Overview of Research Area 3: Computer Algebra

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RA3 Plenary Meeting
Siegen
14th of July 2026

color meets flavor

UNIVERSITÄT BONN

Universität Siegen / TP1 Theoretical Particle Physics / CPPS Center for Particle Physics Siegen

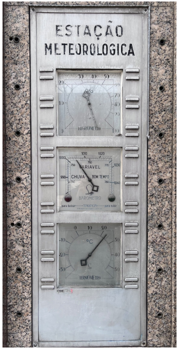
technische universität dortmund

JÜLICH Forschungszentrum

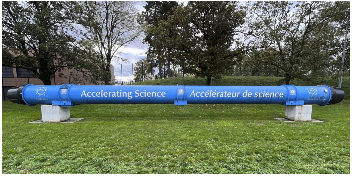
- Goal: Development and implementation of novel algorithms for Feynman diagram calculus
- Methods: Modern techniques from mathematics and computer algebra

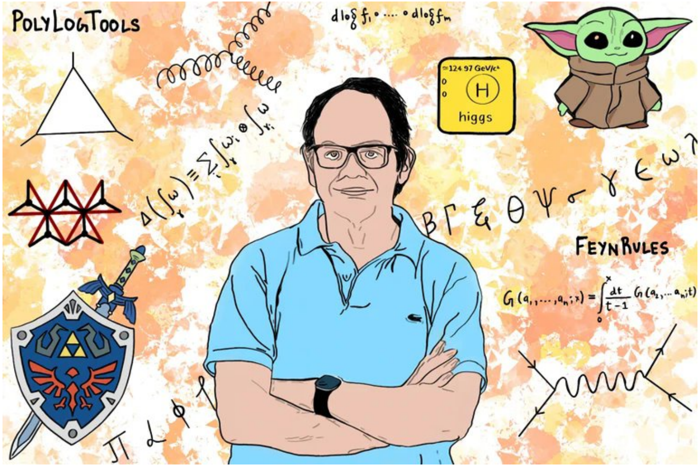
- First kick-off meeting held on 12 May in Siegen
- Discussion of ongoing work and future project directions

- Feynman's diagrammatic approach: cornerstone of modern perturbative calculations
- High luminosity LHC + possible future colliders: Precise theoretical predictions are in high demand
- Higher precision on the theory side: more loops, harder algebra, more complicated integrals to solve



- Here things quickly get technical
- Faster computers (more CPU+RAM) alone are not sufficient
- We also need new ideas from physics and mathematics
- Better and more efficient algorithms/codes are essential
- RA3 is the perfect format to support their development

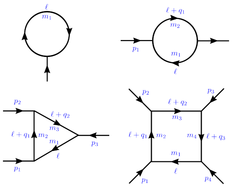


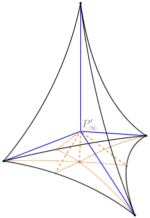


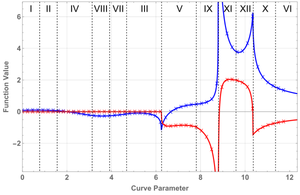
Bonn University - Research Group of Prof. Claude Duhr

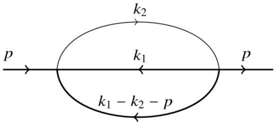
- One-loop Feynman integrals: reducible to a basis of tadpoles, bubbles, triangles and boxes
- $\mathcal{O}(\varepsilon^0)$-results available in the literature - enough for 1-loop
- Higher loops: also need higher orders beyond ε^0
- Difficult to get for most generic kinematic configurations

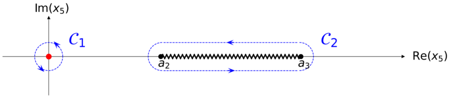
- Key concept: At $\mathcal{O}(\varepsilon^0)$ these integrals are geometric volumes in hyperbolic space
- Recent advances in pure mathematics: express such integrals directly through familiar functions (MPLs)
- Idea: All order results in ε from 1-fold integrals over ε^0-results
- Analytic results publicly available [Duhr and Mork, 2025]











- Differential equations: powerful technique to efficiently calculate large numbers of master integrals
- Key concept: Canonical bases (ε-factorized bases)

$$d\vec{I}(\vec{x},\varepsilon) = \Omega(\vec{x},\varepsilon)\vec{I}(\vec{x},\varepsilon)$$
$$\to d\vec{J}(\vec{x},\varepsilon) = \varepsilon A(\vec{x})\vec{I}(\vec{x},\varepsilon)$$

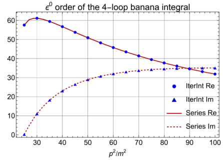
- Matrix $A(\vec{x})$ in dlog form: can solve the system iteratively to any ε-order

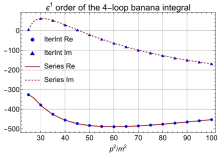
- "Simpler integrals": Get the dlog form for free once the ε-basis is found
- "Complicated integrals": Finding the ε-basis alone is **not enough**
- Algorithm to bring integrals with **any underlying geometry** to canonical form [Duhr, Maggio, Nega, Sauer, Tancredi, and Wagner, 2025]

- Many Feynman integrals expressible as iterated integrals

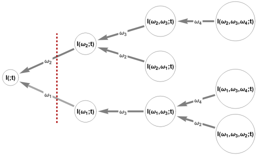
$$I_\gamma(\omega_1, \ldots, \omega_n) = \int_{0 \le \xi_1 \le \ldots \le x_n \le t} \times d\xi_n \ldots d\xi_1 f_n(\xi_n) \ldots f_1(\xi_1)$$

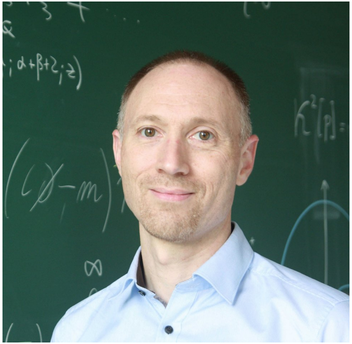
- Tailored algorithms for evaluating *special* classes of such integrals
- Problem: Every new class of special functions has to be implemented from scratch

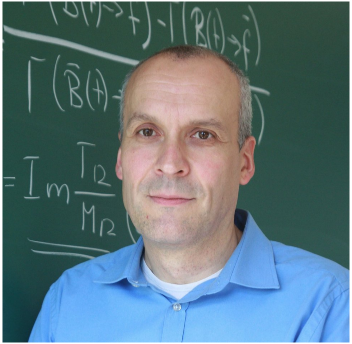




- Idea: Convert nested integrals into a *system of differential equations*
- New algorithm to evaluate **very general** classes of iterated integrals numerically
- Optimized for calculating large numbers of iterated integrals
- Treatment of singularities using shuffle regularization
- Implemented in publicly available package **IterInt** (**Mathematica** & **C++**) [Baur and Duhr, 2026]

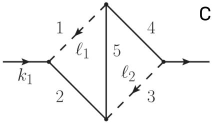






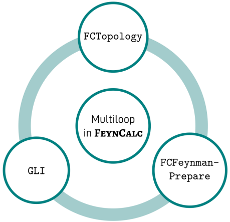
Siegen University - Prof. Tobias Huber, Dr. Vladyslav Shtabovenko, Prof. Guido Bell

- IBP-reduction of loop integrals to master integrals: main tool behind almost every multiloop calculation
- Mostly done by solving a huge sparse system of linear equations symbolically via Gaussian elimination (Laporta algorithm)
- Becomes intractable for involved kinematics, new ideas are needed

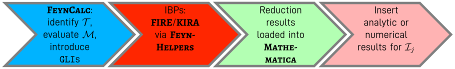


- Idea: Gröbner basis technique plus non-commutative shift operator algebra to bypass existing bottlenecks [Barakat, Brüser, Fieker, Huber, and Piclum, 2023; Barakat, Brüser, Huber, and Piclum, 2022]
- Advantage: Compute Gröbner basis for a topology **once and for all**, after that reduction becomes **fast and cheap**
- Challenges: Gröbner basis computations can be *very expensive*

- Feynman diagram calculus is impossible without automation
- Public frameworks for multiloop calculations are scarce
- Interfacing single tools with each other is tedious and time-consuming
- Practical solution: **FeynCalc** package is easy to use and implements relevant multiloop algorithms in **Mathematica**



- **Latest release FeynCalc 10.2**: interfaces to most useful public tools and parallelization [Shtabovenko, 2025]
- Plans within CmF
  - Turn **FeynCalc** into a multipurpose tool for master integral calculations (Feynman parameters, differential equations)
  - Create a library of analytic results for selected multiloop integrals



- Automation of NNLO calculations in Soft-Collinear Effective Theory
- Prerequisite: extract IR divergences in phase-space integrals for arbitrary measurements on the soft/collinear radiation
- Some applications: rapidity regulator beyond dimensional regularization
- Numerical integration: currently with Divonne/Vegas, quasi-Monte Carlo under development

- Automated framework for the calculation of soft, jet and beam functions [Bell, Brune, Das, and Wald, 2022, 2025; Bell, Rahn, and Talbert, 2018]
- **SoftSERVE**: public codes for calculation of NNLO soft functions
- WIP: Develop a similar code for jet and beam functions
- Automated setup crucial for high-precision resummations at the LHC

Summary

- RA3: Computer Algebra combines expertise from two hubs: Bonn and Siegen
- Bonn: new methods for Feynman integrals
  - Geometric interpretation of one-loop integrals
  - Canonical forms for integrals with arbitrary underlying geometry
  - Numerical evaluation of iterated integrals
- Siegen: automation of large-scale multiloop calculations
  - IBP reduction with Gröbner bases
  - Automatic Feynman diagram evaluation (FeynCalc)
  - Precision collider phenomenology (SoftServe)
- Advances in mathematics, algorithms and software helping to automatize and improve theory predictions