

**Color meets Flavor Symposium**

Siegen - July 15, 2026

# **Effective Field Theories for Color and Flavor**

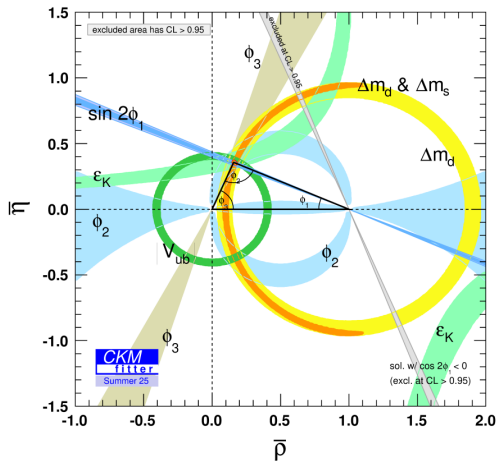
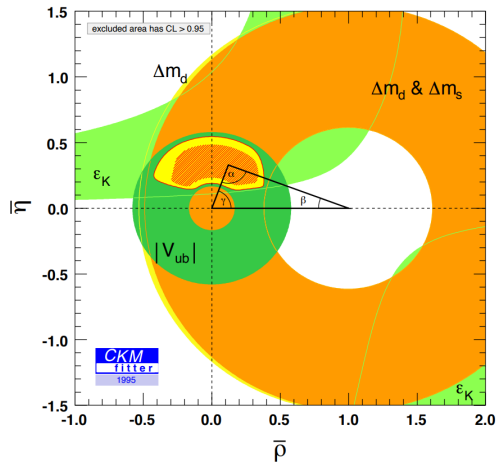
**Philipp Böer**



**Funded by  
the European Union**



# The precision era of flavor physics

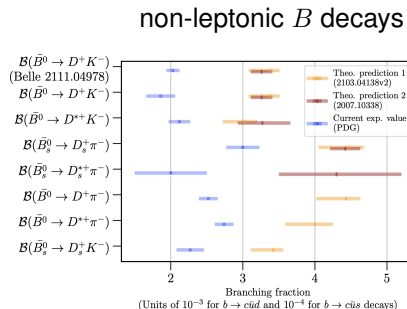
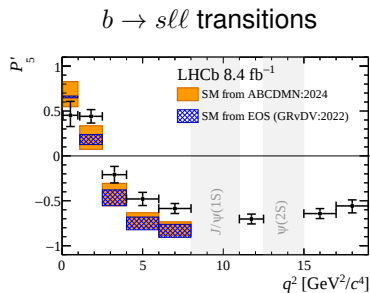


Amazing progress, but data will improve dramatically in the coming decade (HL-LHC, Belle II, ...)

→ Theory needs to catch up!

# Anomalies in $B$ -meson decays

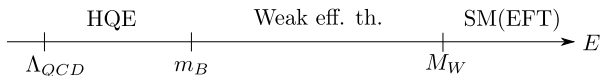
- Precision flavour experiments test the CKM paradigm and **search for new physics**.
- Various **tensions** between theory predictions and measurements



New physics or underestimated hadronic effects?

# Effective field theories as a precision tool

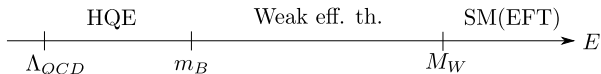
Hierarchy of energy scales:



$$\Lambda_{QCD} \sim 0.5 \text{ GeV} \ll m_B \sim 5 \text{ GeV} \ll M_W \sim 80 \text{ GeV}$$

# Effective field theories as a precision tool

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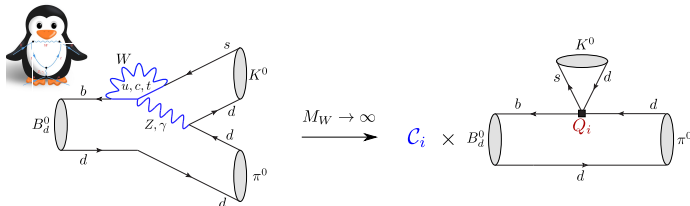


$$\Lambda_{QCD} \sim 0.5 \text{ GeV} \ll m_B \sim 5 \text{ GeV} \ll M_W \sim 80 \text{ GeV}$$

1) Effective low-energy description:  $\mathcal{L}_{SM} \rightarrow \mathcal{L}_{eff} = \sum_i \mathcal{C}_i(\mu) Q_i(\mu)$

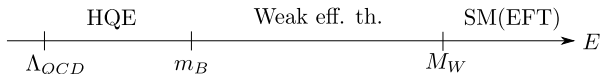
→ Wilson coefficients  $\mathcal{C}_i$  encode short-distance physics

→ Main challenge is to compute hadronic matrix elements  $\langle Q_i \rangle$



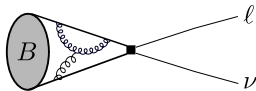
# Effective field theories as a precision tool

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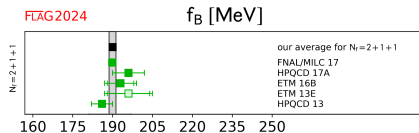


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## 2) Hadronic matrix elements (leptonic decays)

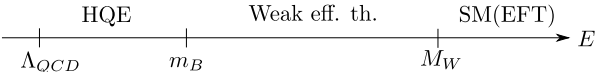


decay constant  $f_B = (190.0 \pm 1.3) \text{ MeV}$



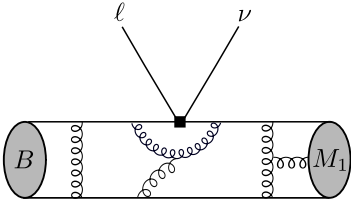
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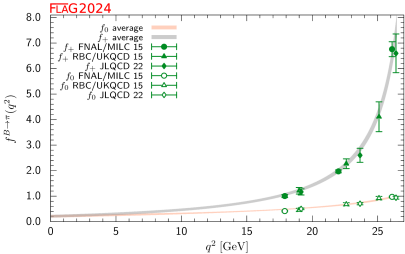


$$\Lambda_{QCD} \sim 0.5 \text{ GeV} \ll m_B \sim 5 \text{ GeV} \ll M_W \sim 80 \text{ GeV}$$

## 2) Hadronic matrix elements (semi-leptonic decays)

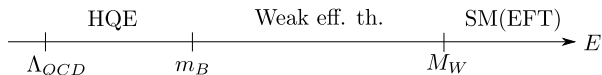


form factors  $f(q^2)$



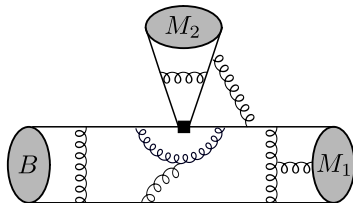
# Effective field theories as a precision tool

Hierarchy of energy scales:



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## 2) Hadronic matrix elements (non-leptonic decays)

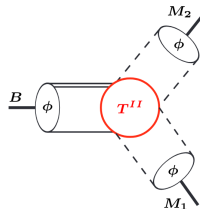
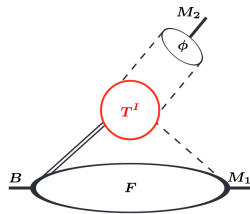


complex hadronic transition

Currently not accessible on the lattice

- Heavy-quark expansion based on EFTs
- Heavy-quark effective theory (HQET)
- Soft-collinear effective theory (SCET) and QCD factorization for energetic particles

# QCD Factorization [BBNS 99-01]

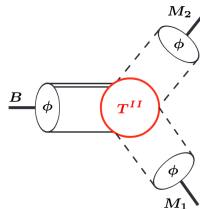
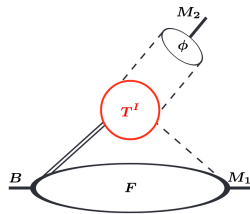


Matrix elements **factorize** in the heavy-quark limit  $m_b \rightarrow \infty$ :

$$\begin{aligned} \langle M_1 M_2 | Q_i | \bar{B} \rangle &\sim F^{B M_1} \int_0^1 du \, T_i^I(u) f_{M_2} \phi_{M_2}(u) \\ &+ \int_0^\infty d\omega \int_0^1 du dv \, T_i^{II}(u, v, \omega) f_{M_1} \phi_{M_1}(v) f_{M_2} \phi_{M_2}(u) f_B \phi_B^+(\omega) + \mathcal{O}(\Lambda/m_b, \alpha_{\text{em}}) \end{aligned}$$

- perturbative hard-scattering kernels  $T_i^{I,II}$
- non-perturbative (universal) decay constants  $f_i$  and light-cone distribution amplitudes  $\phi_i$

# QCD Factorization [BBNS 99-01]



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- Perturbative corrections known to NNLO
- Decay constants and first Gegenbauer moments known from lattice
- parameters of  $\phi_B^+(\omega)$  **poorly constrained**

e.g. [Bell, Beneke, Huber, Li]

e.g. [FLAG, Braun et al.]

# Improving QCDF for non-leptonic $B$ decays with EFT methods

1) Electromagnetic effects

[Beneke, PB, Finauri, Toelstede, Vos 20-22]

2) Constraining  $\phi_B^+(\omega)$  from lattice data

[Beneke, PB, Braun, Ji, Toelstede 26 (to appear)]

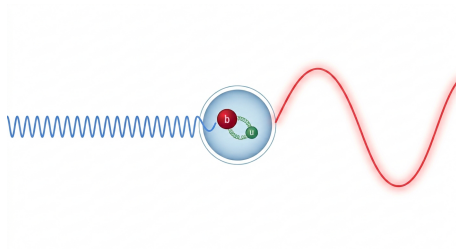
3) Power corrections in  $\Lambda/m_b \sim 0.1$

[PB, Finauri 26 + PB, Neubert, Stillger 26 (both to appear)]

# QED effects in exclusive $B$ decays [Beneke, PB, Finauri, Toelstede, Vos 20-22]

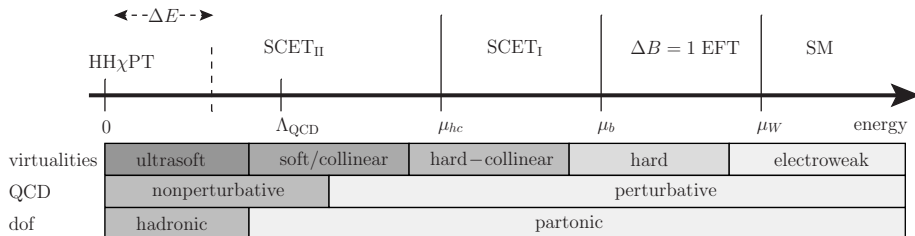
Prime example for an EFT analysis: (Photon wavelength  $\lambda$ , meson radius  $r \sim 1/\Lambda_{\text{QCD}}$ )

- $\lambda \gg r$ : photons couple effectively to point charge
- $\lambda \lesssim r$ : photons resolve the partonic substructure



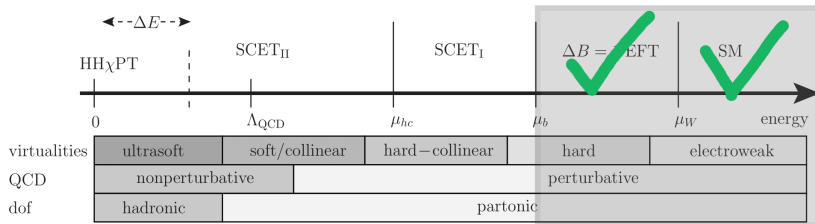
# QED effects in exclusive $B$ decays [Beneke, PB, Finauri, Toelstede, Vos 20-22]

$$\Delta E \sim 60 \text{ MeV} \ll \text{few times } \Lambda_{\text{QCD}} \ll m_b \sim 4.2 \text{ GeV} \ll M_W \sim 80 \text{ GeV}$$



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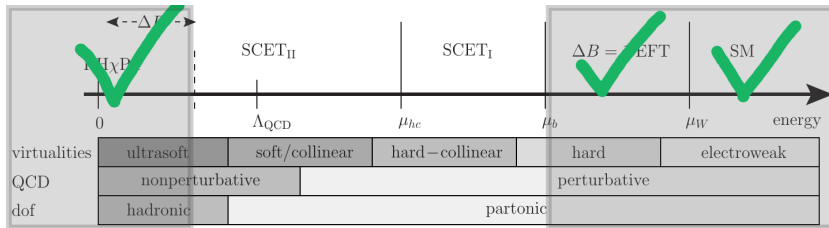
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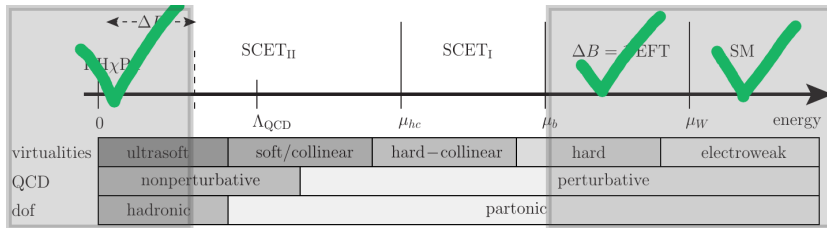
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- ✓ short-distance QED at  $\mu \gtrsim m_b \rightarrow$  Wilson coefficients of weak eff. Lagrangian
- ✓ Far IR (ultrasoft) region  $\mu_{us} \ll \Lambda_{\text{QCD}}$  described by point-like hadrons ( $\rightarrow$  PHOTOS)

# QED effects in exclusive $B$ decays [Beneke, PB, Finauri, Toelstede, Vos 20-22]

$$\Delta E \sim 60 \text{ MeV} \ll \text{few times } \Lambda_{\text{QCD}} \ll m_b \sim 4.2 \text{ GeV} \ll M_W \sim 80 \text{ GeV}$$



- ✓ We developed a theory for “structure dependent QED effects” using EFTs.
  - Theoretically interesting, with applications to (semi-)leptonic channels
  - Numerical impact small for non-leptonic decays. Cannot explain anomalies.

# The $B$ -meson light-cone distribution amplitude $\phi_B^+(\omega)$

$$\langle 0 | \bar{q}(tn) [tn, 0] \not{n} \gamma_5 h_v(0) | \bar{B}_v \rangle = i \tilde{f}_B(\mu) \int_0^\infty d\omega e^{-i\omega t} \phi_B^+(\omega; \mu), \quad n^2 = 0$$

- **Poorly constrained** QCDF input. Its most important parameter is the first inverse moment:

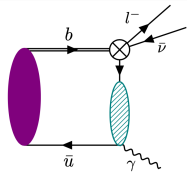
$$\lambda_B^{-1} = \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega) \sim 200 \text{ MeV} - 600 \text{ MeV}$$

- Use  $B \rightarrow \ell \nu \gamma$  e.g. [Beneke, Rohrwild 11] or  $B \rightarrow \ell \nu \ell \bar{\ell}$  data [Beneke, PB, Rigatos, Vos 21] to constrain  $\phi_B(\omega)$ .

# Euclidean correlators as lattice probes of $\phi_B^+(\omega)$ [Beneke, PB, et al. 26 (to appear)]

We use lattice data for  $P \rightarrow \gamma^*$  form factors for Euclidean  $p^2 < 0$  [e.g. Giusti et al. 25] to constrain  $\phi_B^+$

$$\int d^4x e^{ip \cdot x} \langle 0 | \mathcal{T} \{ j_{\text{em}}^\mu(x) j_{\text{weak}}^\nu(0) \} | \bar{B}(p+q) \rangle \sim \int_0^\infty d\omega \frac{\phi_+^B(\omega; \mu)}{2E_\gamma \omega - p^2} + \mathcal{O}(\alpha_s, \Lambda/m_b),$$



[Fig from Galda, Neubert, Wang 22]

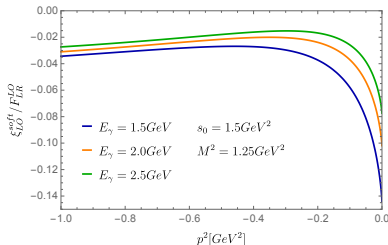
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Theoretically under good control

- ✓ radiative corrections and power corrections up to twist-6 included
- ✓ soft power correction strongly suppressed for  $p^2 < 0$  (sum rule estimate)



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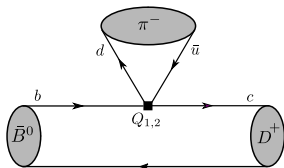
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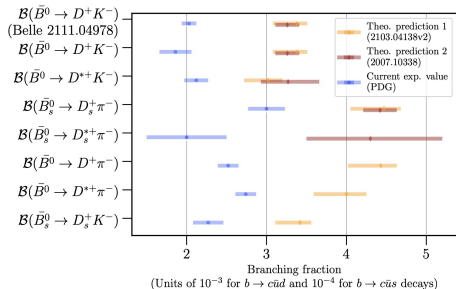
Exploratory stage, but promising new approach to constrain  $\phi_B^+(\omega)$ .

# Power corrections in $B \rightarrow D\pi$ and $B \rightarrow DK$ [PB, Finauri 26 (to appear)]

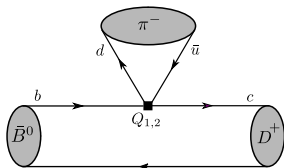


$$\langle DL | Q_{1,2} | \bar{B} \rangle \sim F^{B \rightarrow D} \int_0^1 du \mathbf{T}_i^I(u) f_L \phi_L(u) + \mathcal{O}(\Lambda/m_b, \alpha_{\text{em}})$$

- Theoretically among the **cleanest non-leptonic channels**. Yet, large  $\sim 4\sigma$  deviations.



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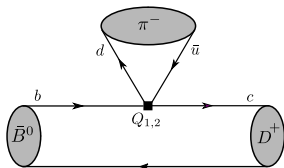
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- Theoretically among the **cleanest non-leptonic channels**. Yet, large  $\sim 4\sigma$  deviations.
- Non-factorizable power correction from soft-gluon exchange [BBNS 00]

$$\sim \mathcal{C}_1 \times \langle D|\bar{c}(0)\gamma^\mu(1-\gamma_5)\tilde{G}_{\mu\nu}(-sn)b(0)|\bar{B}\rangle$$

- LCSR estimate yields **permille** effect. Cannot explain -15% amplitude deficit. [Bordone et al. 20]

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Clear power counting rules of the EFT shows this picture is **incomplete**:

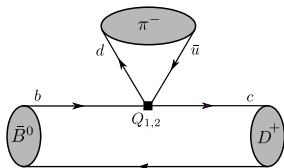
→ **Additional hadronic matrix elements** contribute at the same order in  $\Lambda/m_b$ , e.g.

$$\sim \mathcal{C}_2 \times \langle D | (\bar{c} \gamma^\mu (1 - \gamma_5) b)(0) \text{tr}\{\mathcal{A}_\perp^\mu \mathcal{A}_{\perp,\mu}\}(-sn) | \bar{B} \rangle$$

→ Naive condensate ansatz yields (**preliminary**)

$$\mathcal{A}_{\text{NLP}}/\mathcal{A}_{\text{LP}} \approx -0.5 \times \Delta \quad \text{with} \quad \Delta = \langle 0 | g_s^2 \text{tr}\{\mathcal{A}_\perp^\mu \mathcal{A}_{\perp,\mu}\} | 0 \rangle / \text{GeV}^2$$

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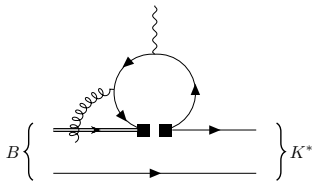
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**Hadronic effects provide potential explanation of the observed deficit.**

# Non-local charm-loop contributions in $b \rightarrow s\gamma$ and $b \rightarrow s\ell\ell$ decays

- 1) Resolved-photon contributions in  $\bar{B} \rightarrow X_s\gamma$  [Bartocci, PB, Hurth 24 + 26 (to appear)]
- 2) A new approach to non-factorisable charm-loop effects in  $B \rightarrow K^{(*)}\ell\ell$  decays [Bartocci, PB, Feldmann, Ferré, Gubernari, Vladimirov 26]



# Resolved-photon contributions in $\bar{B} \rightarrow X_s \gamma$ [Bartocci, PB, Hurth 24 + 26 (to appear)]

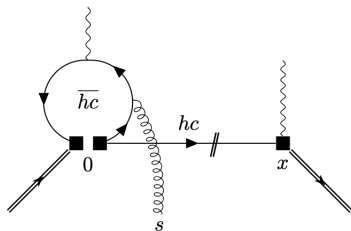
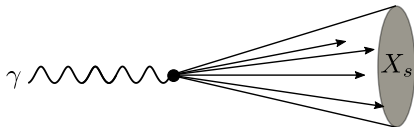
Tensions in mesonic  $b \rightarrow s$  transitions motivate study of modes affected by **independent uncertainties**:

- Baryonic  $\Lambda_b$  decays [PB, Feldmann, van Dyk et al. 14-19]
- Inclusive  $\bar{B} \rightarrow X_s \gamma$  and  $\bar{B} \rightarrow X_s \ell \ell$  decays

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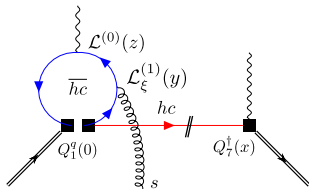
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**Jet-like** configuration close to kinematic endpoint  $m_b - 2E_\gamma \sim \mathcal{O}(\Lambda)$

- photon-energy spectrum described by SCET factorization formula.
- $Q_1$ - $Q_{7\gamma}$  interference constitutes the **largest uncertainty**:  $\mathcal{F}_{17} \in [2.9\%, 11.7\%]$  [Benzke, Garzelli, Hurth '25]

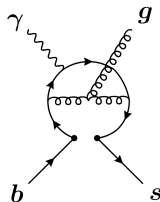
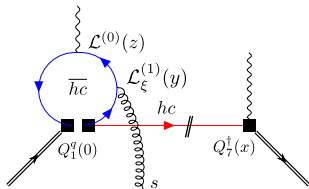
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Factorization formula [Benzke, Lee, Neubert, Paz 10 + Bartocci, PB, Hurth 26]

$$d\Gamma(\bar{B} \rightarrow X_s \gamma) \sim \int_0^1 du \mathbf{H}(u; \mu) \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1 + i0} \bar{J}(u, \omega_1; \mu) \int_{-\infty}^{\bar{\Lambda}} d\omega J(\omega; \mu) g_{17}(\omega, \omega_1; \mu),$$

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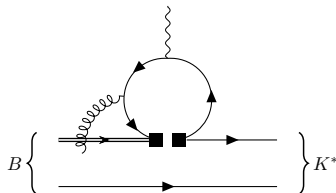
- ✓ Hard matching contributions  $\mathbf{H}(u; \mu)$  at NLO
- ✓ Quark jet function  $J(\omega; \mu)$  at NLO
- ✓ One-loop renormalization of subleading shape function  $g_{17}(\omega, \omega_1; \mu)$  [Bartocci, PB, Hurth 24]
- (✓) two-loop “charm-loop jet function”  $\bar{J}(u, \omega_1; \mu)$  (finished for  $m_c \rightarrow 0$  [Bartocci, PB, Hurth 26 (to appear)])

# A new approach to non-factorizable charm-loop effects [Bartocci, PB, et al. 26]

Use similar approach to describe non-factorisable soft-gluon effects in exclusive rare decays. [Qin et al. 22]

→  $B$ -meson **Di-light-cone distribution amplitudes** (DLCDAs)

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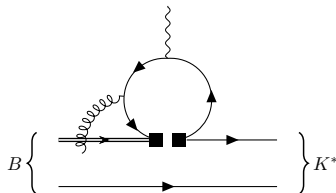


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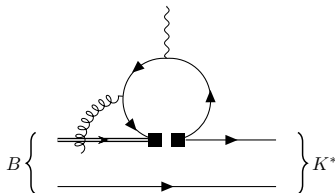
- ✓ eight three-particle DLCDA's: 1 at twist-3; 3 at twist-4 and twist-5; 1 at twist-6
- ✓ tree-level relations for normalisation and first moments from eom's
- ✓ simple **momentum-space models** for all DLCDA's
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**Application to rare  $B \rightarrow K^{(*)} \ell \ell$  decays could shed new light on anomalies!**

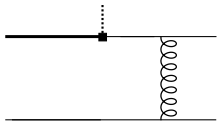
[w.i.p. with Bartocci, PB, Feldmann, Ferré, Gubernari, Vladimirov]

## From flavor physics to high-energy physics

- 1)  $B_c \rightarrow \eta_c$  form factors at large recoil [Bell, PB, Feldmann, Horstmann, Shtabovenko 22-26]
- 2) Resummation of double-logarithms in Di-Higgs production [PB 26 (to appear)]
- 3) Multi-leg amplitudes with massive quarks [PB, Jaskiewicz, Monni (in progress)]

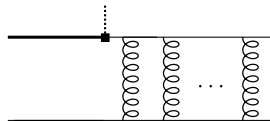
# Heavy-to-light form factors for NR bound states [Bell, PB et al. 22-26]

Hard-scattering picture well understood



$$\int_0^1 \frac{dx}{x} \phi_\pi(x) = \text{finite}$$

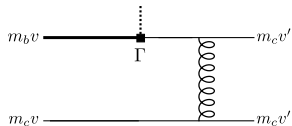
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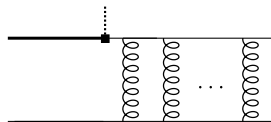
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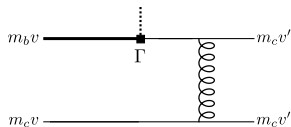
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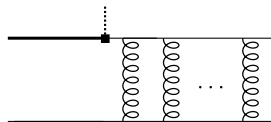
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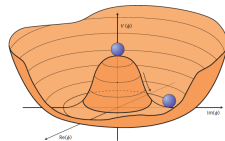
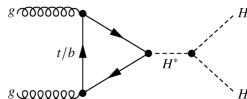
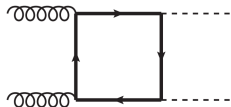
→ perturbative setup with scale hierarchy  $m_b \gg m_c \gg \Lambda_{\text{QCD}}$

We performed an **all-order resummation** of the leading log-enhanced contributions from **soft-quark exchange**

→ For the first time, we have unraveled the dynamics of this mechanism.

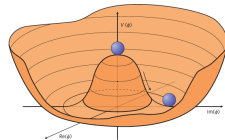
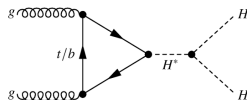
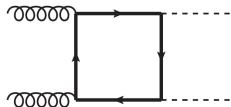
Important step to overcome this decades-old problem of QCDF for power corrections.

# Double Logarithms in Higgs-Boson Pair Production [PB 26 (to appear)]



- Higgs-boson pair production is a **flagship process** at the HL-LHC (and beyond)
- To measure trilinear Higgs self-coupling need good control over QCD corrections

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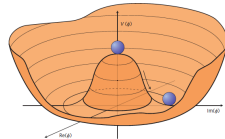
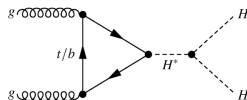
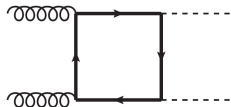


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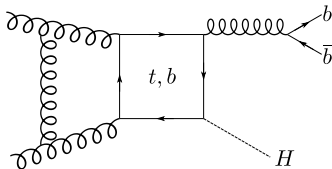
$$F_1|_{\text{Box}} \simeq \rho^2 L^2 {}_2F_2(1,1; \frac{3}{2}, 2; -\frac{z}{4}) \left( 8 - \frac{\alpha_s}{2\pi} L^2 \left( C_F - \frac{C_A}{2} \right) \frac{s^2}{t(s+t)} {}_2F_2(1,1; \frac{3}{2}, 2; -\frac{z}{4}) \right)$$

→  $z = \frac{\alpha_s}{2\pi} (C_F - C_A) L^2$  and  $\rho = m_q^2/s$

# Multi-leg amplitudes with massive quarks [PB, Monni, Jaskiewicz (in progress)]

Two-loop amplitudes with massive quarks constitute **bottleneck calculations** for important LHC processes.

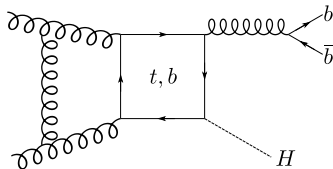
→ e.g.  $pp \rightarrow (b\bar{b}H, b\bar{b}W, t\bar{t}H, HH)$



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We use EFT methods to derive **controlled approximations** around the high-energy limit.

- explore the universality of the subleading soft and jet functions across different channels.
- exploit methodological **synergies with  $B_c \rightarrow \eta_c$  analysis**

# Conclusion

- 1) Flavor physics and HEP have entered the **precision era**. Great success of the Standard Model. Intriguing **tensions** in various  $B$  decays could show evidence for physics beyond the SM.
  - Sensitivity to new physics is determined by the accuracy of SM predictions.
  - Control over **complicated QCD dynamics** of utmost importance.
  
- 2) My research is centered around the use of **Effective Field Theories of color** as a tool for precision predictions for **flavor** observables within the SM.
  - Systematically improvable, and clear scaling behaviour of subleading contributions
  - Disentangle energy scales from hadronic to electroweak physics
  - Synergies between heavy flavor and Higgs physics
  
- 3) With the HL-LHC and Belle II, data will improve dramatically.
  - Fruitful future ahead, but **theory has to catch up**.

**Thank you!**

Backup-Slides

# Numerical Estimates of QED effects for $\pi K$ Final States

## Universal contributions:

- from ultra-soft radiation and universal part of virtual corrections
- resums large soft  $\sim \ln \Delta E/m_B$  and collinear  $\sim \ln m_i/m_B$  logarithms

$$U(M_1 M_2) = \left( \frac{2\Delta E}{m_B} \right)^{-\frac{\alpha_{\text{em}}}{\pi}} \left( Q_B^2 + Q_1^2 \left[ 1 + \ln \frac{m_1^2}{m_B^2} \right] + Q_2^2 \left[ 1 + \ln \frac{m_2^2}{m_B^2} \right] \right)$$

For  $\Delta E = 60$  MeV:

( $\Delta E = \pi K$  invariant mass window around  $m_B$ )

$$U(\pi^+ K^-) = 0.914$$

$$U(\pi^0 K^-) = 0.976$$

$$U(\pi^- \bar{K}^0) = 0.954$$

$$U(\bar{K}^0 \pi^0) = 1$$

Structure-dependent (virtual) corrections in non-rad. amplitude partly included

- ✓ Electroweak scale to  $m_B$ : QED corrections to Wilson coefficients
- ✓  $m_B$  to  $\Lambda_{\text{QCD}}$ :  $\mathcal{O}(\alpha_{\text{em}})$  corrections to short-distance kernels
- ⚡ QED effects in LCDAs/form factors, but log-enhanced effects  $\sim \ln \Lambda/m_B$  under control.

# Ratios and Isospin Sum Rules

1. Consider ratios where QCD uncertainties drop out:

$$R_L = \frac{2\text{Br}(\pi^0 K^0) + 2\text{Br}(\pi^0 K^-)}{\text{Br}(\pi^- K^0) + \text{Br}(\pi^+ K^-)} = R_L^{\text{QCD}} + \cos \gamma \text{Re } \delta_E + \delta_U$$

[Beneke, Neubert '03]

$$R_L^{\text{QCD}} - 1 \approx (1 \pm 2)\% \quad \delta_E \approx 0.1\% \quad \delta_U \approx 5.8\%$$

→ QED corrections larger than QCD and QCD uncertainty, but short-distance QED negligible

2. Isospin sumrule

$$\begin{aligned} \Delta(\pi K) &\equiv A_{\text{CP}}(\pi^+ K^-) + \frac{\Gamma(\pi^- \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^- \bar{K}^0) - \frac{2\Gamma(\pi^0 K^-)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^0 K^-) - \frac{2\Gamma(\pi^0 \bar{K}^0)}{\Gamma(\pi^+ K^-)} A_{\text{CP}}(\pi^0 \bar{K}^0) \\ &\equiv \Delta(\pi K)^{\text{QCD}} + \delta\Delta(\pi K) \end{aligned}$$

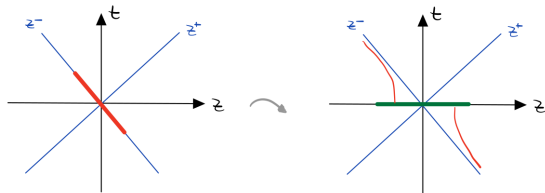
[Gronau, Rosner '06]

$$\Delta(\pi K)^{\text{QCD}} = (0.5 \pm 1.1)\% \quad \delta\Delta(\pi K) \approx -0.4\%$$

→ Isospin sumrule robust against QED effects (QED of similar size but small)

# The $B$ -meson light-cone distribution amplitude

$$\langle 0 | \bar{q}(tn)[tn,0] \not{n} \gamma_5 h_v(0) | \bar{B}_v \rangle = i \tilde{f}_B(\mu) \int_0^\infty d\omega e^{-i\omega t} \phi_B^+(\omega; \mu)$$



First-principles lattice determination desirable, but lattice intrinsically **Euclidean** (no light-cone)!

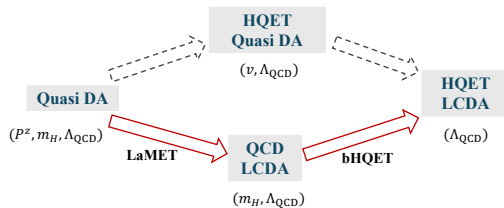
Ji's proposal [Ji '13+'14]:

→ Euclidean equal-time correlators (“**Quasi-DAs**” with small  $z^2 < 0$ ) (Quasi-/Pseudo-PDFs etc.)

$$\tilde{\phi}(x, P^z) \sim \int \frac{dz}{2\pi} e^{-ixP^z z} \langle 0 | \bar{q}(z) \Gamma[z, 0] Q(0) | H(P^z) \rangle$$

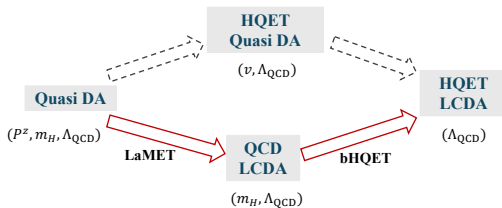
→ ... are related to light-like correlators in the large-momentum limit  $P^z \gg m_H, \Lambda_{\text{QCD}}$  via a factorization formula in large momentum effective theory (“**LaMET**”)

# Two-step matching [Han et al. '24+'25]



Scale hierarchy:  $P^z \gg m_H \gg \Lambda_{\text{QCD}}$

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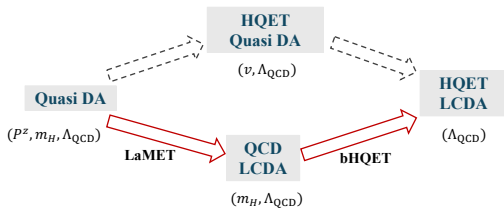


Scale hierarchy:  $P^z \gg m_H \gg \Lambda_{\text{QCD}}$

1.)  $P^z \gg m_H$ : match quasi-DA  $\rightarrow$  QCD-LCDA in LaMET

$$\tilde{\phi}(x, P^z) = \int_0^1 dy C\left(x, y; \frac{\mu}{P^z}\right) \phi(y; \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

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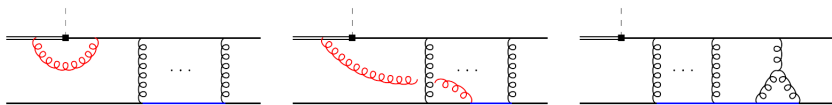
2.)  $m_H \gg \Lambda_{\text{QCD}}$ : match QCD-LCDA  $\rightarrow$  HQET-LCDA (boosted HQET) [Beneke et al. '23]

$$\phi(y; \mu; m_H) = m_H \frac{\tilde{f}_H}{f_H} \left[ \mathcal{J}_{\text{peak}} \phi_B^+(\omega; \mu) + \phi_B^+(\omega; \mu)|_{\text{tail}} \right] + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_H}\right) \quad (\omega = ym_H)$$

$\rightarrow$  two **perturbative** matching steps:  $C(x, y) = \delta(x - y) + \mathcal{O}(\alpha_s)$ ,  $\mathcal{J}_{\text{peak}} = 1 + \mathcal{O}(\alpha_s)$



## Double logarithms in $B_c \rightarrow \eta_c$ form factors



Soft-overlap form factor can be expressed in terms of two functions  $f_{1,2}$ :

$$(\bar{u}_0 = m_2/m_\eta = 1/2)$$

$$F(\gamma) = \xi_0 \exp \left\{ -\frac{\alpha_s C_F}{4\pi} L^2 \right\} \left( 2 \frac{1 + \bar{u}_0}{\bar{u}_0^3} f_2(m_2, m_2) + \frac{C_A}{2C_F \bar{u}_0^3} (1 - f_1(m_2, m_2)) - \frac{1}{\bar{u}_0^2} \right)$$

which obey a novel type of **implicit integral equations**:

$$f_1(q_+, q_-) = 1 + \frac{\alpha_s}{4\pi} \int_{q_-}^{p_2-} \frac{dk_-}{k_-} \int_{m_2^2/k_-}^{q_+} \frac{dk_+}{k_+} \exp \left\{ -S \left( \frac{q_+}{k_+}, \frac{p_-}{k_-} \right) \right\} 2C_F f_1(k_+, k_-)$$

$$f_2(q_+, q_-) = 1 + \frac{\alpha_s}{4\pi} \int_{q_-}^{p_2-} \frac{dk_-}{k_-} \int_{m_2^2/k_-}^{q_+} \frac{dk_+}{k_+} \exp \left\{ -S \left( \frac{q_+}{k_+}, \frac{p_-}{k_-} \right) \right\} \left[ 2C_F f_2(k_+, k_-) - \left( C_F - \frac{C_A}{2} \right) f_1(k_+, k_-) + \frac{C_A}{2} \right]$$

→ **Sudakov factor**  $S(x, y) = \frac{\alpha_s C_F}{2\pi} \ln x \ln y$