \frac{1}{2}\pi e^{-\frac{\pi s^2}{4}} s \qquad \text{GOE}$$



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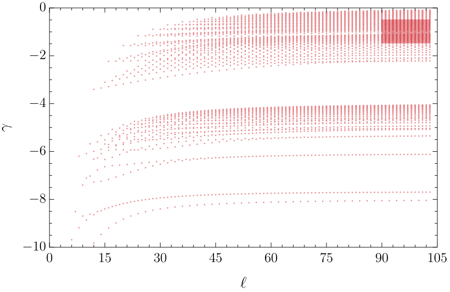
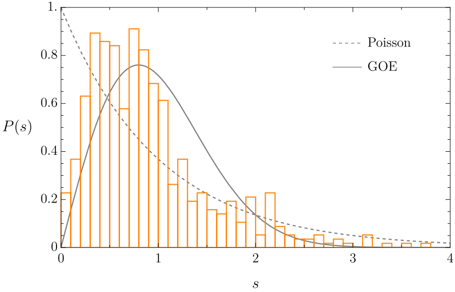
**Q = 4**



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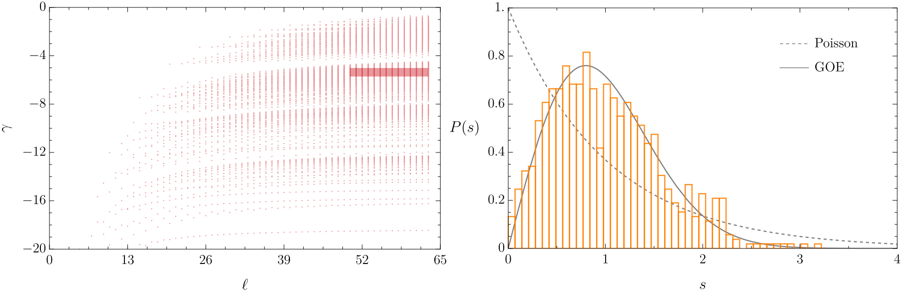

# Table of Contents

# Summary and future directions

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- Uncharged triple-trace operators $[\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2]_\ell$?
- Extend the method to OPE coefficients
- Combine with correlator approach
- Sharpen the chaos diagnostics: larger sectors and more spins, eigenvector statistics

Thank you!