

Meson Spectroscopy and Amplitude Analyses at the COMPASS and Belle II Experiments

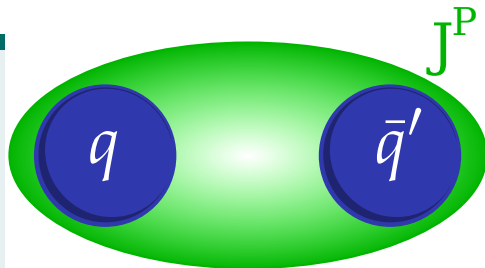
Stefan Wallner
(swallner@mpp.mpg.de)

Max Planck Institute for Physics

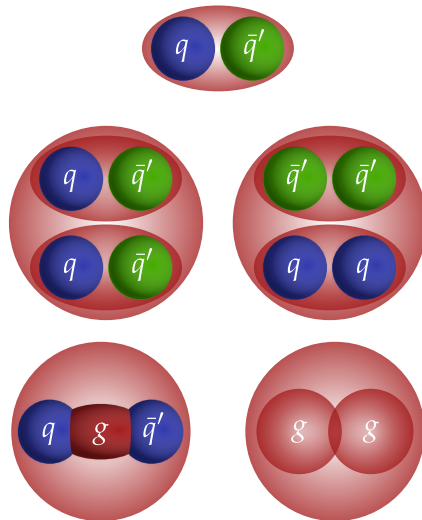
Color Meets Flavor Symposium
July 15, 2026



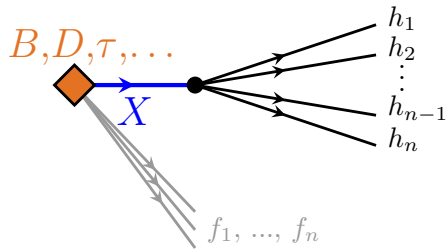
- ▶ Study systems bound by the strong interaction
 - ➡ Improve our understanding of QCD at low energies
- ▶ Simplest systems:
 - ▶ quark-antiquark state $\Rightarrow$ meson
- ▶ Properties include
 - ▶ Total spin: J
 - ▶ Parity: P
 - ▶ Charge-conjugation: C
 - ▶ Mass and width
- ▶ QCD predicts more complex configurations
 - ➡ Exotic mesons



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 - ➡ **Exotic mesons**



- ▶ Hadrons appear as resonances in multi-body hadronic decays of heavy mesons or τ leptons
- ▶ Studied in
 - ▶ precision measurements of the Standard Model
 - ▶ $R(D^{(*)})$, V_{cb} , ...
 - ▶ searches for physics beyond the Standard Model
 - ▶ CP violation, $b \rightarrow s$ transitions, ...
- ▶ Often rare decays
 - ▶ Small data samples
 - ▶ Large backgrounds
- ▶ Precise knowledge of the hadron excitation spectrum needed as input for decay models



Meson Spectroscopy at COMPASS

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The COMPASS Experiment



- Fixed-target experiment at M2 beam line in CERN North Area

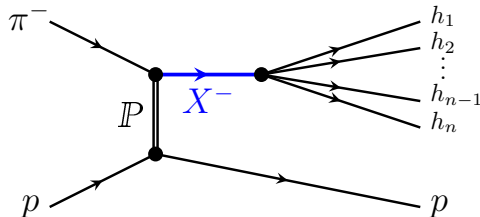
Light-Meson Spectroscopy

► Final States

- $\pi^- \pi^- \pi^+$ [PRD 95 (2017) 032004], [M. Aghasyan, ... SW, ... PRD 98 (2018) 092003]
- $\eta \pi^-$ [PLB 740 (2015) 303]
- $K_S^0 K^-$ [PoS HADRON2025 (2026) 155]
- $\omega \pi^- \pi^0$ [PoS HADRON2025 (2026) 155]
- $\eta \pi^- \pi^- \pi^+$ [PoS HADRON2025 (2026) 153]

► Observation of numerous light mesons

- Clarified existence of spin-exotic $\pi_1(1600)$ [PRL 122 (2019) 042002]



Light-Meson Spectroscopy

Final States

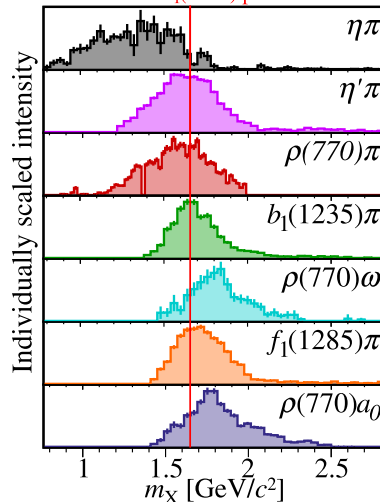
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▶ Observation of **numerous light mesons**

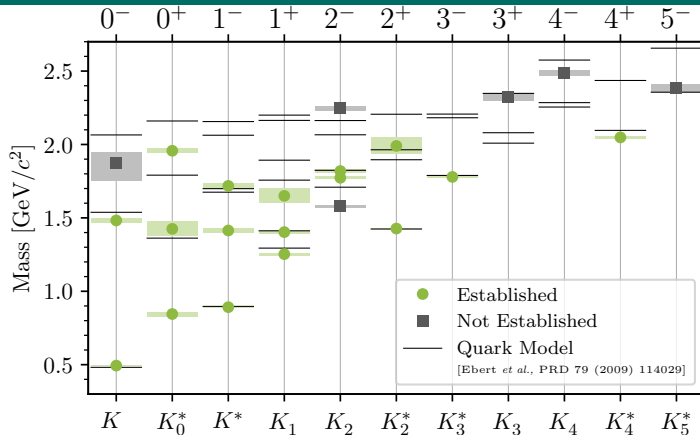
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Spin-exotic $J^{PC}=1^{-+}$ waves at COMPASS
preliminary

Nominal $\pi_1(1600)$ position

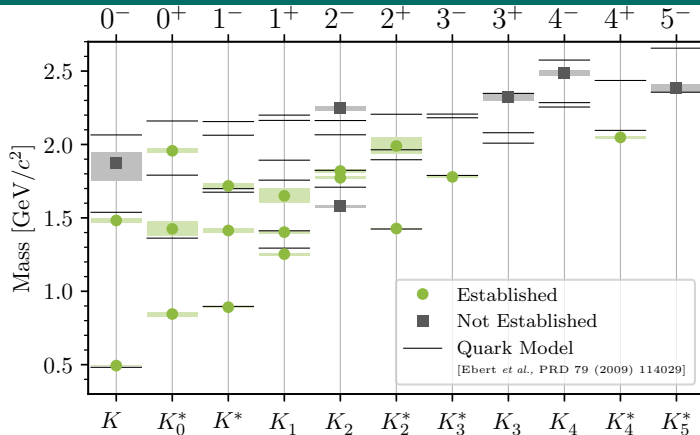


[Courtesy D. Spülbeck]



Box heights indicate mass uncertainties

- ▶ PDG lists 17 **established states**, 8 need **further confirmation**
- ▶ **Dense spectrum** of expected quark-model states
- ▶ Many predicted states have not yet been observed

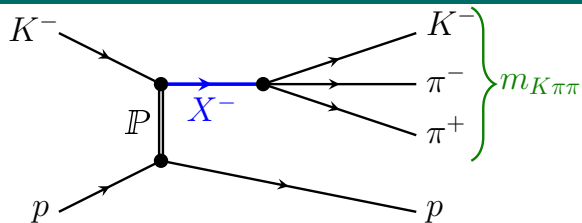


- ▶ No exotic quantum numbers for strange mesons
 - ➡ Appear only as **supernumerary states**, so-called crypto-exotic states
- ▶ No clear candidates for crypto-exotic strange mesons except for $K_0^*(700)/\kappa$

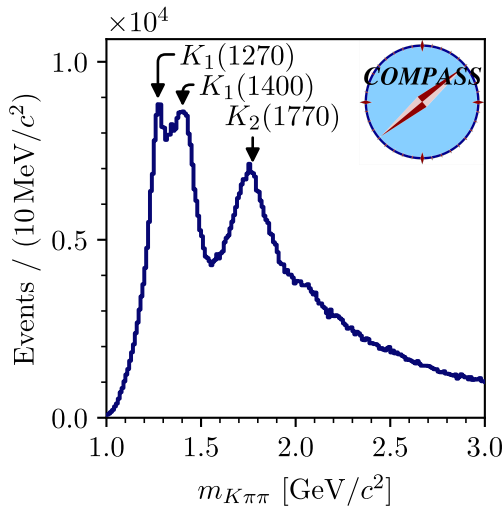
The Strange-Meson Spectrum

The $K^-\pi^-\pi^+$ Data Sample

[SW, NCC 47 (2024) 4]



- ▶ World's largest data set of about 720 k events
 - ▶ About $3.5\times$ larger than ACCMOR data (1981)
- ▶ Rich spectrum of **overlapping and interfering X^-**
 - ▶ Dominant well-known states
 - ▶ States with lower intensity are "hidden"



Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

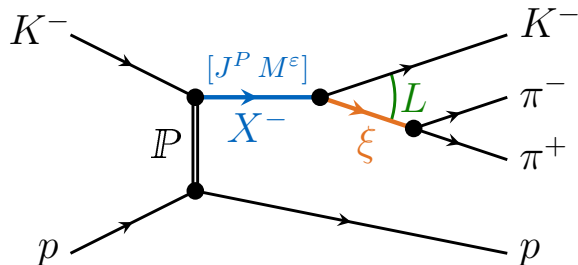
Partial-Wave Decomposition

[SW, NCC 47 (2024) 4]



Partial wave: $J^P M^\epsilon \xi b^- L$

- ▶ J^P spin and parity
- ▶ M^ϵ spin projection
- ▶ ξ two-body resonance
- ▶ b^- spectator particle
- ▶ L orbital angular momentum



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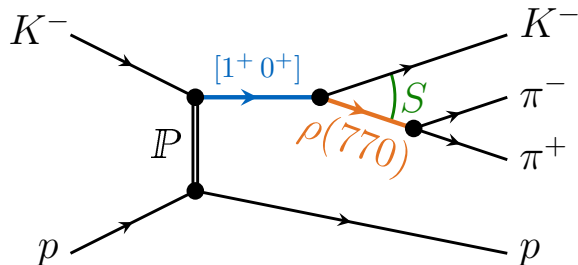
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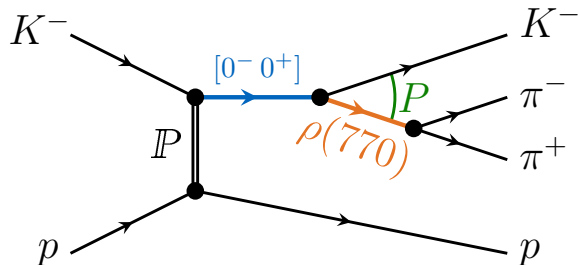
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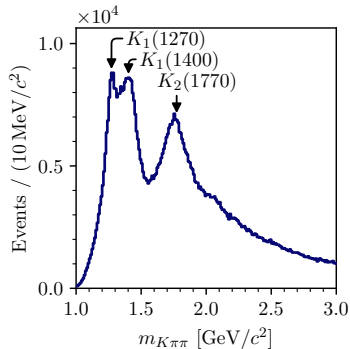
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Partial-Wave Analysis of the $K^-\pi^-\pi^+$ Final State

Analysis Scheme

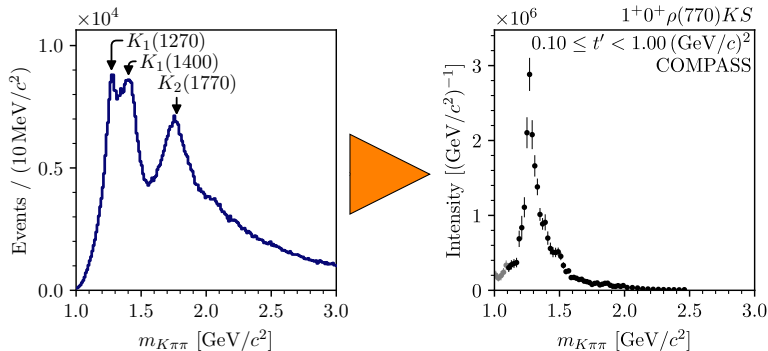
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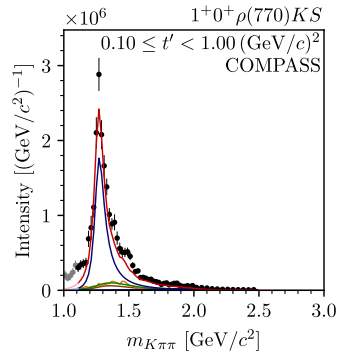
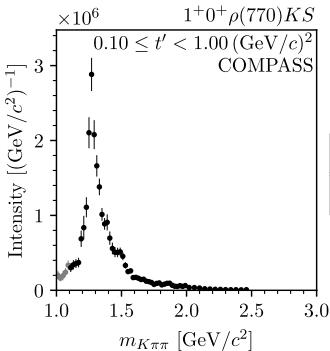
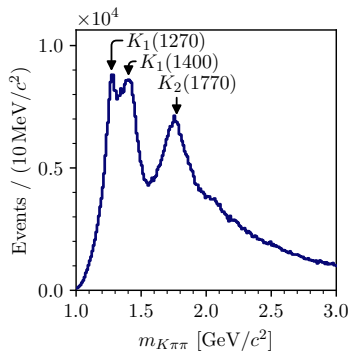
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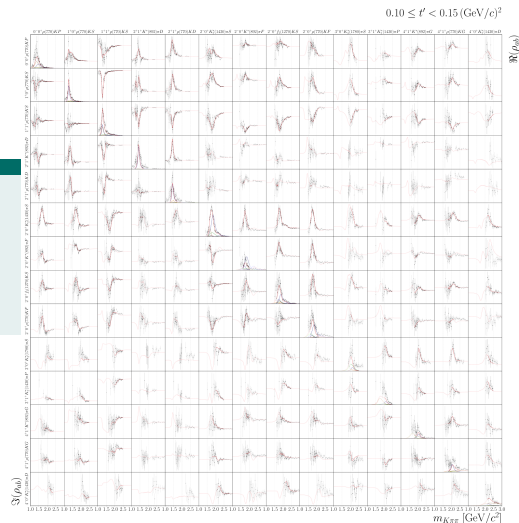
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Resonance-Model Fit

[SW, NCC 47 (2024) 4]



- ▶ Simultaneously fit 14 partial waves
- ▶ Model parameters constrained by partial-wave intensities and interference terms (relative phases)



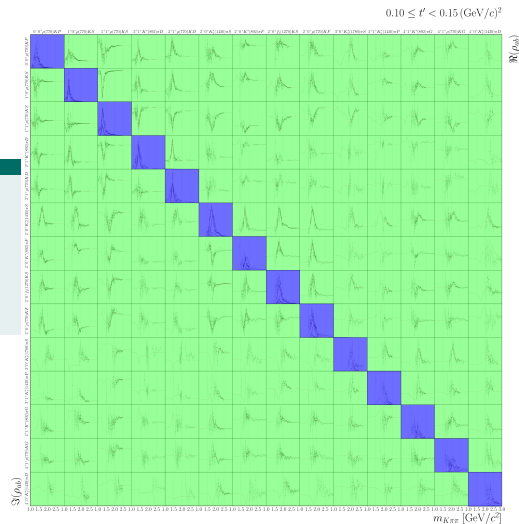
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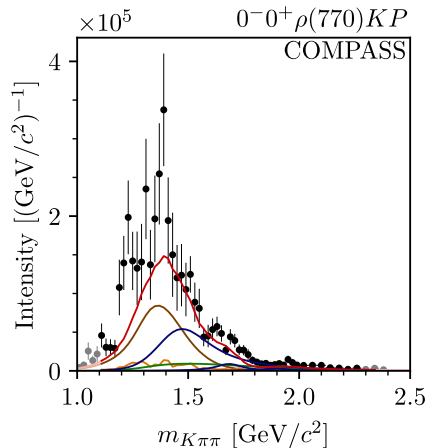
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Intensities; Interference terms

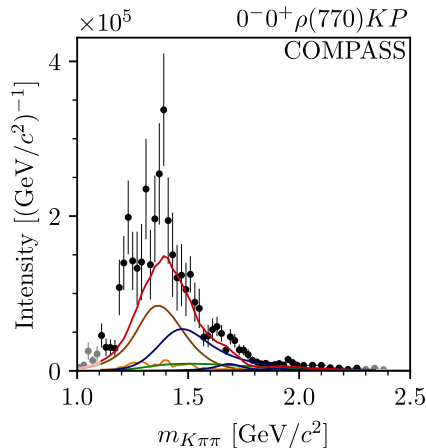
COMPASS $K^-\pi^-\pi^+$ data

- ▶ Peak at about $1.4 \text{ GeV}/c^2$
 - ▶ Established $K(1460)$
 - ▶ But, $m_{K\pi\pi} \lesssim 1.5 \text{ GeV}/c^2$ region weakly affected by known analysis artifacts
- ▶ Second peak at about $1.7 \text{ GeV}/c^2$
 - ▶ Excited $K(1690)$ signal
 - ▶ About 8σ statistical significance
 - ▶ Accompanied by rising phase
- ▶ Third signal at about $2.0 \text{ GeV}/c^2$
 - ▶ $K(1830)$ signal, about 5σ statistical significance
 - ▶ Most precise measurement of $K(1830)$ resonance parameters



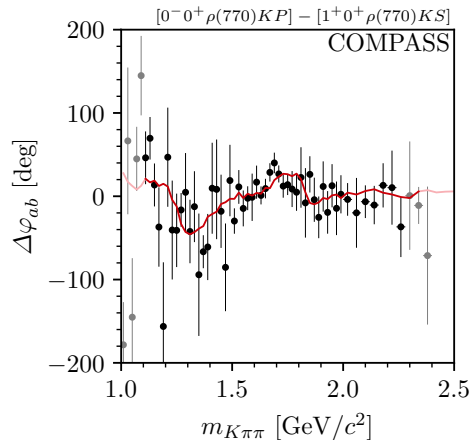
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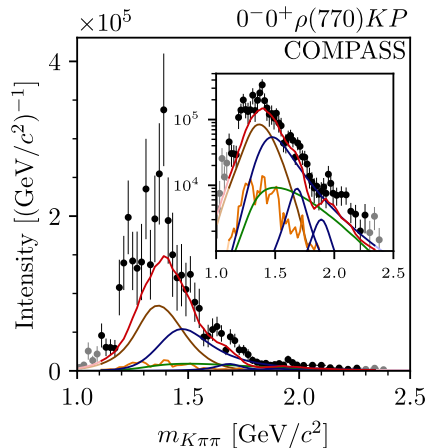
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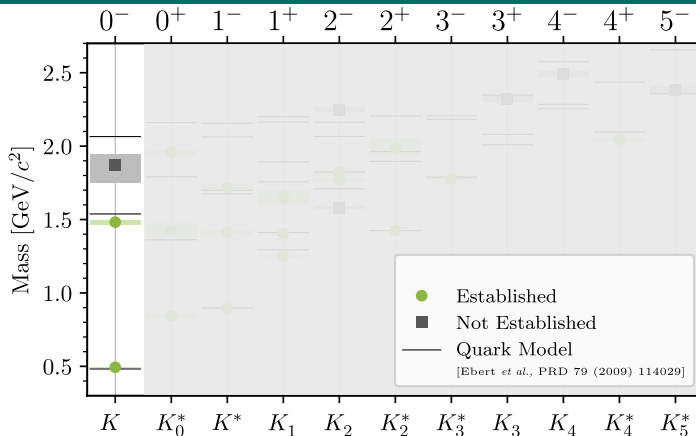
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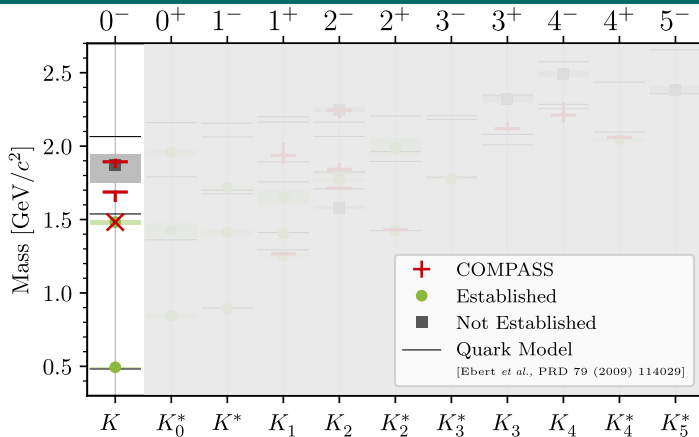
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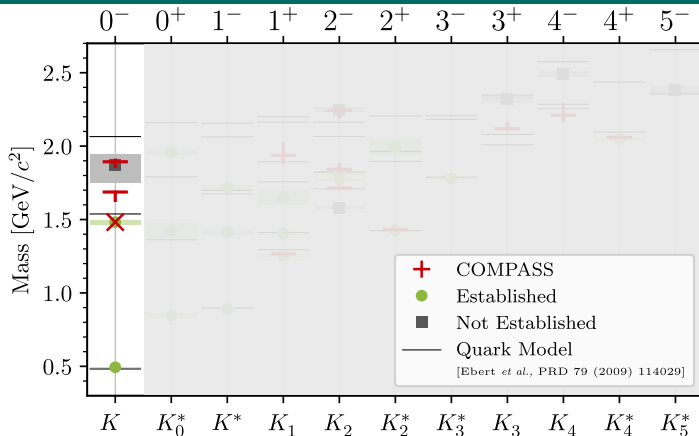




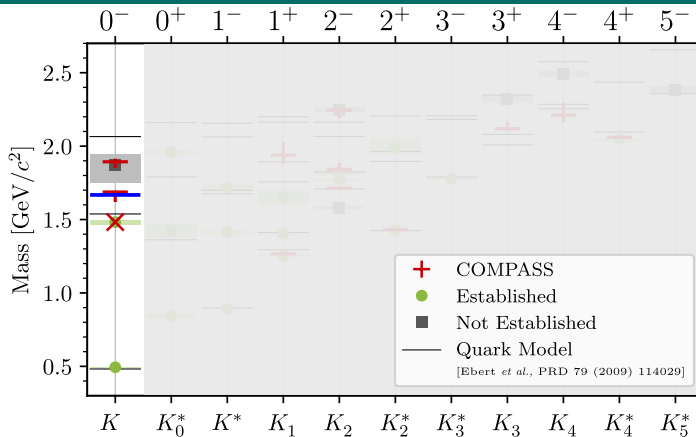
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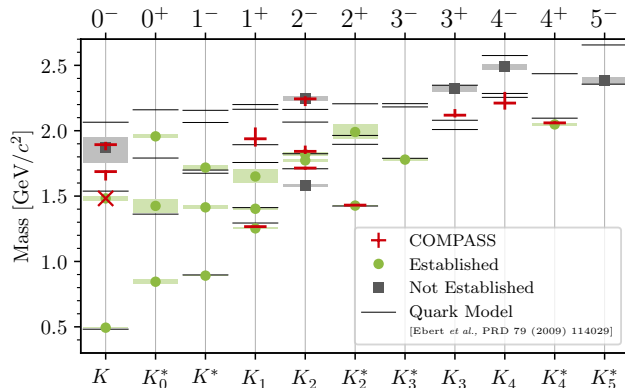
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The Strange-Meson Spectrum from COMPASS

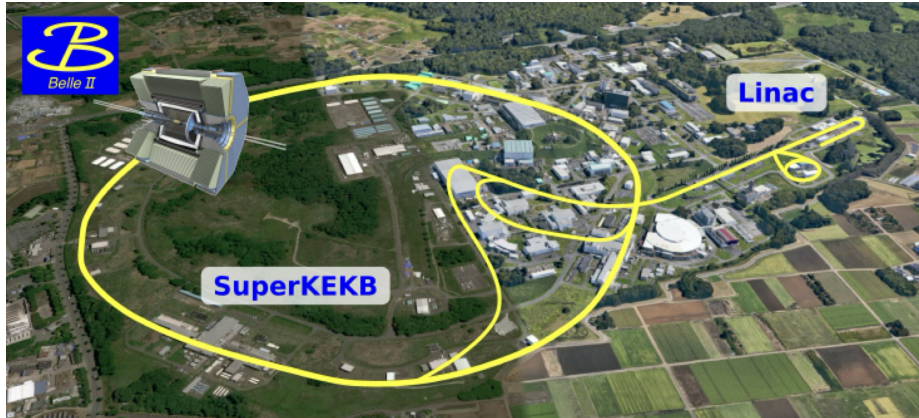
[SW, NCC 47 (2024) 4]



- ▶ Most detailed and comprehensive analysis of $K^-\pi^-\pi^+$ final state using world's largest sample
 - ▶ Measured **12 strange mesons** covering numerous J^P sectors
 - ▶ **First candidate for exotic strange-meson signal with $J^P = 0^-$**
- ▶ Accessible partial waves **limited by kinematic range of final-state particle identification**

Amplitude Analyses at Belle II

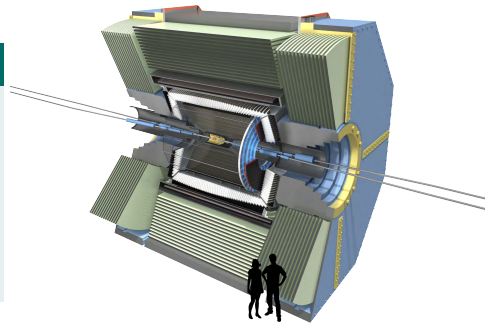
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Belle II at SuperKEKB accelerator (2019–)

- ▶ Run 1 (2019–2022)
 - ▶ Collected about $\frac{1}{2} \times$ Belle data-sample size
 $1 \times$ BaBar data-sample size
- ▶ Run2 started in spring 2024
 - ▶ Record luminosity: $5.319 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
 - ▶ Collected about Belle data-sample size

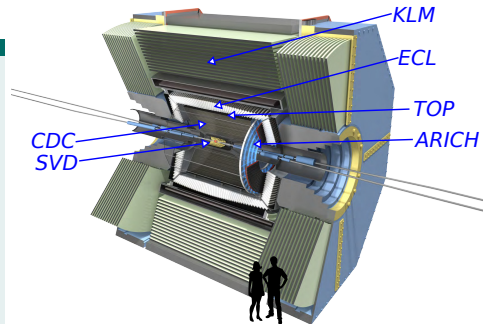


Charged Particle Identification

- ▶ Information from six subdetectors combined for charged particle identification

[I. Adachi, ... SW, ... EPJC 85 (2025) 1237], [M. Abumusabh, ... SW, ... arXiv:2606.26274]

- ▶ **Neural Network to optimally combine** this information
- ▶ **Improved K efficiency** by over 10 % at low fake rates
[I. Adachi, ... SW, ... EPJC 85 (2025) 1237]
- ▶ **Extending approach** to all six charged particle species (e , μ , π , K , p , d)



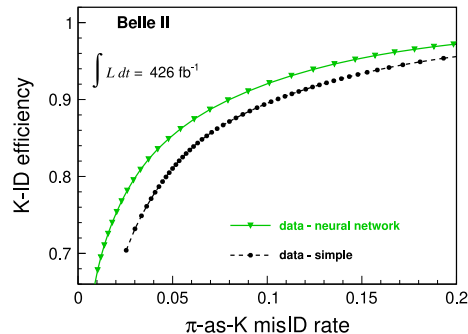
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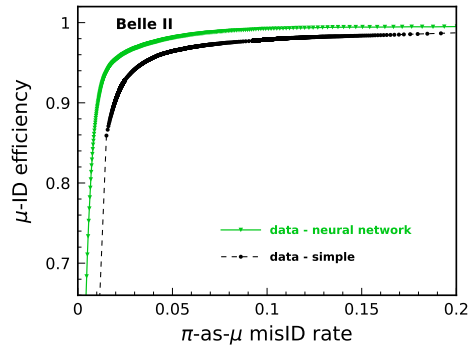
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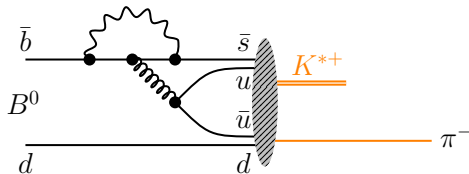


CP Asymmetries in $B \rightarrow K\pi\pi$ Decays from Dalitz Analyses

- ▶ $b \rightarrow s$ transitions have strong contribution from penguin diagrams sensitive to potential contributions from new physics
- ▶ Cancel large hadronic uncertainty in predictions for $\mathcal{B}$ and $\mathcal{A}^{CP}$ by combining decays related by isospin symmetry

$$I_{K^*\pi} = \mathcal{A}_{K^{*+}\pi^-}^{CP}$$

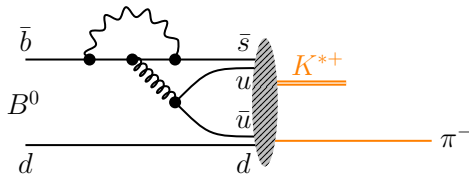
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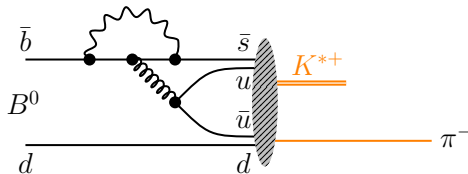
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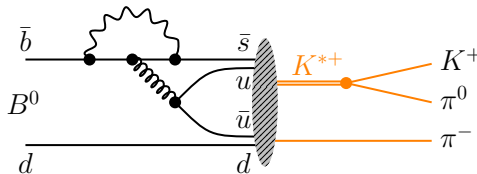
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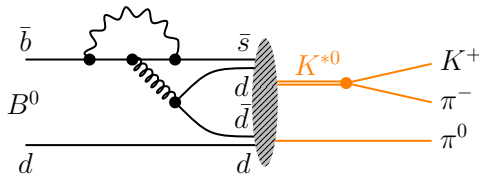
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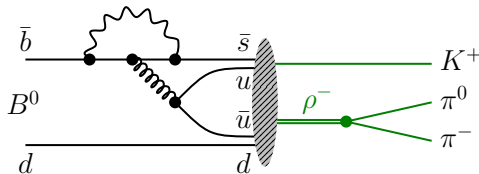
- ▶ Requires Amplitude Analysis to separate intermediate resonances



- ▶ $b \rightarrow s$ transitions have strong contribution from penguin diagrams sensitive to potential contributions from new physics
- ▶ Cancel large hadronic uncertainty in predictions for $\mathcal{B}$ and $\mathcal{A}^{CP}$ by combining decays related by isospin symmetry

$$I_{K^*\pi} = \mathcal{A}_{K^{*+}\pi^-}^{CP} + \mathcal{A}_{K^{*0}\pi^+}^{CP} \frac{\mathcal{B}_{K^{*0}\pi^+}}{\mathcal{B}_{K^{*+}\pi^-}} \frac{\tau_0}{\tau_+} - \mathcal{A}_{K^{*+}\pi^0}^{CP} \frac{2\mathcal{B}_{K^{*+}\pi^0}}{\mathcal{B}_{K^{*+}\pi^-}} \frac{\tau_0}{\tau_+} - \mathcal{A}_{K^{*0}\pi^0}^{CP} \frac{2\mathcal{B}_{K^{*0}\pi^0}}{\mathcal{B}_{K^{*+}\pi^-}} = \mathcal{O}(1\%)$$

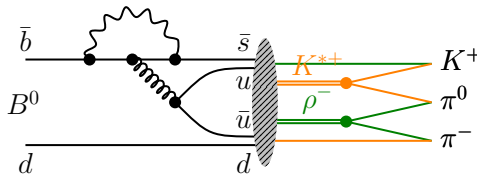
- ▶ K^* decays to $K\pi$
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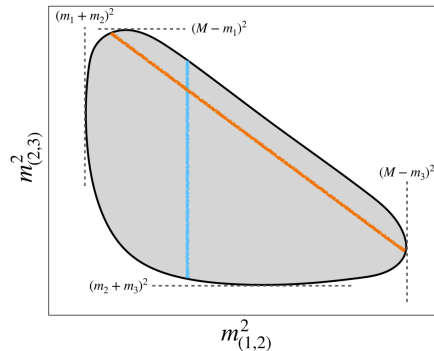
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- ▶ $K\pi\pi$ dynamics encoded in 2D Dalitz-Plot distribution
- ▶ Model for Dalitz-Plot distribution

$$I(m_{12}^2, m_{23}^2) = \left| \sum_a c_a \psi_a(m_{12}^2, m_{23}^2) \right|^2$$

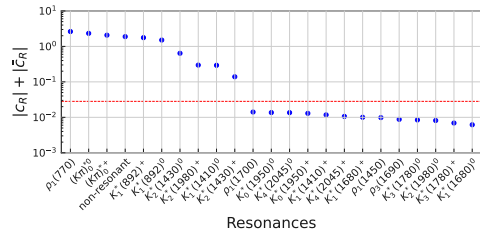
- ▶ Determine c_a separately for B and $\bar{B}$ decays
- ▶ Set of considered amplitudes $\sum_a$ determined from data
 - ▶ Based on large pool of amplitudes with **minimal assumptions**
 - ▶ Using regularization-based model-selection techniques
 - ▶ Agnostic to CP asymmetry



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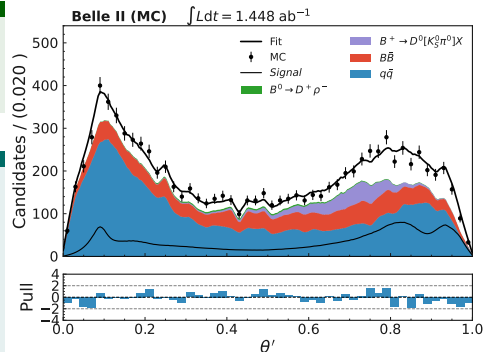
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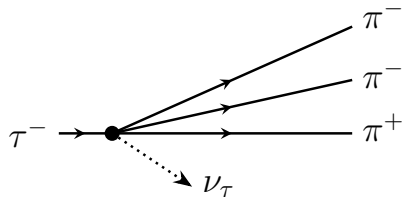
$$I_{K^*\pi} = x \pm 0.29 \text{ (stat)} \pm 0.10 \text{ (sys)}$$

- ▶ First self-consistent determination of $I_{K^*\pi}$
- ▶ Almost doubled number of considered resonances compared to previous measurements
 - ▶ 9 resonances in $B^+ \rightarrow K_S^0 \pi^+ \pi^0$
 - ▶ 13 resonances in $B^0 \rightarrow K^+ \pi^- \pi^0$
- ▶ Limited by statistical precision

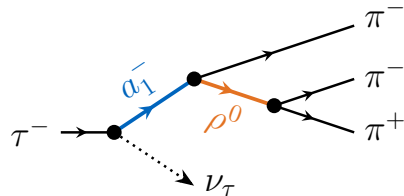


Meson Spectroscopy in $\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$ Decays

- ▶ $\pi^- \pi^- \pi^+$ system forms meson resonances
- ▶ Dominated by a_1 decays
 - ▶ Parameters of $a_1(1260)$ poorly known
 - ▶ Important input, e.g., for measuring angle ϕ_2 of CMK unitarity triangle in $B \rightarrow \rho\rho$
- ▶ Search for isospin-violating decays

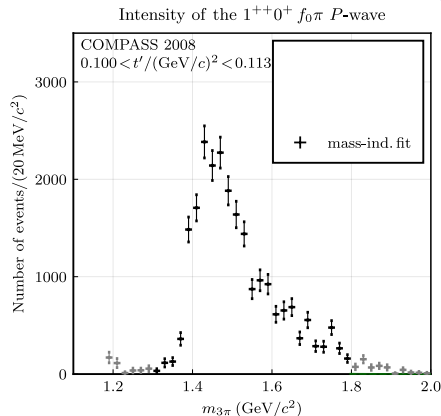


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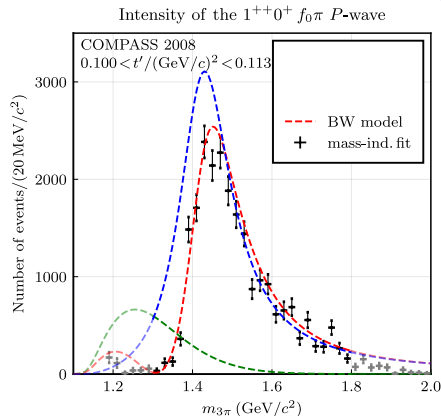
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- ▶ $a_1(1420)$ signal observed by COMPASS in $\pi^- + p \rightarrow \pi^- \pi^- \pi^+ + p$
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 - ▶ Decay to $f_0 \pi$
- ▶ Can be parameterized by genuine resonance (Breit-Wigner amplitude)
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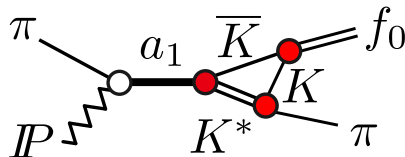
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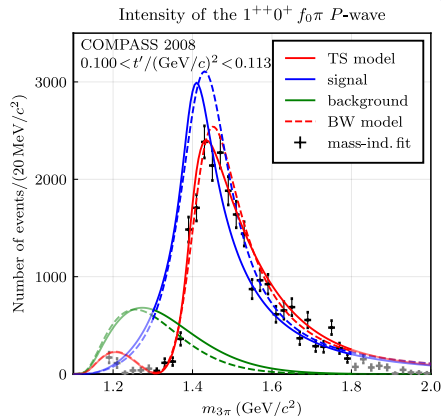
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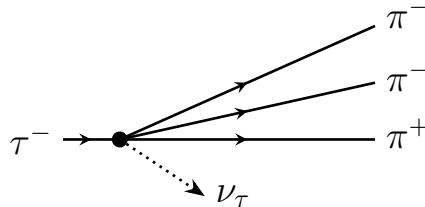
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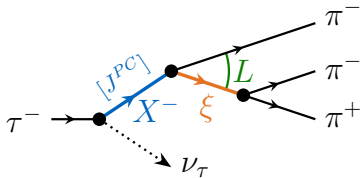
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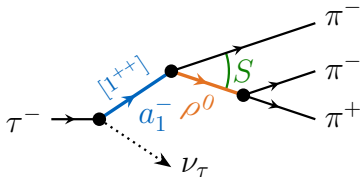
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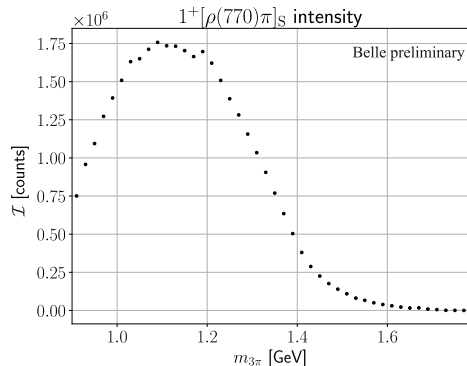


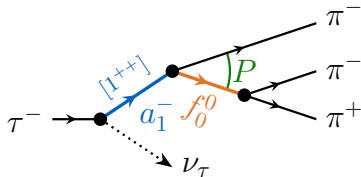


- ▶ Perform amplitude analysis to measure $m_{3\pi}$ dependence of partial-wave amplitudes
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 - ▶ Clear $a_1(1260)$ signal in $1^{++}[\rho(770)\pi]_S$ wave
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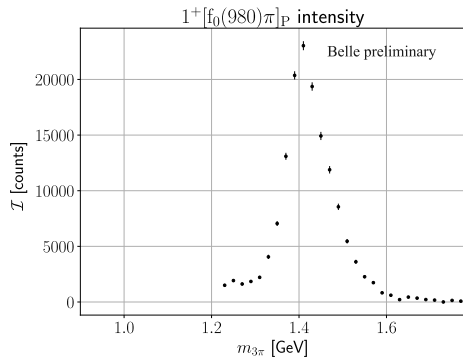


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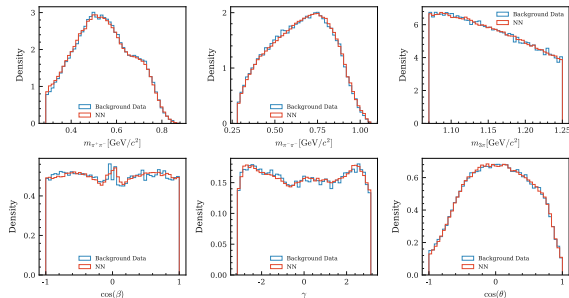


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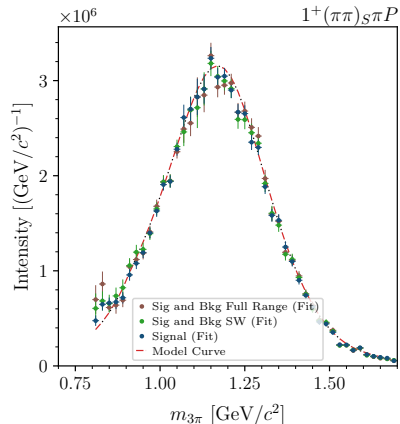
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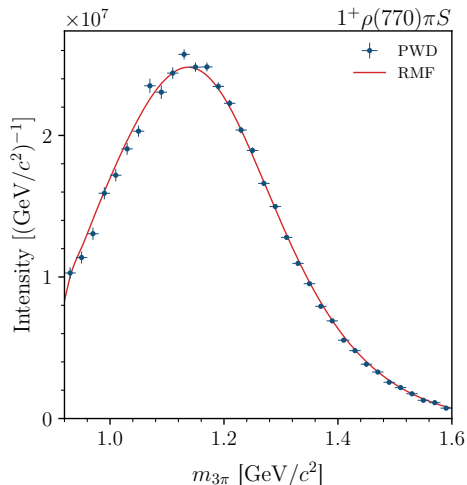


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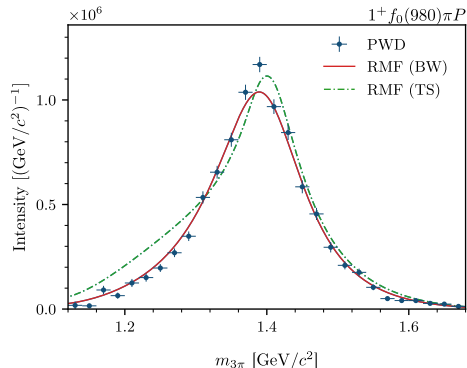
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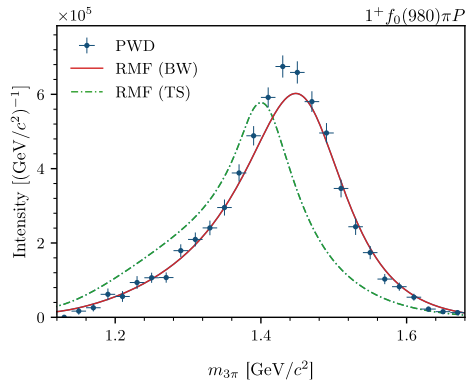
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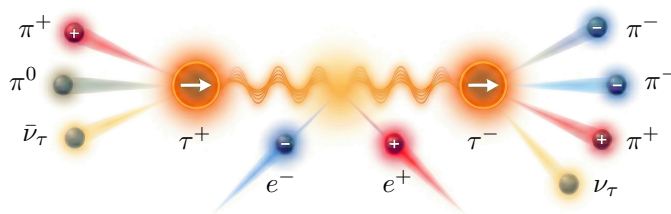
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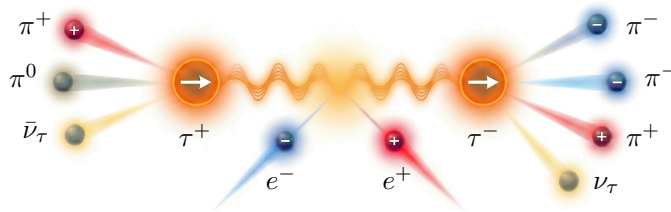
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Quantum Spin Tomography of τ Pairs

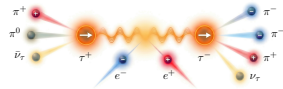
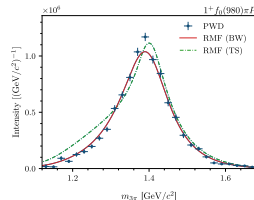
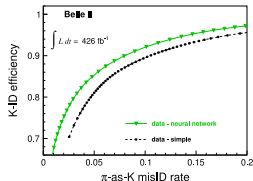
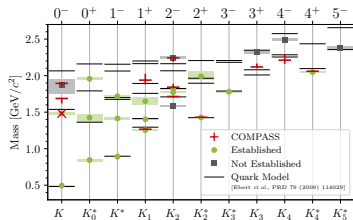


- ▶ τ leptons are **produced in pairs** and **decay in well-separated hemispheres**
- ▶ Described by a **joint spin quantum state**
- ▶ Measuring this quantum state (tomography)
 - Test fundamental properties of quantum state such as entanglement
 - Measure properties of τ leptons (EDM/MDM)
 - Search for new physics



- ▶ Measure τ spin state **through they decays to hadrons** (assuming SM decay)
- ▶ Requires to know hadronic dynamics (hadronic current) of τ decays
 - ▶ Known for $\tau^- \rightarrow \pi^- \nu_\tau$ and $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ (neglecting scalar part)
 - ▶ **Requires modeling of hadronic dynamics for $\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$**
- ▶ Toy studies demonstrate that Belle II is sensitive to observe entanglement of τ pair state
[PRD 109 (2024) 032005]

- ▶ Era of high-precision hadron spectroscopy at various experiments
- ▶ Measured **12 strange-mesons** using world's largest data sample on $K^- \pi^- \pi^+$ from COMPASS
 - ▶ First candidate for exotic strange-meson signal with $J^P = 0^-$
- ▶ Precision tests of **multi-body hadronic decays** rely on understanding the hadronic dynamics
- ▶ **Hadronic τ decays** provide unique possibility for meson spectroscopy and beyond
- ▶ **Comprehensive measurements and close collaboration between theory and experiment** required
 - ▶ DEmocratizing MOdelS (DEMOS) and SHARP to make hadronic models FAIR



Backup

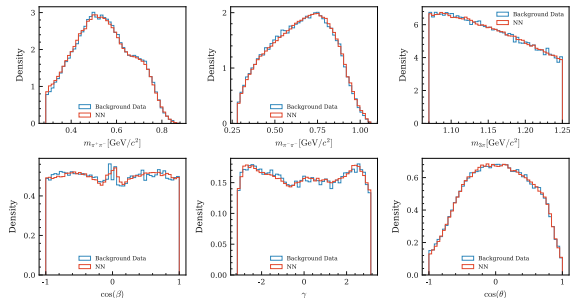
- 14 $\tau \rightarrow 3\pi\nu_\tau$ Amplitude Analysis at Belle II
- 15 Quantum Spin Tomography of τ Pairs
 - Quantum Observables
- 16 $K^-\pi^-\pi^+$ Partial-Wave Analysis
- 17 The COMPASS Experiment
- 18 Partial-Wave Decomposition
 - Treating the $\pi^-\pi^-\pi^+$ and Other Backgrounds
- 19 Resonance-Model Fit
 - Modeling the $K^-\pi^-\pi^+$ Signal
 - Modeling the $\pi^-\pi^-\pi^+$ Background
 - Modeling the Effective Background
 - χ^2 Fit Procedure
- 20 Wave-Set Selection
 - Regularization: LASSO
 - Regularization: Generalized Pareto
 - Regularization: Cauchy
- For the $K^-\pi^-\pi^+$ Final State
- 21 14-Wave Resonance-Model Fit
 - Searching for Exotic Strange Mesons with $J^P = 0^-$
 - Partial Waves with $J^P = 2^+$
 - Partial Waves with $J^P = 2^-$
 - K_3 and K_4 Mesons
 - K_4^* Mesons
- 22 Kinematic Distribution of $K^-\pi^-\pi^+$ Events
 - Subsystem
 - $m_{K^-\pi^-}$
 - t' Spectrum
 - Exclusivity
- 23 Systematic Studies of the Partial-Wave Decomposition
 - 14 Waves
 - Leakage Waves
- 24 Leakage Effect
- 25 Incoherent $\pi^-\pi^-\pi^+$ Background

$\tau \rightarrow 3\pi\nu_\tau$ Amplitude Analysis at Belle II



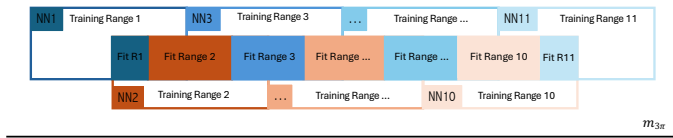
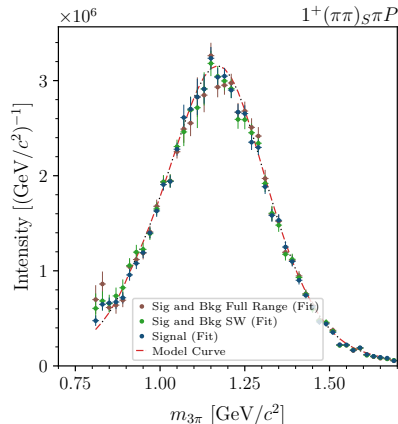
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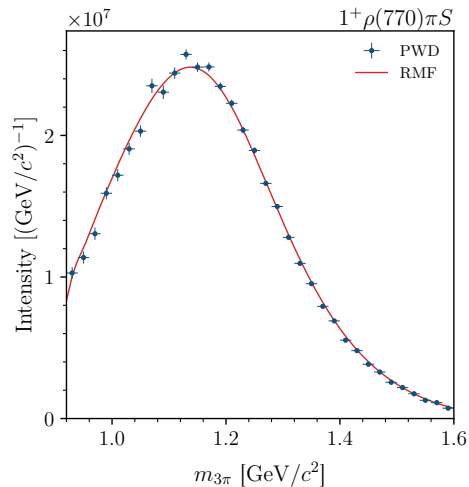


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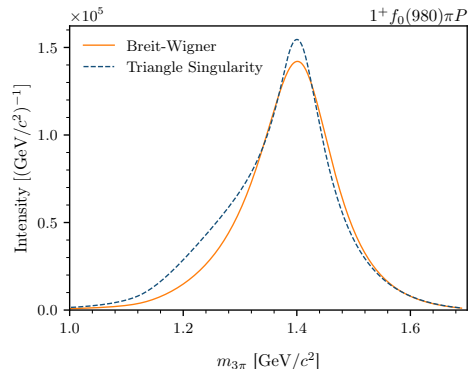
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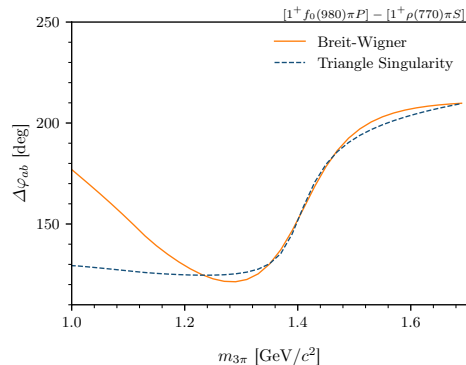
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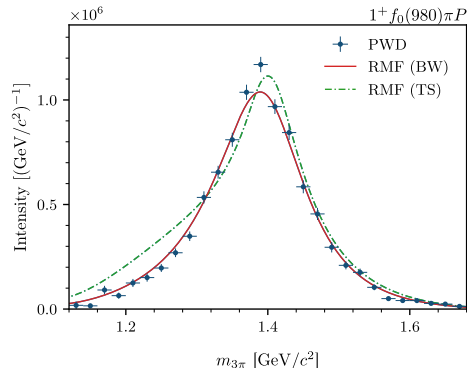
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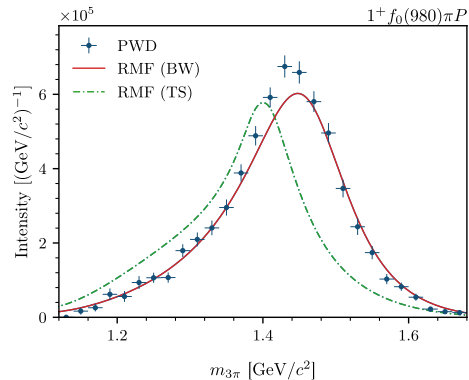
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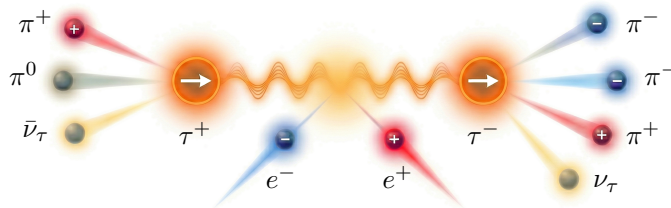


- ▶ Spin quantum state described by **spin-density matrix**

$$\rho = \frac{1}{4} \left[\mathbb{1} \otimes \mathbb{1} + \sum_i \mathbb{B}_i^+ (\sigma_i \otimes \mathbb{1}) + \sum_j \mathbb{B}_j^- (\mathbb{1} \otimes \sigma_j) + \sum_{i,j} \mathbb{C}_{ij} (\sigma_i \otimes \sigma_j) \right].$$

- ▶ Parameterized in terms of
 - ▶ $\tau^\mp$ polarization vectors $\mathbb{B}^\mp$
 - ▶ Spin-correlation matrix $\mathbb{C}$

Goal: Measure $\mathbb{B}^\mp$ and $\mathbb{C}$

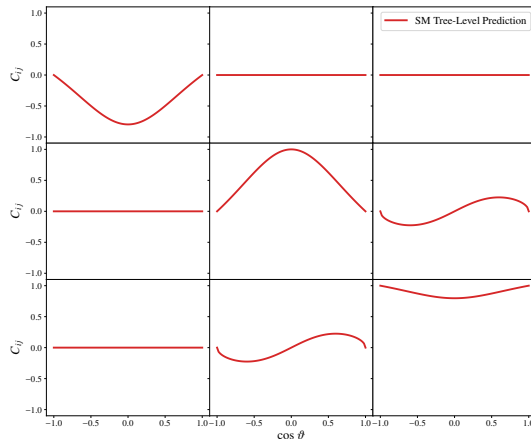


- Measure τ spin state **through they decays to hadrons** (assuming SM decay) [[PRD 109 \(2024\) 032005](#)]

$$\frac{d\sigma}{d\Phi} = |\bar{\mathcal{M}}_p|^2 |\bar{\mathcal{M}}_d^+|^2 |\bar{\mathcal{M}}_d^-|^2 (1 + \mathbb{B}^+ \cdot h^+ + \mathbb{B}^- \cdot h^- + h^+ \cdot \mathbb{C} \cdot h^-).$$

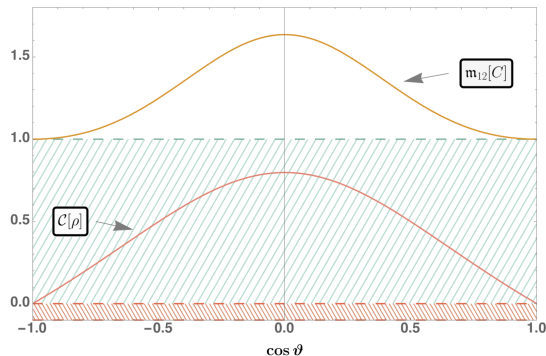
- Polarimeter vectors $h^\mp$ calculated from dynamics (hadronic current) of $\tau^\mp$ decay
 - Known for $\tau^- \rightarrow \pi^- \nu_\tau$ and $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ (neglecting scalar part)
 - **Requires modeling of hadronic dynamics for $\tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau$**
- Toy studies by [K. Ehatäht et al.](#) demonstrate that Belle II is sensitive to observe entanglement and Bell inequality violation of τ -pair spin state

- ▶ Produced quantum state depends on τ -pair production angle ϑ
- ▶ Calculate observables sensitive to
 - ▶ inseparable nature of the joint quantum state (Entanglement): concurrence $\mathcal{C}[\rho]$
 - ▶ Bell inequality violation of the quantum state (non-local nature of QM): $m_{12}[\rho]$
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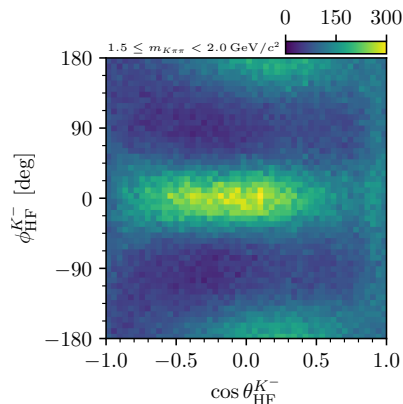
[PRD 109 (2024) 032005]

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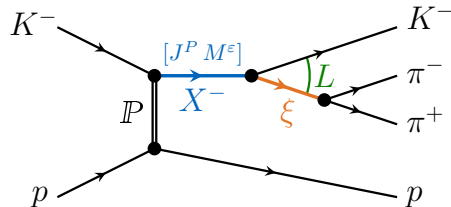
[[PRD 109 \(2024\) 032005](#)]

- ▶ Model intensity distribution in $K^-\pi^-\pi^+$ phase-space τ
- ▶ Various partial waves a contribute to data and interfere
- ▶ Factorize transition and decay of X^-
 - ▶ Decay amplitude $\psi_a(\tau)$ known
 - ▶ Transition amplitude $\mathcal{T}_a$
 - ▶ contains information on X^- resonance
 - ▶ measured from data using unbinned maximum-likelihood fit to 5D phase-space distribution
- ▶ Set of considered waves in $\sum_a$ determined using regularization-based model-selection techniques



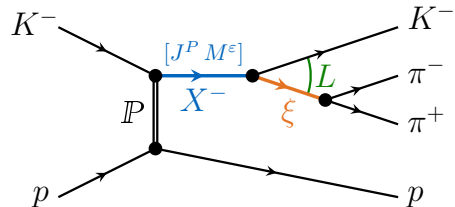
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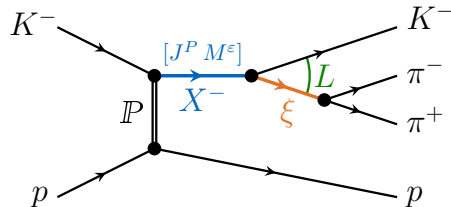
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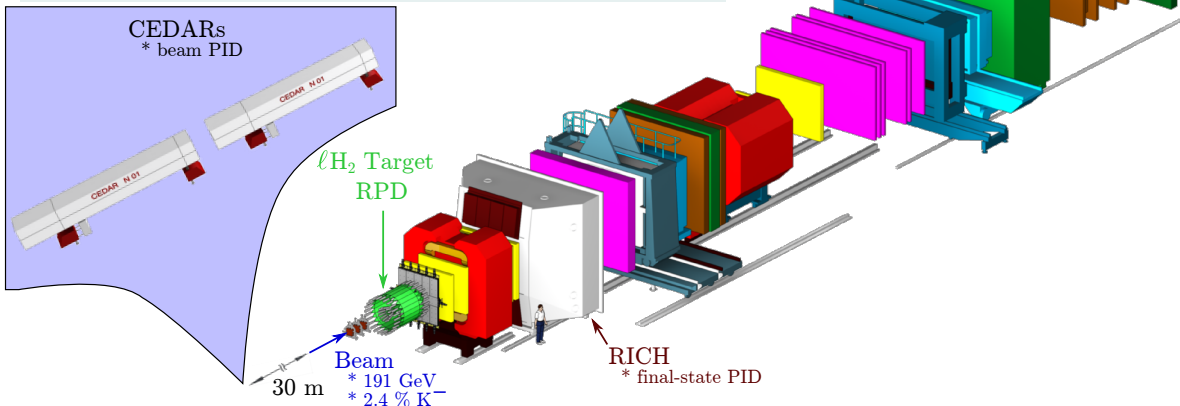


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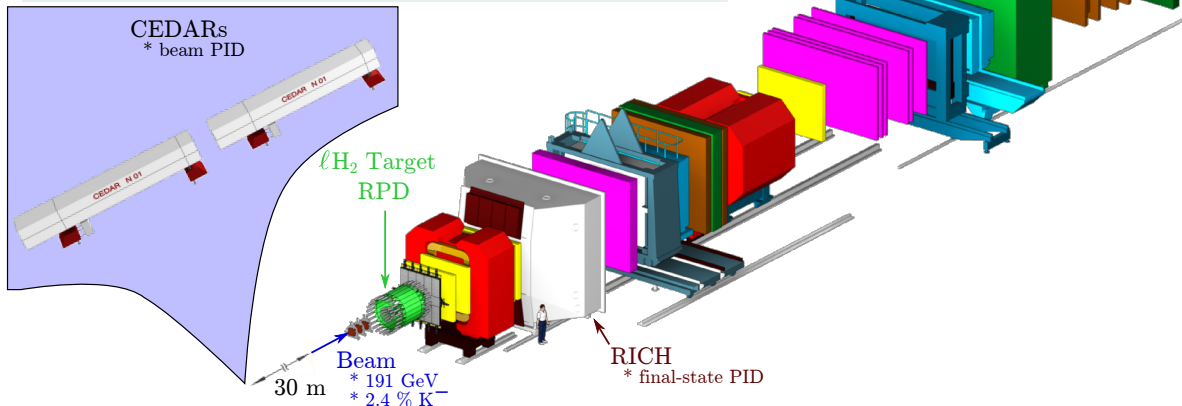


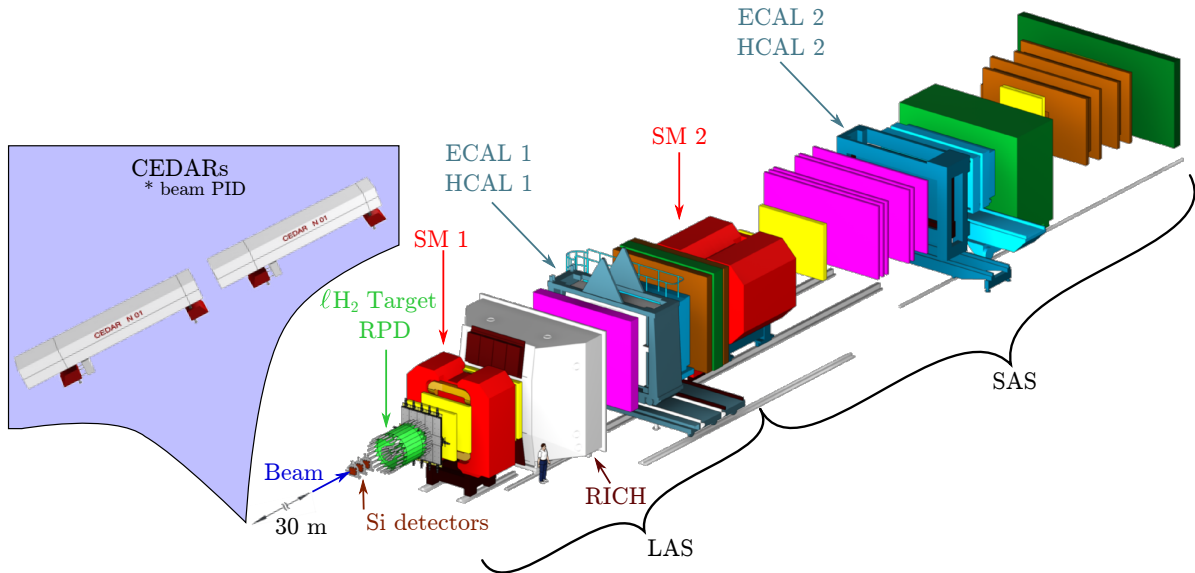
- Fixed-target experiment at M2 beam line; CERN North Area

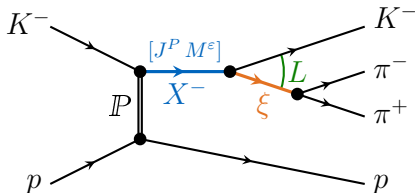
- ▶ 191 GeV negative hadron beam from M2 beam line
- ▶ About 97 % π^-
- ▶ About 2.4 % K^- identified by two Cherenkov counters



- ▶ Large acceptance two-stage magnetic spectrometer
- ▶ Electromagnetic and hadronic calorimeters
- ▶ Final-state particle identification with RICH detector



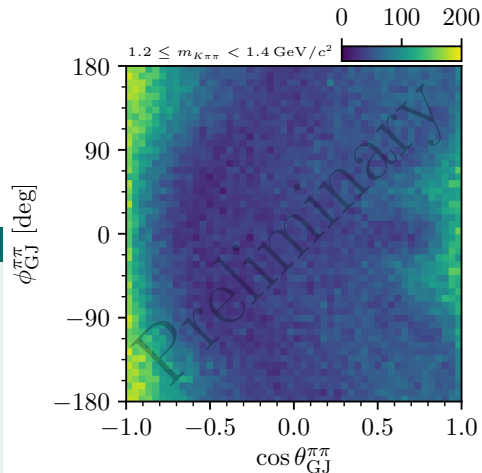




Partial wave

$$J^P M^\epsilon \xi b L$$

- ▶ $J^P M^\epsilon$: Spin, parity, and spin projection of X^-
- ▶ ξ : Isobar
- ▶ b : Bachelor particle. Here: Spectator K^-
- ▶ L : Angular momentum between bachelor and isobar



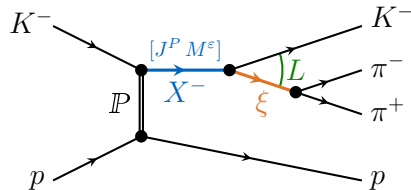
Model intensity

$$\mathcal{I}(\tau, m_{K\pi\pi}, t') = \sum_z \left| \sum_{a \in \mathbb{W}_z(m_{K\pi\pi}, t')} \mathcal{T}_a^z(m_{K\pi\pi}, t') \Psi_a^z(\tau; m_{K\pi\pi}) \right|^2$$

► Model intensity distribution

- in 5D $K^- \pi^- \pi^+$ phase-space
- for a given $(m_{K\pi\pi}, t')$ cell
- as **incoherent sum** over **coherent sectors** z
 - “Rank” of the partial-wave model = number of coherent sectors

- Ψ_a^z known, assuming the isobar model
- Wave set $\mathbb{W}_z(m_{K\pi\pi}, t')$ inferred from data using regularization-based model-selection techniques
- $\mathcal{T}_a^z$ extracted in maximum-likelihood fit, independently for each $(m_{K\pi\pi}, t')$ cell



Spin-Density Matrix

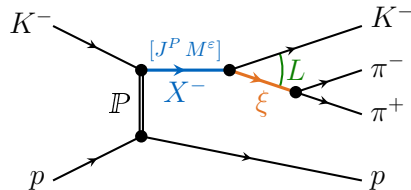
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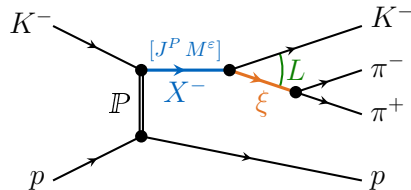
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Spin-Density Matrix

$$\rho_{ab} = \sum_z \mathcal{T}_a^z [\mathcal{T}_b^z]^*$$

Approach

- ▶ Effectively take into account in partial-wave decomposition by **incoherently adding additional coherent sectors z**
(Model background by $K^-\pi^-\pi^+$ partial waves)
 - ➡ Increasing the rank of the spin-density matrix ρ_{ab}
 - ➡ Signal not separated from background in partial-wave decomposition
 - ➡ Partial-wave amplitudes include background
- ▶ Model signal and background contributions in resonance-model fit using more constrained signal model
 - ➡ Separate signal from background

$$\mathcal{I}(\tau, m_{K\pi\pi}, t') = \sum_z \left| \sum_{a \in \mathbb{W}_z(m_{K\pi\pi}, t')} \mathcal{T}_a^z(m_{K\pi\pi}, t') \Psi_a^z(\tau; m_{K\pi\pi}) \right|^2$$

$$\rho_{ab} = \sum_z \mathcal{T}_a^z [\mathcal{T}_b^z]^*$$

True physics intensity distribution

$$\mathcal{I}(\tau) = \left| \sum_a^{\text{waves}} \mathcal{T}_a \Psi_a(\tau) \right|^2$$

Experimentally measured intensity distribution

$$\mathcal{I}_{\text{measured}}(\tau) = \eta(\tau) \mathcal{I}(\tau)$$

- ▶ Take into account different processes p
 - ▶ Different model intensities $\mathcal{I}^p$
 - ▶ Different experimental acceptance η^p
 - ▶ Formulated in terms of different phase-space variables τ^p
 - ▶ Jacobian terms $J(\tau^{K\pi\pi} \rightarrow \tau^p)$ from variable transformation

True physics intensity distribution for process p

$$\mathcal{I}^p(\tau) = \left| \sum_a^{\text{waves}} \mathcal{T}_a^p \Psi_a^p(\tau) \right|^2$$

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- ▶ $\eta^{\pi\pi\pi}$ from detector simulation

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- ▶ $\eta^{\pi\pi\pi}$ computationally expensive
- ▶ Different $m_{3\pi}$ bins enter one $m_{K\pi\pi}$ bin
- ▶ Other background channels: $K^-K^-K^+$, ...
 - ▶ $\mathcal{I}^?$ unknown
 - ▶ Unknown background channels

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Approximate model for process $\mathfrak{p}$ by $K^-\pi^-\pi^+$ partial waves

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Total true physics intensity distribution

$$\mathcal{I}(\tau^{K\pi\pi}) = \sum_{\mathfrak{p}} \left| \sum_a^{\text{waves}} \mathcal{T}_a^{\mathfrak{p}} \Psi_a^{K\pi\pi}(\tau^{K\pi\pi}) \right|^2$$

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Spin-density matrix with rank $N_r > 1$

$$\rho_{a,b} = \sum_p^{N_r} \mathcal{T}_a^p [T_b^p]^*$$

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► Experimentally measurable quantities are spin-density matrix elements

- ➡ Transition amplitudes $\mathcal{T}_a^p$ are only effective parameters
- ➡ Cannot determine $\mathcal{T}_a^p$ of individual processes
- ➡ Cannot separate different processes

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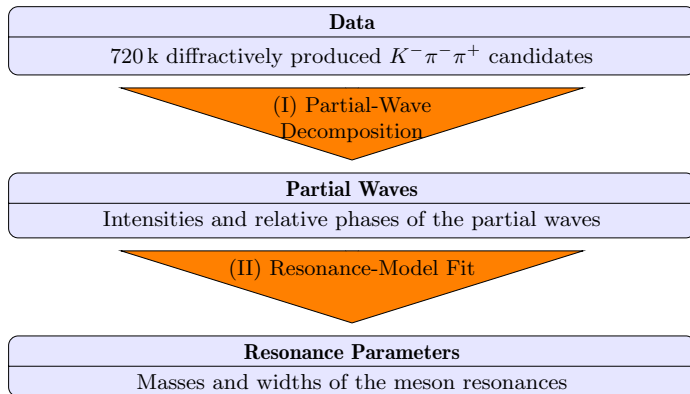
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Spin-density matrix with rank $N_r > 1$

$$\rho_{a,b} = \sum_{\mathbf{r}}^{N_r} \mathcal{T}_a^{\mathbf{r}} [T_b^{\mathbf{r}}]^*$$

- ▶ Large number of fit parameters: $N_{\text{para}} = N_r(2N_{\text{waves}} - N_r)$
- ▶ Sufficient rank of spin-density matrix must be determined
 - ▶ Rank two needed to describe pure $\pi^-\pi^-\pi^+$ Monte Carlo sample using $K^-\pi^-\pi^+$ partial waves
 - ▶ Used rank three to model $K^-\pi^-\pi^+$ sample



- ▶ Spin-density matrix $\rho_{ab}(m_{K\pi\pi}, t')$ measured in partial-wave decomposition
- ▶ Model spin-density matrix in resonance-model fit

$$\hat{\rho}_{ab}(m_{K\pi\pi}, t') = \hat{\rho}_{ab}^{K\pi\pi}(m_{K\pi\pi}, t') + \hat{\rho}_{ab}^{3\pi}(m_{K\pi\pi}, t') + \hat{\rho}_{ab}^{\text{Bkg}}(m_{K\pi\pi}, t')$$

Model transition amplitudes as coherent sum over various components

$$\hat{\mathcal{T}}_a^z(m_{K\pi\pi}, t') = \sum_{k \in \mathbb{S}_a} K(m_{K\pi\pi}, t')^k \mathcal{C}_a^{K\pi\pi}(t') \mathcal{D}_k(m_{K\pi\pi}; \zeta_k)$$

- ▶ Dynamic functions $\mathcal{D}_k(m_{K\pi\pi}; \zeta_k)$
 - ▶ For resonances: rel. Breit-Wigner
 - ▶ For non-resonant terms: $\mathcal{D}_k^{\text{NR}}(m_{K\pi\pi}; a_k, c_k) = (m_{K\pi\pi} - m_{\text{thr}})^{a_k} e^{-b(c_k)} \tilde{q}_k^2(m_{K\pi\pi})$
- ▶ “Coupling amplitudes”: ${}^k\mathcal{C}_a^z(t')$
 - ▶ Independent coupling amplitude for each t' bin
- ▶ Kinematic factor $K(m_{K\pi\pi}, t')$
- ▶ Coherently summed over all assumed model components

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 - ▶ For non-resonant terms: $\mathcal{D}_k^{\text{NR}}(m_{K\pi\pi}; a_k, c_k) = (m_{K\pi\pi} - m_{\text{thr}})^{a_k} e^{-b(c_k)} \tilde{q}_k^2(m_{K\pi\pi})$
- ▶ “Coupling amplitudes”: ${}^k\mathcal{C}_a^z(t')$
 - ▶ Independent coupling amplitude for each t' bin
- ▶ Kinematic factor $K(m_{K\pi\pi}, t')$
- ▶ Coherently summed over all assumed model components

Model transition amplitudes as coherent sum over various components

$$\hat{\mathcal{T}}_a^z(m_{K\pi\pi}, t') = \sum_{k \in \mathbb{S}_a} K(m_{K\pi\pi}, t')^k \mathcal{C}_a^{K\pi\pi}(t') \mathcal{D}_k(m_{K\pi\pi}; \zeta_k)$$

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3π spin-density matrix

$$\hat{\rho}_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t') = \left| \mathcal{C}^{\pi\pi\pi} \right|^2 \rho_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t')$$

- ▶ $\rho_{ab}^{\pi\pi\pi}(m_{K\pi\pi}, t')$ obtained from PWD of $\pi^-\pi^-\pi^+$ pseudodata sample
 - ▶ $m_{K\pi\pi}$ dependence fixed
 - ▶ t' dependence fixed
 - ▶ Rel. strength between partial waves fixed (freed in a study)
- ▶ One global real-valued yield parameter $\left| \mathcal{C}^{\pi\pi\pi} \right|^2$

Background spin-density matrix

- ▶ Additional incoherent contribution from other processes: $K^- K^- K^+$, ...
- ▶ Transition amplitudes modeled by non-resonant parameterizations for each partial wave

$$\hat{\mathcal{T}}_a^{\text{eBKG}}(m_{K\pi\pi}, t') = K(m_{K\pi\pi}, t') \mathcal{C}_a^{\text{eBKG}}(t') \mathcal{D}_{k_a}^{\text{eBKG}}(m_{K\pi\pi}; a_{k_a}, c_{k_a})$$

- ▶ χ^2 fit of the real and imaginary parts of the spin-density matrix
 - ▶ Taking into account correlations between spin-density matrix elements
 - ▶ Shape parameters (m_0 , Γ_0 , ...) and coupling amplitudes are free parameters
- ▶ For the main fit, we performed 2000 fit attempts with random start-parameter values for the shape parameters, e.g. mass and width parameters, and the coupling and branching amplitudes.
- ▶ Start-parameter ranges for the shape parameters are chosen according to previous measurements (see note)
- ▶ The best result is the one which yielded the smallest χ^2 value

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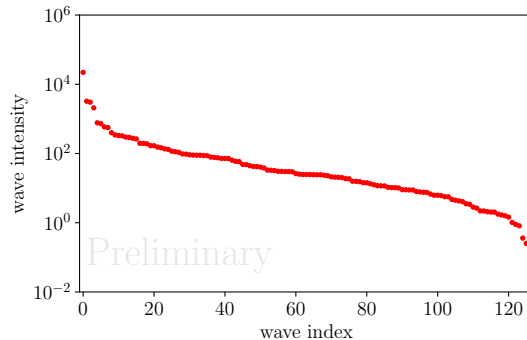
$$\mathcal{I}(\tau, m_{K\pi\pi}, t') = \left| \sum_{a \in \mathbb{W}(m_{K\pi\pi}, t')} \mathcal{T}_a(m_{K\pi\pi}, t') \Psi_a(\tau; m_{K\pi\pi}) \right|^2$$

Challenge: Find the “best” set of waves that describes the data

- ▶ If the wave set is too large
 - ➡ Starting to describe statistical fluctuations
- ▶ If waves that contribute to the data are missing
 - ➡ Intensity can be wrongly attributed to other waves
 - ➡ Model leakage

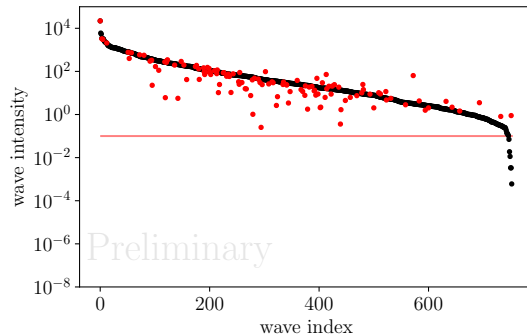
Infer wave set from data

- ▶ **Systematically construct** large set of allowed partial waves
 - ➡ “Wave pool”
- ▶ Fit wave pool to data
 - ▶ Impose penalty on $|\mathcal{T}_a|^2 \Rightarrow$ **regularization**
 - ▶ Suppress insignificant waves
- ▶ **Select waves** that significantly contribute to data
 - ➡ “Best” subset of waves that describe the data



- ▶ $\pi^- \pi^- \pi^+$ Monte Carlo mock data set with 126 partial waves
- ▶ Fitting wave pool of 753 waves
 - Massive overfitting
 - Almost all waves pick up intensity

Courtesy F. Kaspar, TUM



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$$\ln \mathcal{L}_{\text{fit}} = \ln \mathcal{L}_{\text{extended}} + \sum_a^{\text{waves}} \ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \{c_{\text{para}}\})$$

LASSO/L1 regularization¹

$$\ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \lambda) = -\lambda |\mathcal{T}_a|$$

- ▶ Maximum at $|\mathcal{T}_a| = 0$
- ▶ Well established²
- ▶ “Smoothing” at $|\mathcal{T}_a| = 0$

$$|\mathcal{T}_a| \rightarrow \sqrt{|\mathcal{T}_a|^2 + \varepsilon}$$

¹ Robert Tibshirani. “Regression Shrinkage and Selection via the Lasso”. In: Journal of the Royal Statistical Society. Series B 58.1 (1996)

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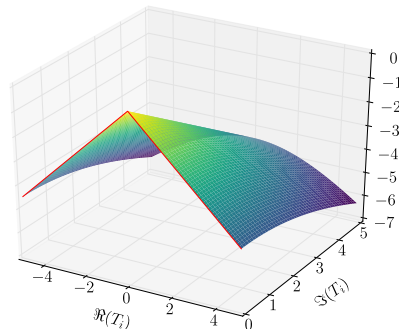
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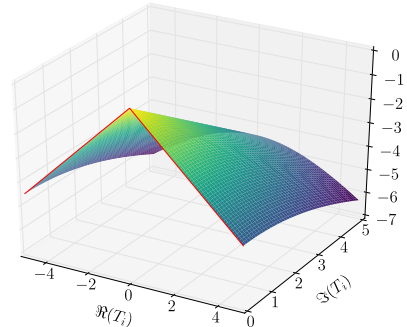
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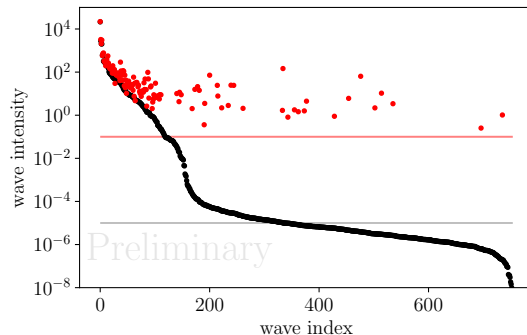
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$$\lambda = 0.3$$

$$\varepsilon = 10^{-5}$$

- Bias also on large transition amplitudes
- Some additional waves
- Some waves missing

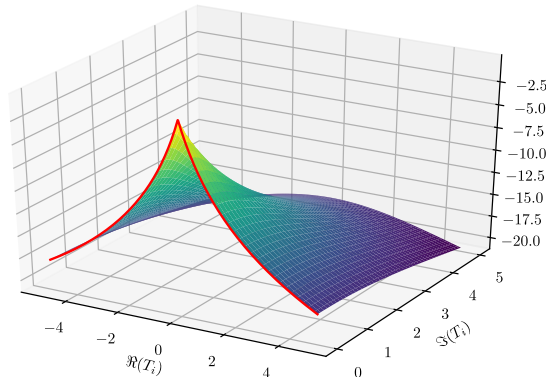
Courtesy F. Kaspar, TUM

Generalized Pareto¹

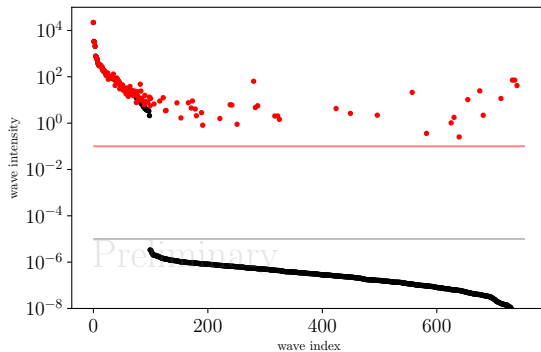
$$\ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \Gamma, \zeta) = -\frac{1}{\zeta} \ln \left[1 + \zeta \frac{|\mathcal{T}_a|}{\Gamma} \right]$$

- ▶ Wave **intensities** spread over **orders of magnitudes**
- ▶ Use **logarithmic prior**
 - ➡ Heavy-tailed
 - ➡ Less bias on large waves
- ▶ LASSO-like for $|\mathcal{T}_a| \rightarrow 0$
- ▶ “Smoothing” at $|\mathcal{T}_a| = 0$

$$|\mathcal{T}_a| \rightarrow \sqrt{|\mathcal{T}_a|^2 + \varepsilon}$$



¹ Artin Armagan, David B. Dunson, and Jaeyong Lee. “Generalized double Pareto shrinkage”. In: *Statistica Sinica* (2013). doi: 10.5705/ss.2011.048.



$$\zeta = 0.5$$

$$\Gamma = 0.1$$

$$\varepsilon = 10^{-5}$$

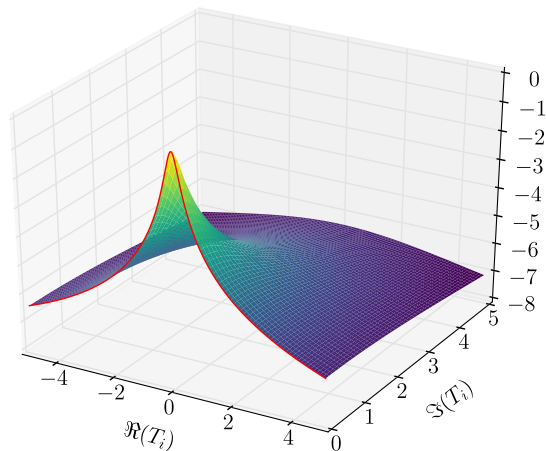
- ▶ Less bias on large transition amplitudes
- ▶ Clear **kink** in intensity distribution to smoothing scale $\Rightarrow$ Selection
- ▶ Less additional waves
- ▶ Some small waves missing

Courtesy F. Kaspar, TUM

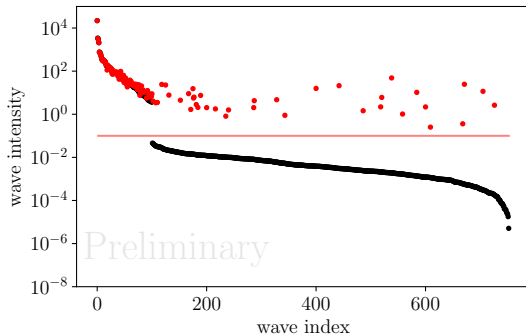
“Cauchy”

$$\ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \Gamma) = -\ln \left[1 + \frac{|\mathcal{T}_a|^2}{\Gamma_a^2} \right]$$

- ▶ Logarithmic prior
- ▶ L2-like for $|\mathcal{T}_a| \rightarrow 0$



$$\Gamma = 0.2$$



- ▶ Less bias on large transition amplitudes
- ▶ Clear kink in intensity distribution
- ▶ Few additional waves
- ▶ Few small waves missing

Courtesy F. Kaspar, TUM

Wave pool

- ▶ Spin $J \leq 7$
- ▶ Angular momentum $L \leq 7$
- ▶ Positive naturality of exchange particle
- ▶ 12 isobars
 - ▶ $[K\pi]_S^{K\pi}$, $[K\pi]_S^{K\eta}$, $K^*(892)$, $K^*(1680)$, $K_2^*(1430)$, $K_3^*(1780)$
 - ▶ $[\pi\pi]_S$, $f_0(980)$, $f_0(1500)$, $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$

⇒ “Wave pool” of 596 waves

“only” 720 k events

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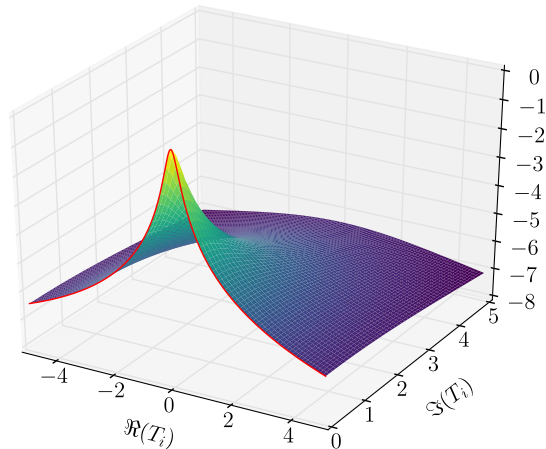
Regularization

$$\ln \mathcal{L}_{\text{reg}}(|\mathcal{T}_a|; \Gamma) = -\ln \left[1 + \frac{|\mathcal{T}_a|^2}{\Gamma_a^2} \right]$$

- Use Cauchy regularization
- Scale of $|\mathcal{T}_a|$ depends on experimental acceptance
 - Apply penalty on expected number $\bar{N}_a$ of observed events

$$\Gamma_a = \frac{\Gamma}{\sqrt{\eta_a}} \Rightarrow \frac{|\mathcal{T}_a|^2}{\Gamma_a^2} = \frac{\bar{N}_a}{\Gamma^2}$$

- Γ is a universal parameter



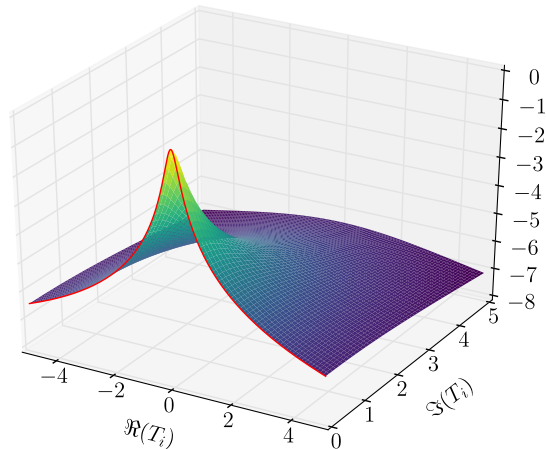
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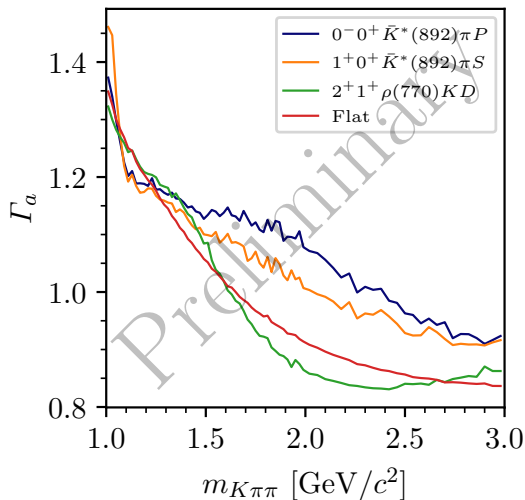
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COMPASS



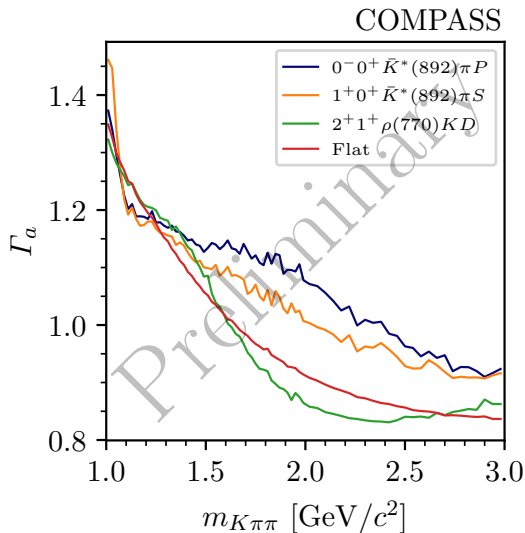
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Imposing continuity of the wave set

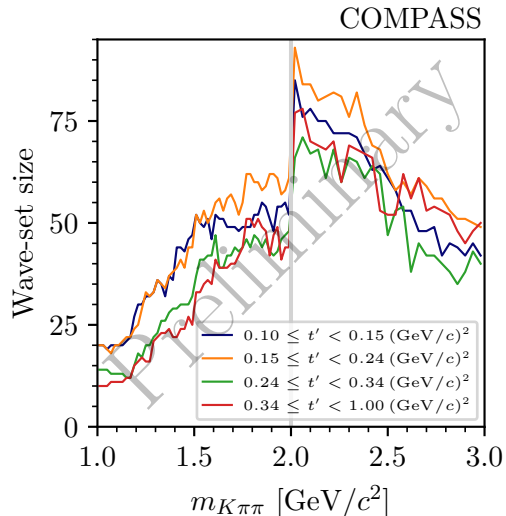
- ▶ Wave-set inferred independently for each $(m_{K\pi\pi}, t')$ cell
- ▶ Impose continuity of the wave set in $m_{K\pi\pi}$ by adding additional regularization term

$$\ln \mathcal{L}_{\text{cont}}(\{\mathcal{T}_a(m_{K\pi\pi}, t')\}; \lambda) = \sum_{j=i-3}^{j=i+3} \lambda \left| \mathcal{T}_a(m_{K\pi\pi}, t')(m_{K\pi\pi}^{j+1}) - \mathcal{T}_a(m_{K\pi\pi}, t')(m_{K\pi\pi}^j) \right|^2,$$

which suppresses fluctuations among neighboring $m_{K\pi\pi}$ bins

Wave-set size

- ▶ 5 to 90 waves per $(m_{K\pi\pi}, t')$ cell
- ▶ Larger wave set for larger binning in $m_{K\pi\pi}$
- ▶ Larger wave set in t' bins with more events

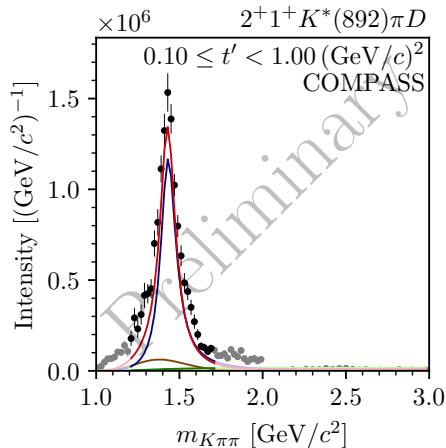


Wave-Set Selection

For the $K^-\pi^-\pi^+$ Final State



- Selection of large signals
- as well as of signals at per-mil level

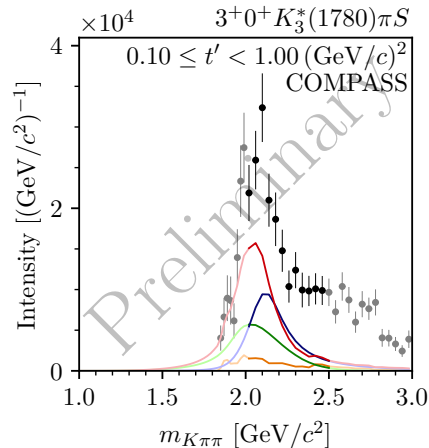


Wave-Set Selection

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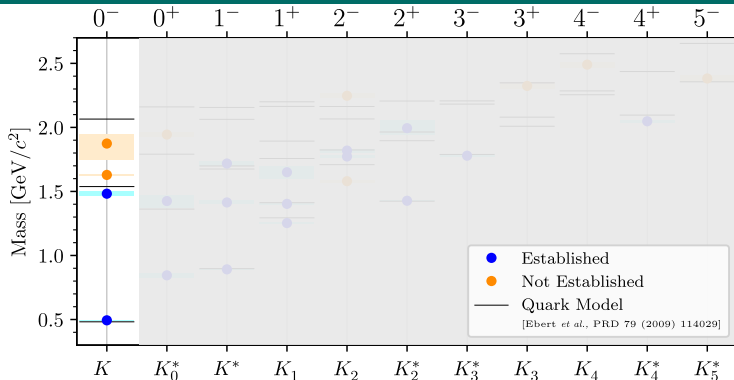


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14-Wave Resonance-Model Fit

Searching for Exotic Strange Mesons with $J^P = 0^-$



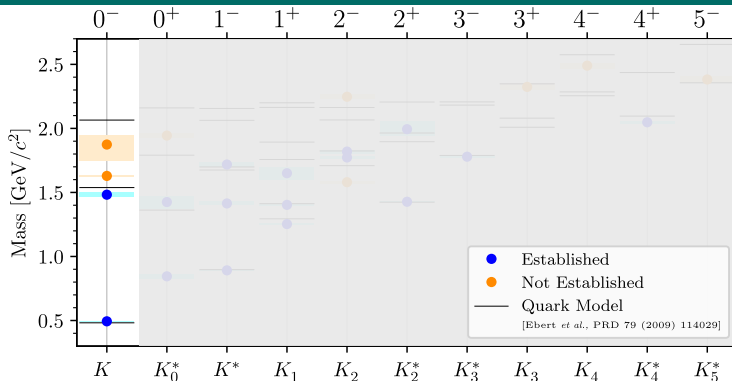
PDG

(2022)

- ▶ $K(1460)$ and $K(1830)$
- ▶ $K(1630)$
 - ▶ Unexpectedly small width of only $16 \text{ MeV}/c^2$
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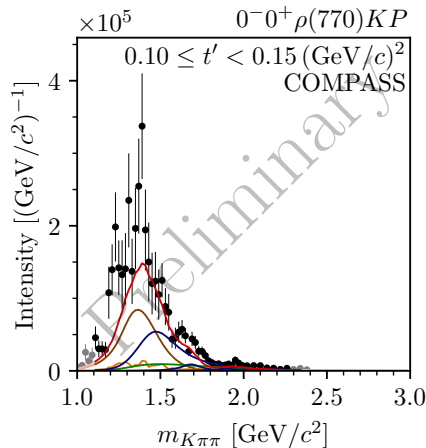
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- ▶ Peak at about $1.4 \text{ GeV}/c^2$
 - ▶ Potentially from established $K(1460)$
 - ▶ But, $m_{K\pi\pi} \lesssim 1.5 \text{ GeV}/c^2$ region affected by analysis artifacts
- ▶ Second peak at about $1.7 \text{ GeV}/c^2$
 - ▶ $K(1630)$ signal with 8.3σ statistical significance
 - ▶ Accompanied by rising phase
- ▶ Weak signal at about $2.0 \text{ GeV}/c^2$
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total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background

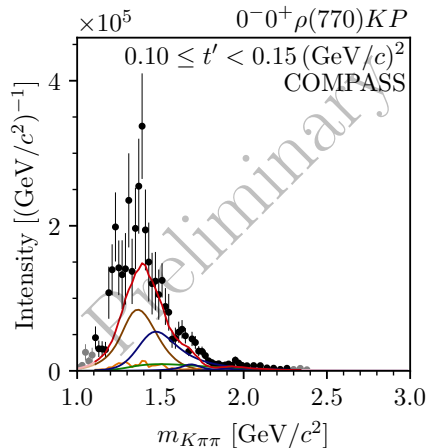
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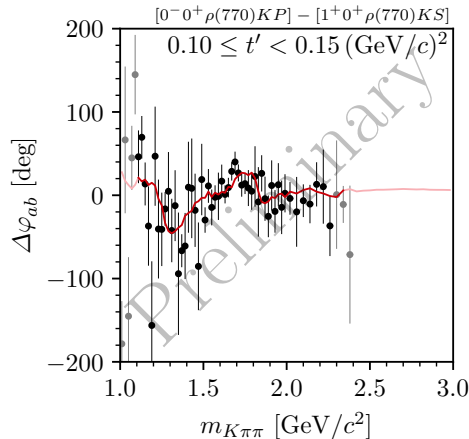
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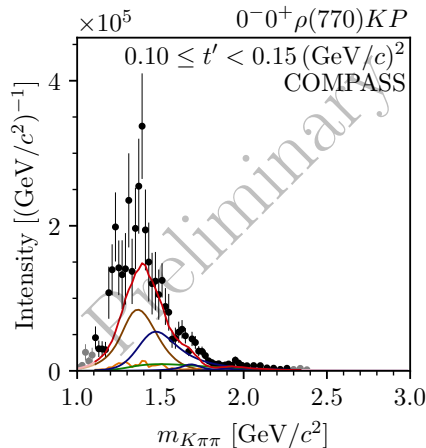
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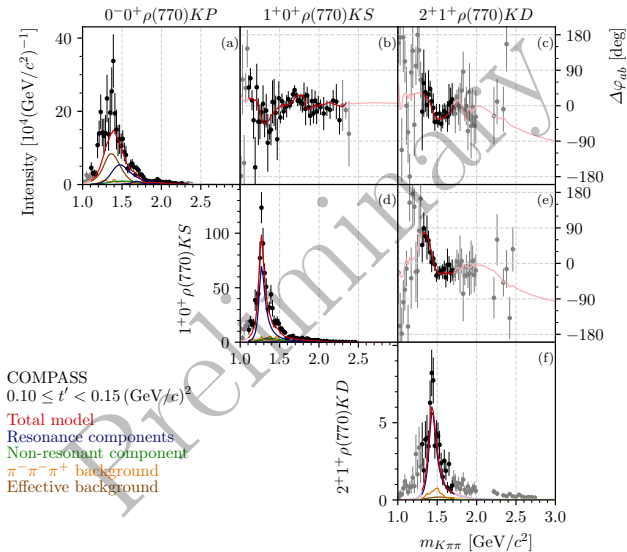
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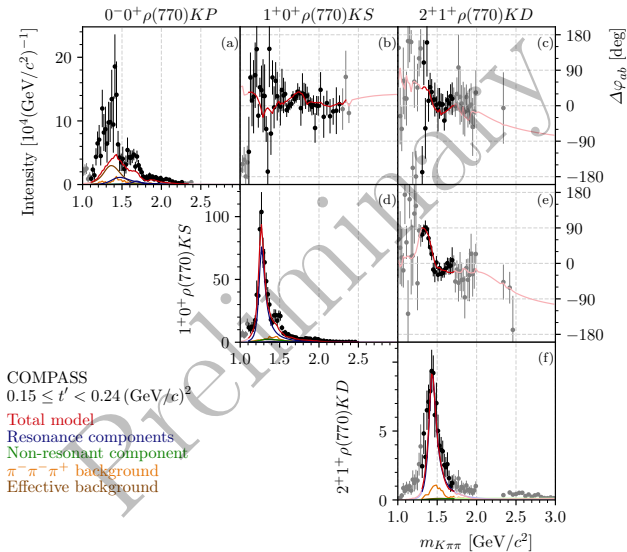
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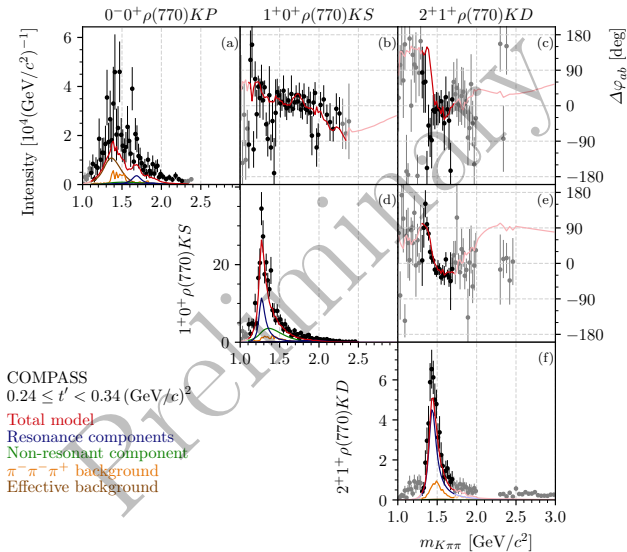
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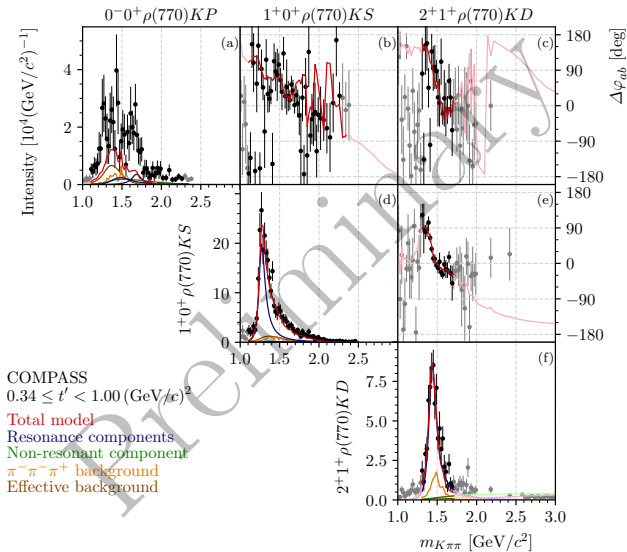
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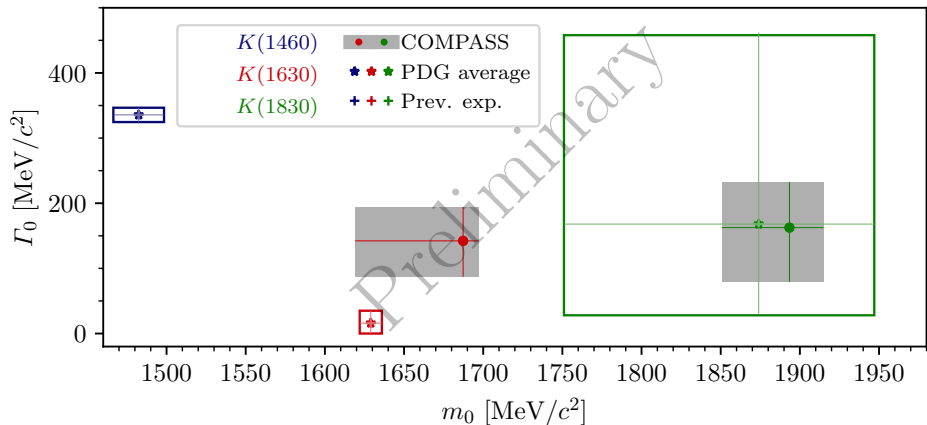
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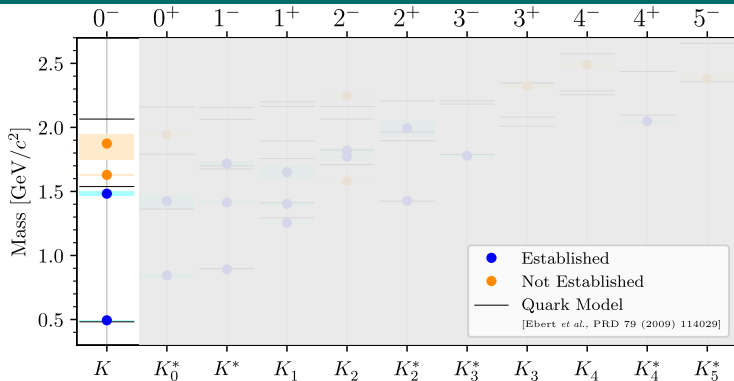
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- ▶ $K(1830)$ parameters in good agreement with LCHb measurement [PRL 118 (2017) 022003]
- ▶ Realistic $K(1630)$ width of about 140 MeV/ c^2

14-Wave Resonance-Model Fit

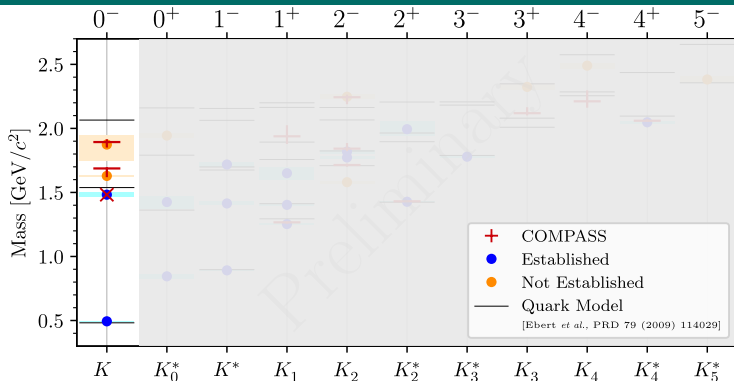
Searching for Exotic Strange Mesons with $J^P = 0^-$



- ▶ Indications for 3 excited K from a single analysis
- ▶ Quark-model predicts only two excited states: potentially $K(1460)$ and $K(1830)$
 - ➡ $K(1630)$ supernumerary signal
 - ➡ Candidate for **exotic non- $q\bar{q}$ state**; other explanations possible ($K^*(892)$ ω threshold nearby)

14-Wave Resonance-Model Fit

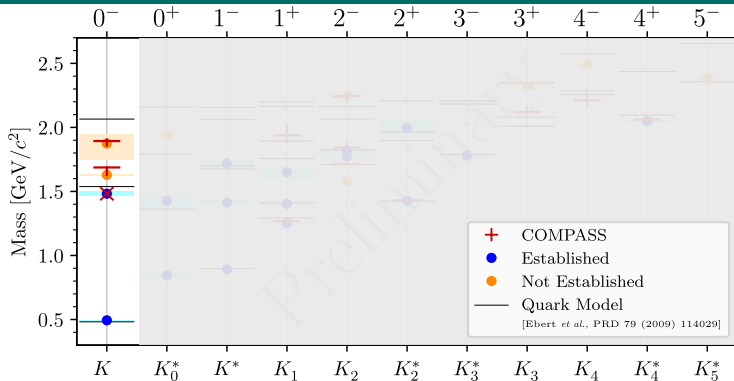
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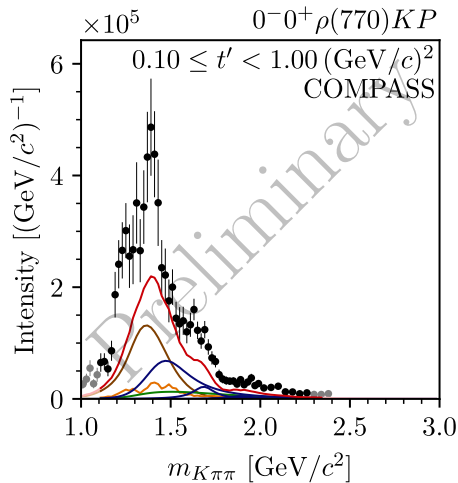
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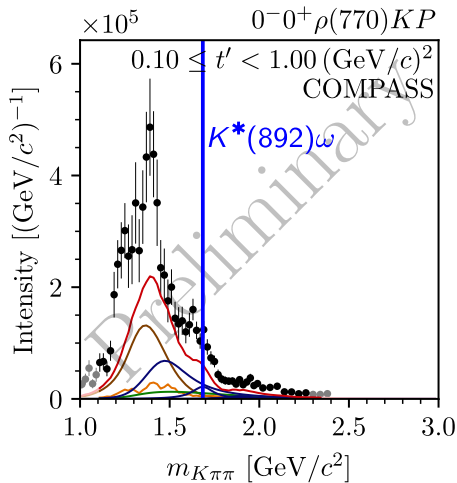
14-Wave Resonance-Model Fit

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14-Wave Resonance-Model Fit

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14-Wave Resonance-Model Fit

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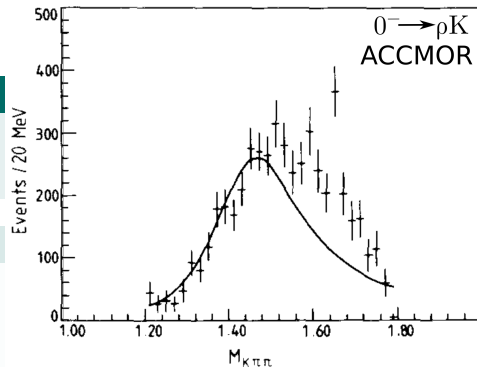


$K^-\pi^-\pi^+$ from ACCMOR

- Potential $K(1630)$ signal already in ACCMOR analysis

$K^-\pi^-\pi^+$ from LHCb

- Measurement of $D^0 \rightarrow K^\mp \pi^\pm \pi^\pm \pi^\mp$ at LHCb
 - Study strange mesons in $K\pi\pi$ subsystem
 - MIPWA of $J^P = 0^-$ amplitude
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 - Limited by kinematic range



14-Wave Resonance-Model Fit

Searching for Exotic Strange Mesons with $J^P = 0^-$

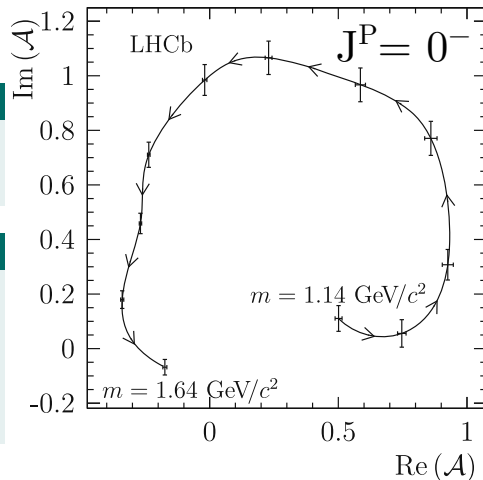


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14-Wave Resonance-Model Fit

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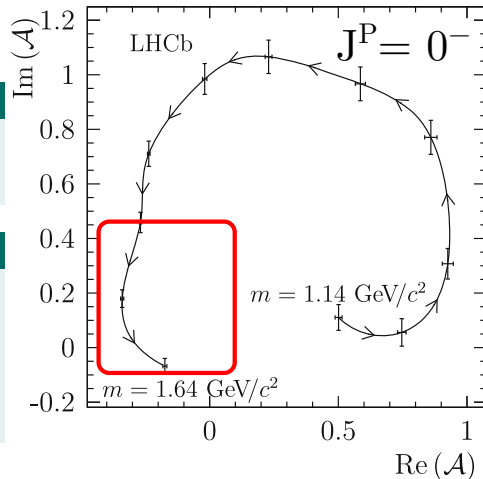


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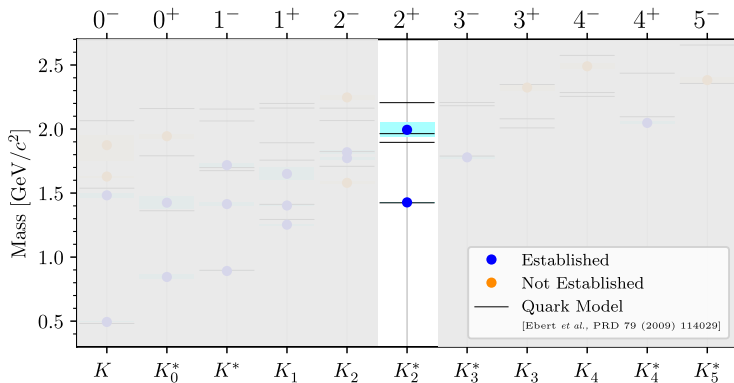
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14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^+$



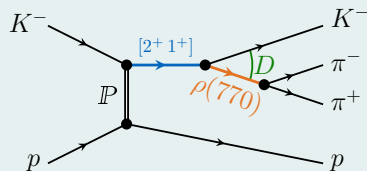
PDG

(2022)

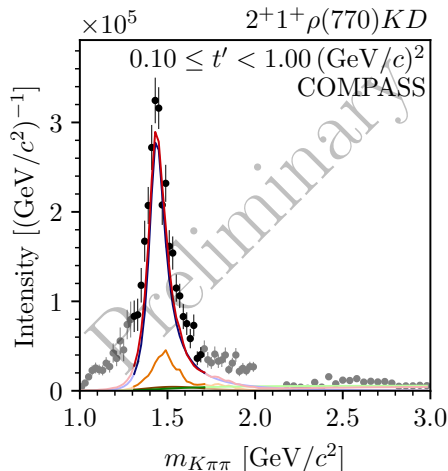
► $K_2^*(1430)$ well known resonance

14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^+$



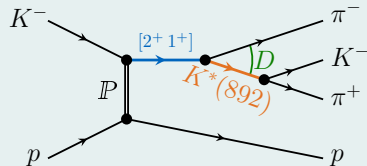
- ▶ Signal in $K_2^*(1430)$ mass region
- ▶ In different decays
 - ▶ $\rho(770) K D$
 - ▶ $K^*(892) \pi D$
- ▶ In agreement with previous measurements
- ▶ Cleaner signal in COMPASS data



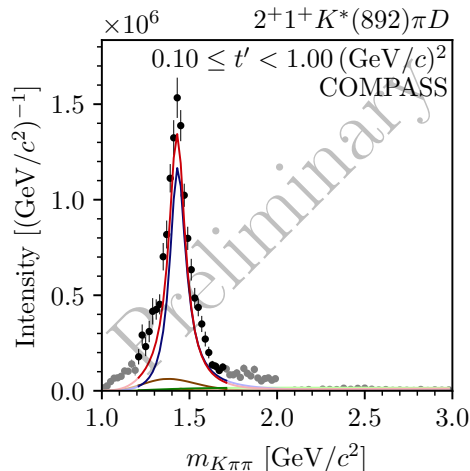
total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background

14-Wave Resonance-Model Fit

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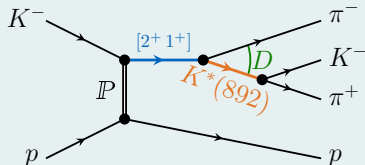
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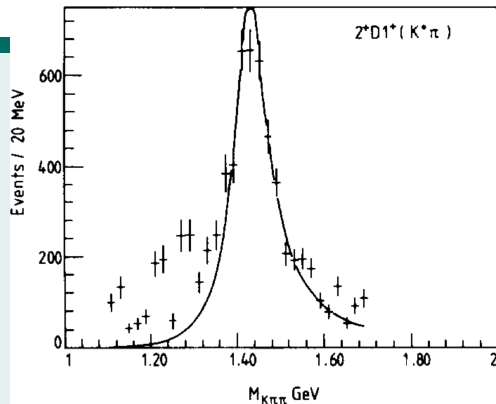
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14-Wave Resonance-Model Fit

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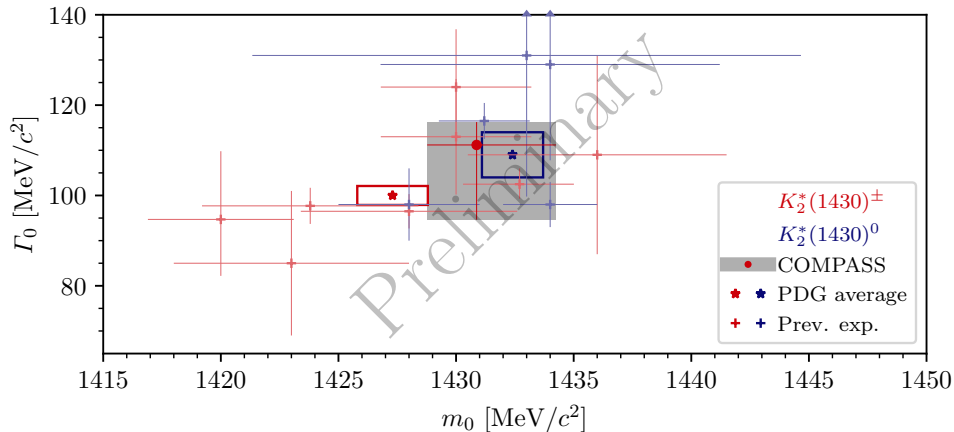


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14-Wave Resonance-Model Fit

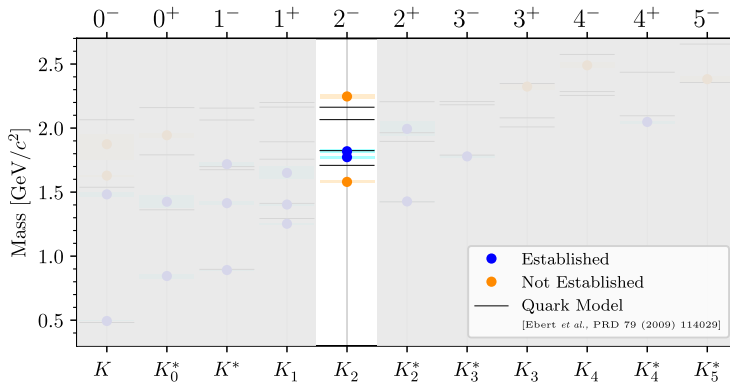
Partial Waves with $J^P = 2^+$



- ▶ $K_2^*(1430)$ parameters consistent with previous observations
- ▶ Better agreement with PDG average values for neutral $K_2^*(1430)$

14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^-$



PDG

(2022)

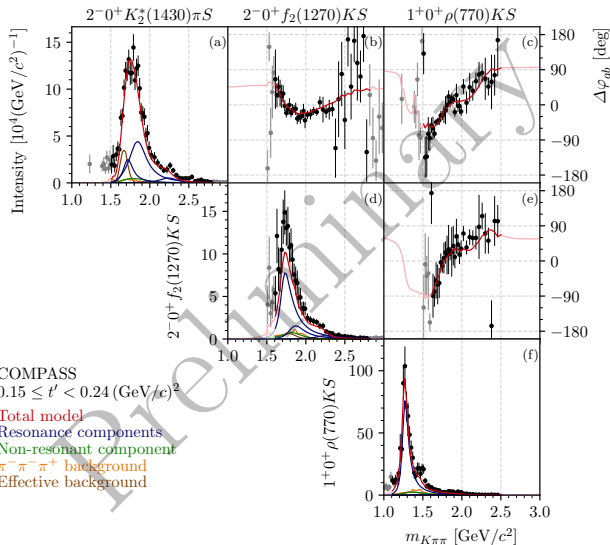
- ▶ Established $K_2(1770)$ and $K_2(1820)$
- ▶ $K_2(2250)$ need further confirmation

14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^-$



- ▶ Simultaneously fit 4 waves with $J^P = 2^-$
- ▶ $1.8 \text{ GeV}/c^2$ peak modeled by $K_2(1770)$, $K_2(1820)$
- ▶ High-mass shoulder modeled by $K_2(2250)$
- ▶ Different intensity spectra and large phase motions among 2^- waves

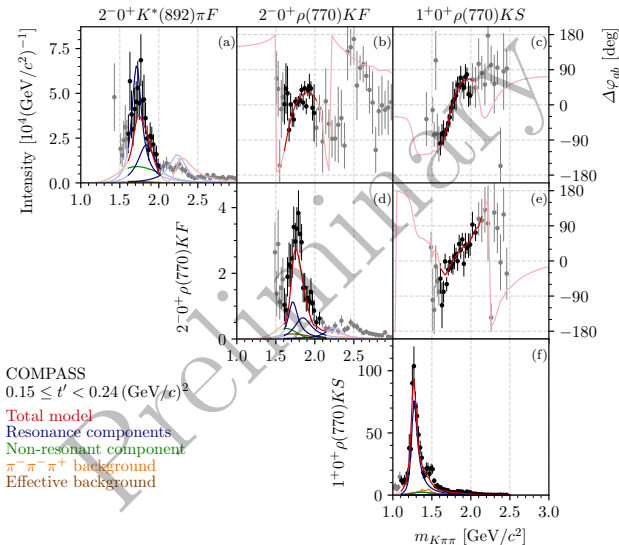


14-Wave Resonance-Model Fit

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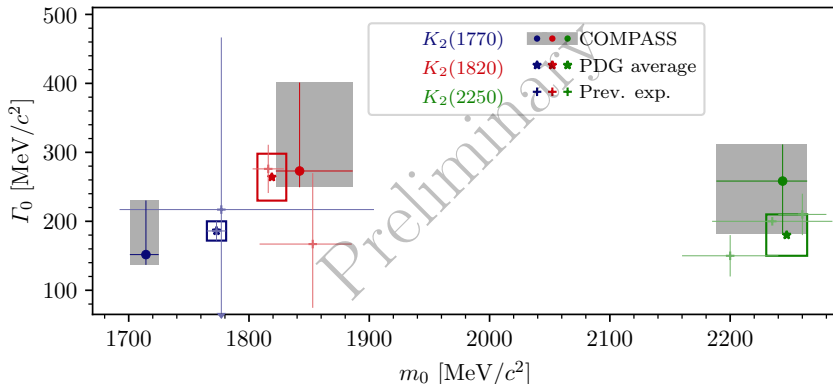


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14-Wave Resonance-Model Fit

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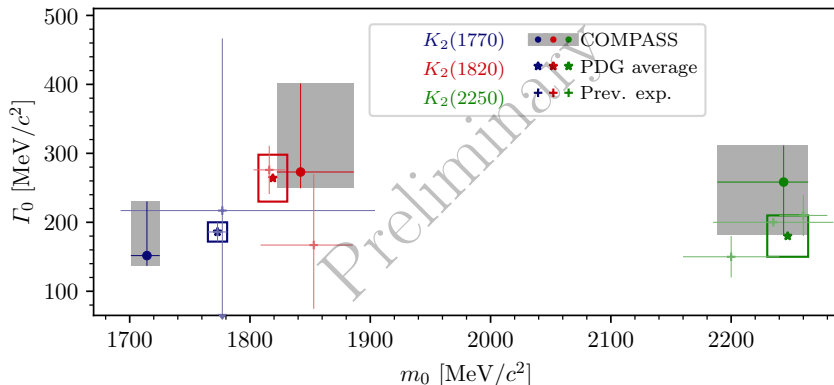


$K_2(1770)$ and $K_2(1820)$

- ▶ Two states were considered by only three measurements ACCMOR, LASS, LHCb
- ▶ Only LHCb measurement could confirm two states (3σ statistical significance)
- ▶ We observe two states with 11σ statistical significance

14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^-$

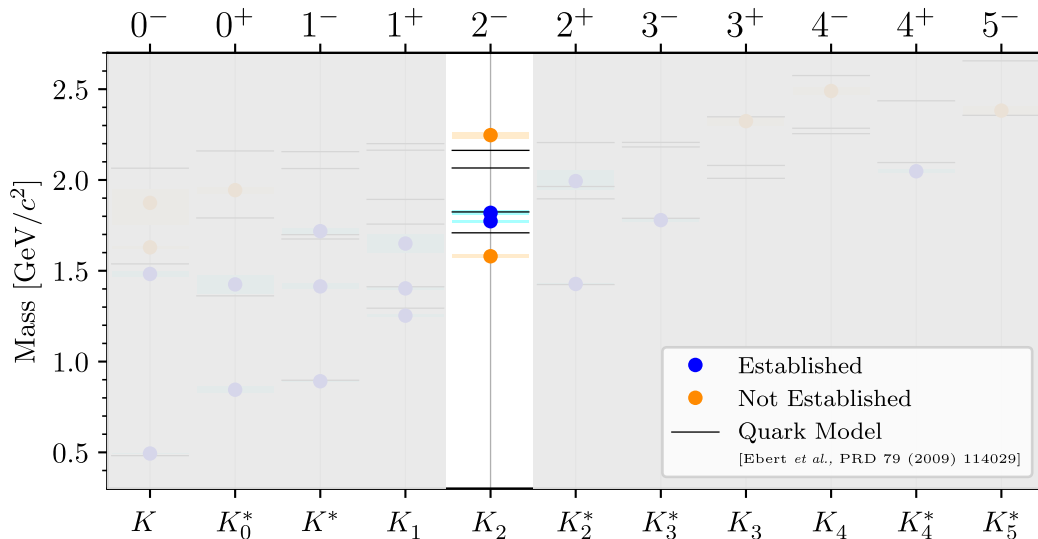


$K_2(2250)$

- ▶ Studied so far mainly in $\bar{\Lambda}(\bar{p})$ final states
- ▶ First simultaneous measurement of $K_2(1770)$, $K_2(1820)$, and $K_2(2250)$
- ▶ Resonance parameters consistent with previous observations

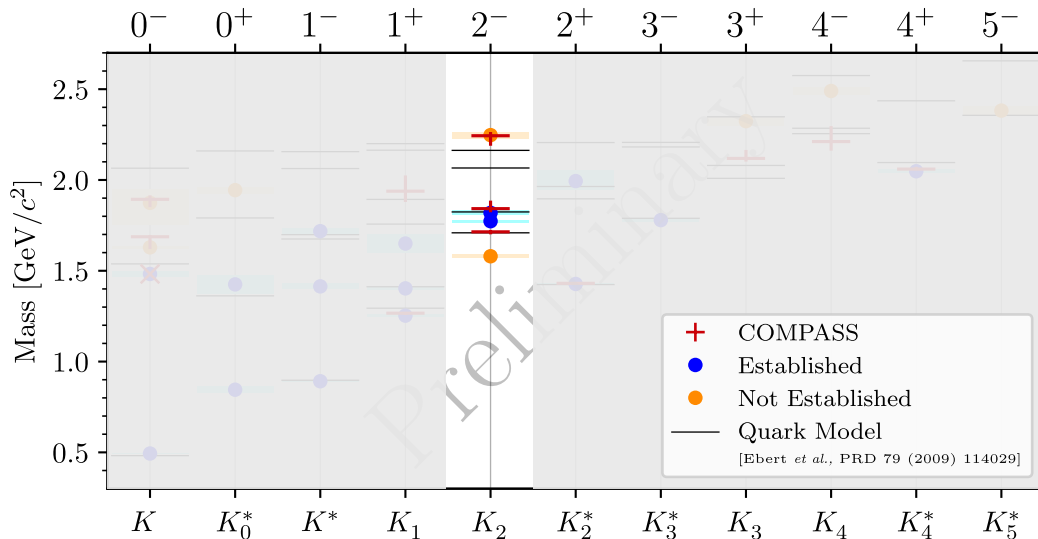
14-Wave Resonance-Model Fit

Partial Waves with $J^P = 2^-$



14-Wave Resonance-Model Fit

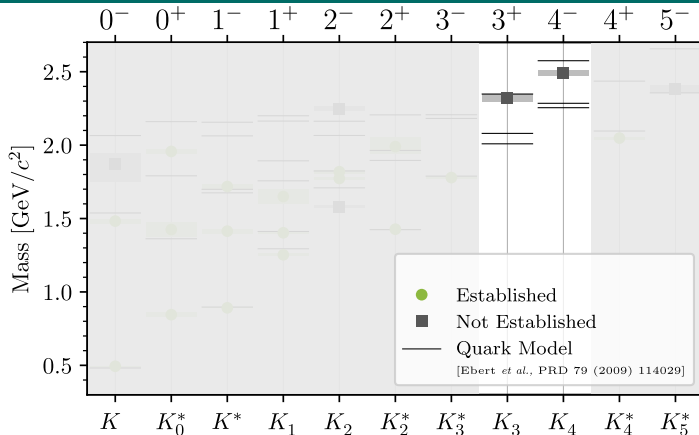
Partial Waves with $J^P = 2^-$



14-Wave Resonance-Model Fit

K_3 and K_4 Mesons

[arXiv:2504.09470]



- ▶ $K_3(2320)$ and $K_4(2500)$ listed by the PDG
- ▶ Need further confirmation
- ▶ Seen only in $\Lambda\bar{p}$ final state by few experiments

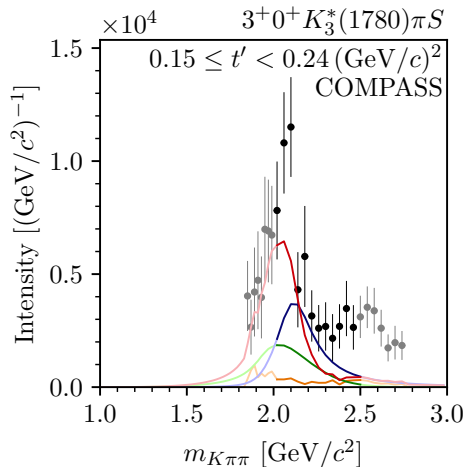
14-Wave Resonance-Model Fit

K_3 and K_4 Mesons

[arXiv:2504.09470]



- Observe K_3 signal at about $2.1 \text{ GeV}/c^2$
 - in $K_3^*(1780)\pi$ and $K_2^*(1430)\pi$ decays
- Evidence for K_4 signal at about $2.2 \text{ GeV}/c^2$
 - in $K_2^*(1430)\pi$ decay



total resonance model, resonances, non-resonant, $\pi\pi\pi$ background, effective background

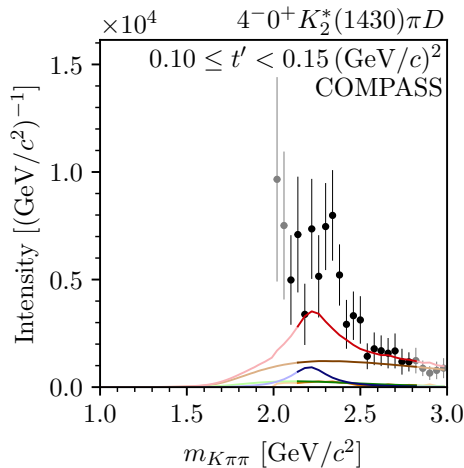
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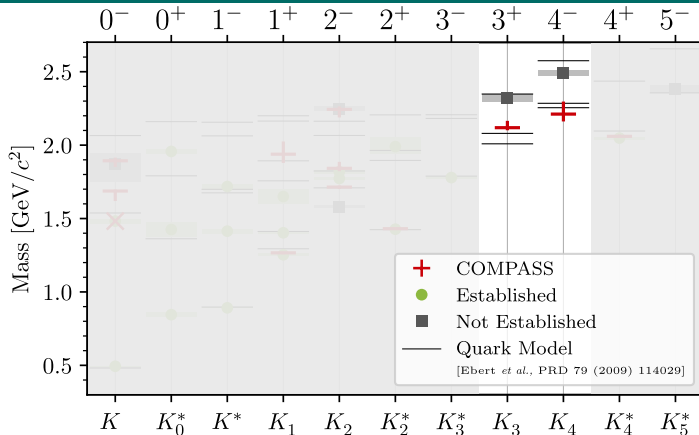


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14-Wave Resonance-Model Fit

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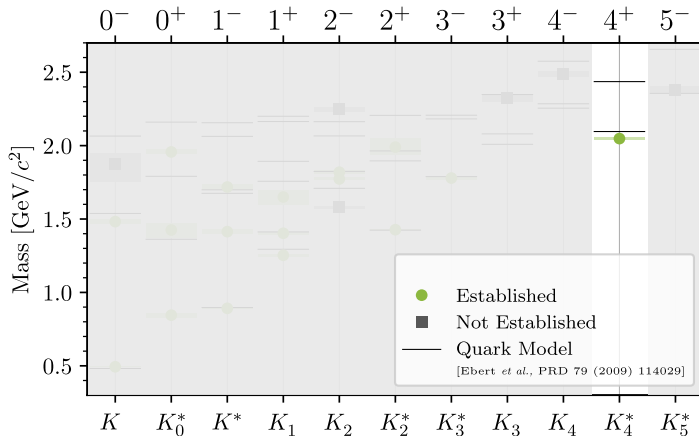


- ▶ Widths of both states in agreement with previous observations
- ▶ Masses significantly lower, however in good agreement with quark-model predictions for ground states
 - ➔ Potential first observation of K_3 and K_4 ground states, while excited states observed in $\Lambda\bar{p}$ decay

14-Wave Resonance-Model Fit

K_4^* Mesons

[arXiv:2504.09470]



PDG

► $K_4^*(2045)$ established state

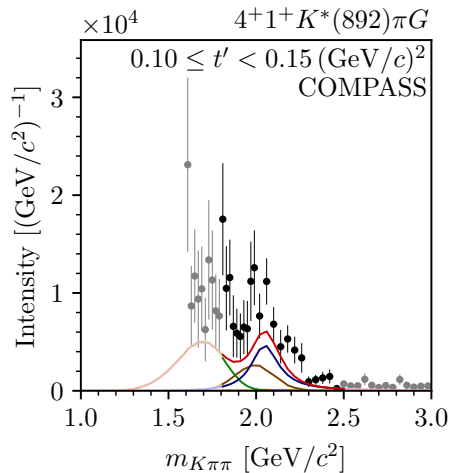
14-Wave Resonance-Model Fit

K_4^* Mesons

[arXiv:2504.09470]



- ▶ $K_4^*(2045)$ signal at per-mil level
 - ▶ in $K^*(892) \pi$ decay
 - ▶ in $\rho(770) K$ decay
- ▶ Interference terms described well
- ▶ Accompanied by rising phase



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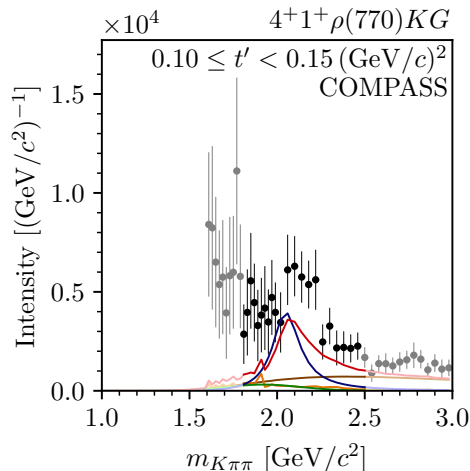
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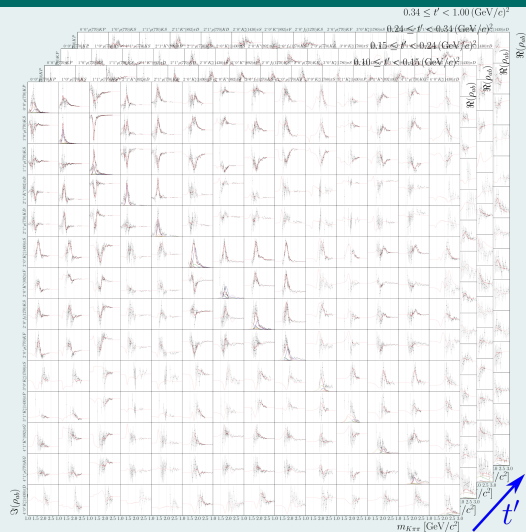


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14-Wave Resonance-Model Fit

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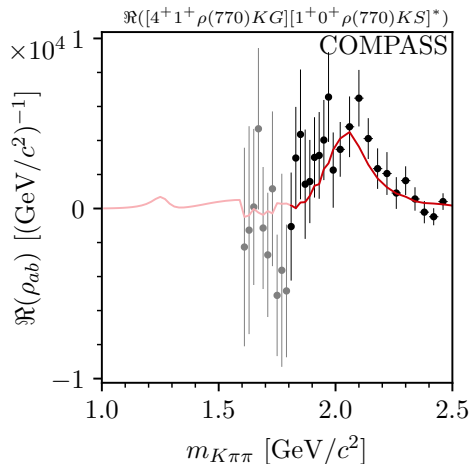
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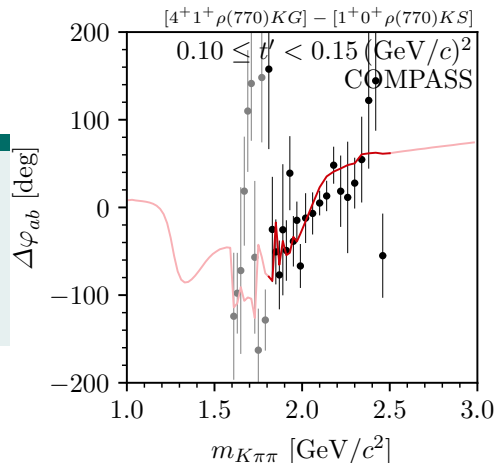


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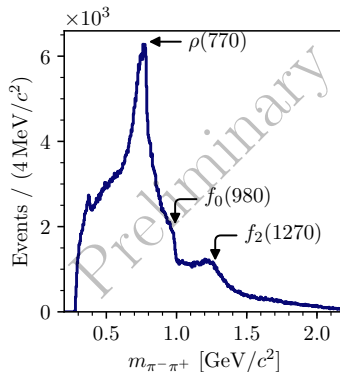
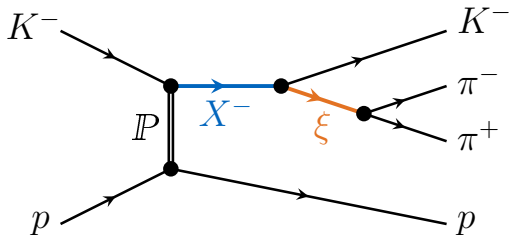
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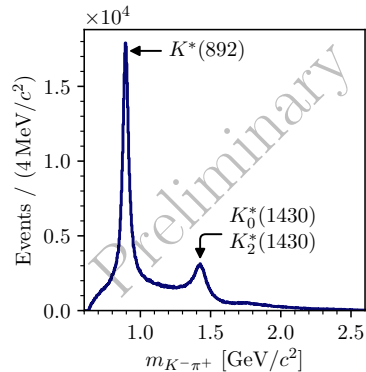
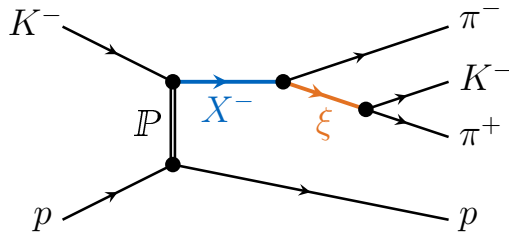


Kinematic Distribution of $K^-\pi^-\pi^+$ Events

Subsystem



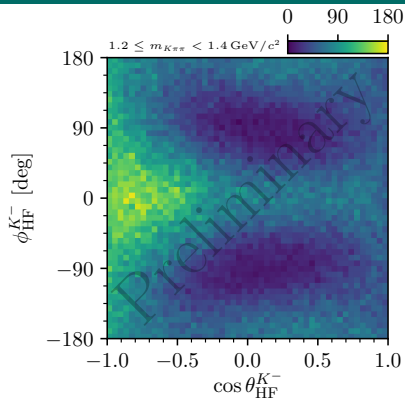
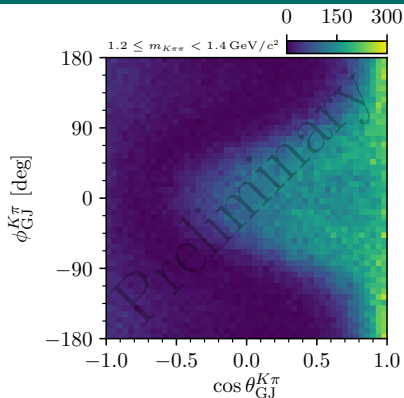
- ▶ Also structure in $\pi^-\pi^+$ and $K^-\pi^+$ subsystems
 - ➡ Successive 2-body decay via $\pi^-\pi^+$ / $K^-\pi^+$ resonance called **isobar**
- ▶ Also structure in angular distributions



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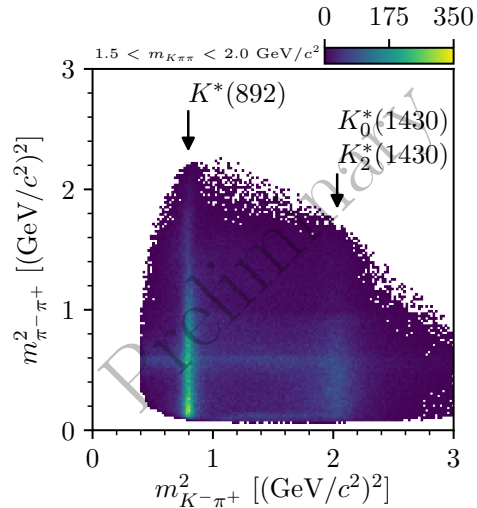
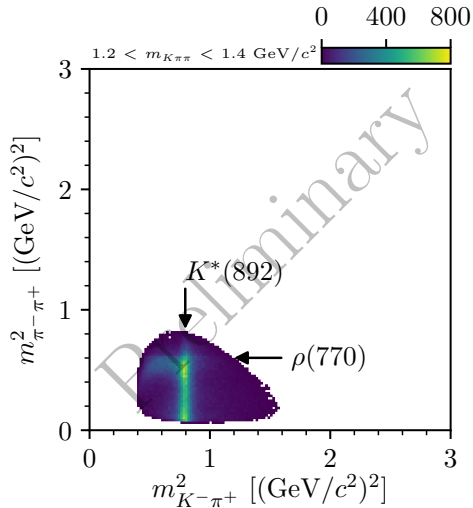
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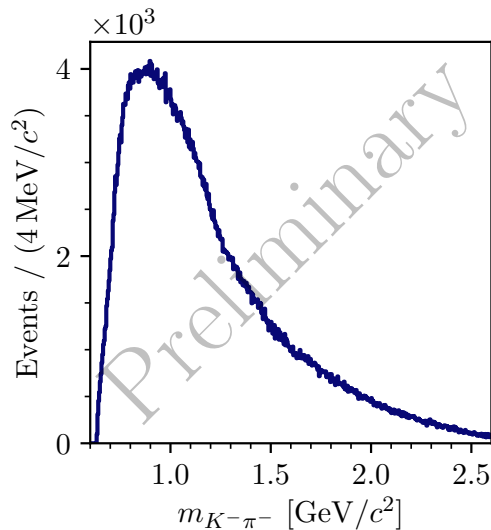


Kinematic Distribution of $K^-\pi^-\pi^+$ Events

$m_{K^-\pi^-}$



- No dominant resonant structures

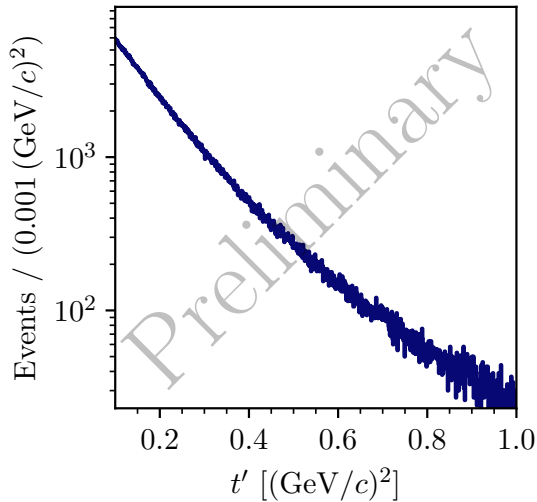


Kinematic Distribution of $K^-\pi^-\pi^+$ Events

t' Spectrum

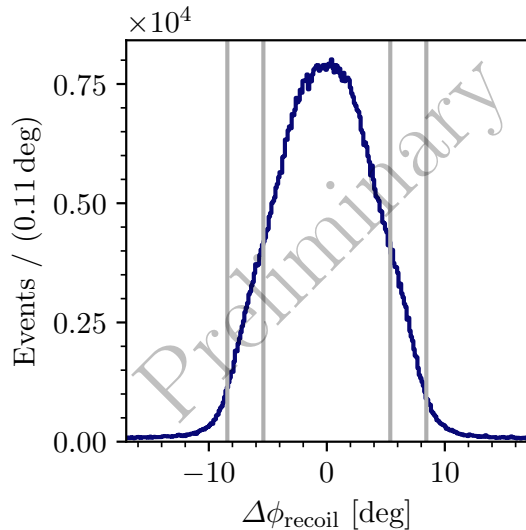
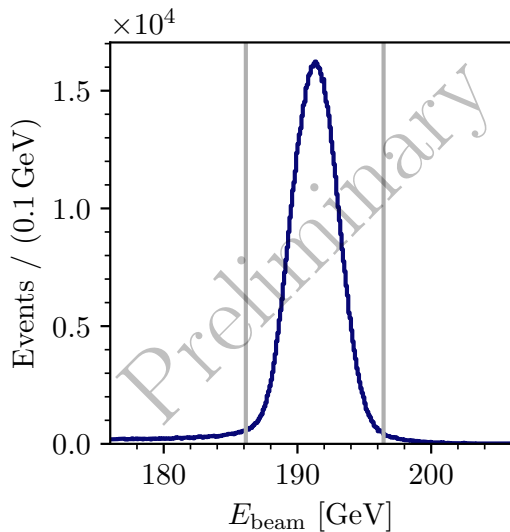


- ▶ Exponential shape
- ▶ Shallower for larger t'



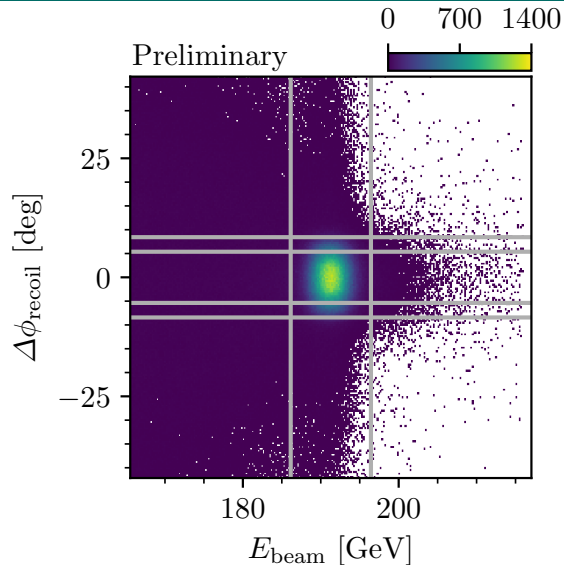
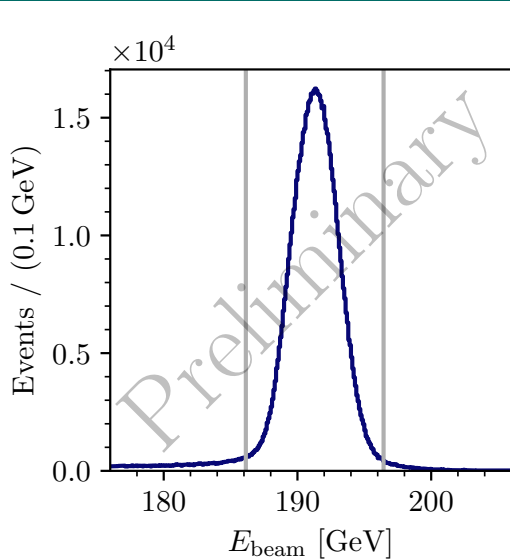
Kinematic Distribution of $K^-\pi^-\pi^+$ Events

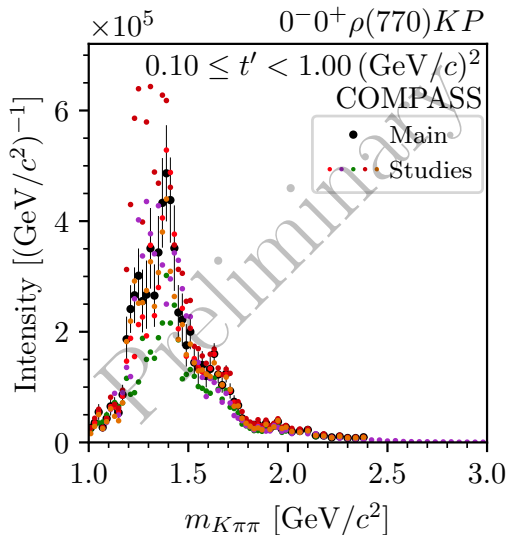
Exclusivity

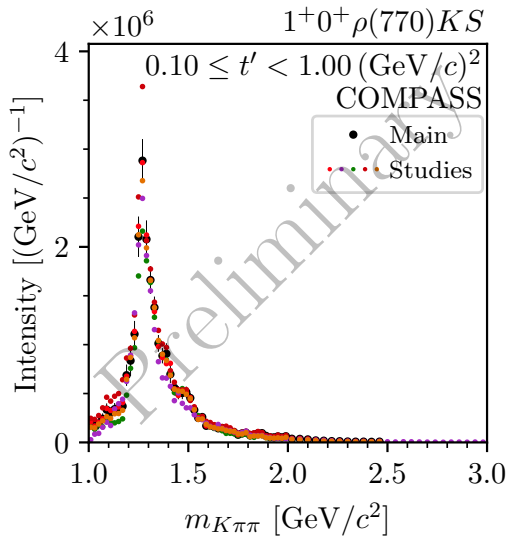


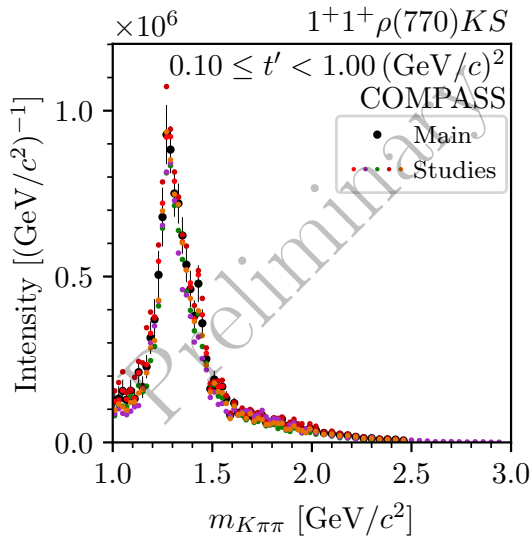
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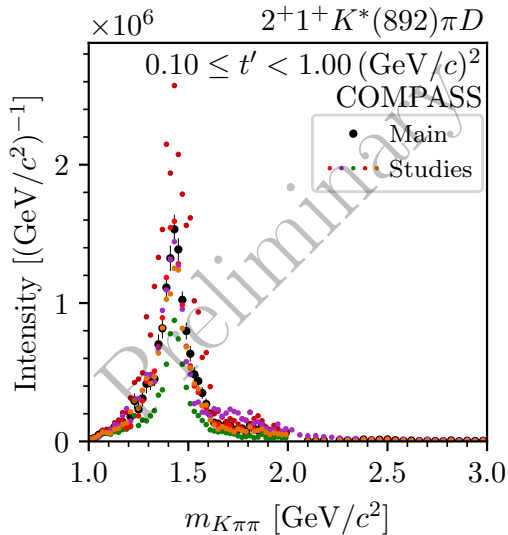
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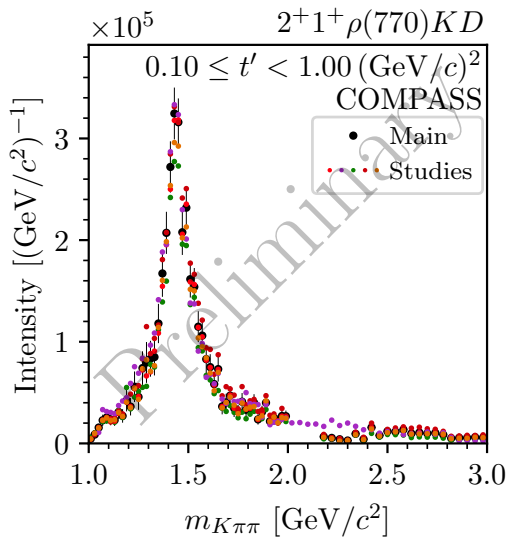


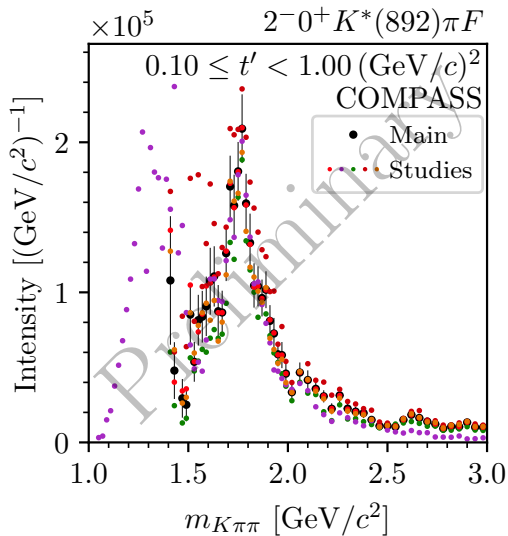


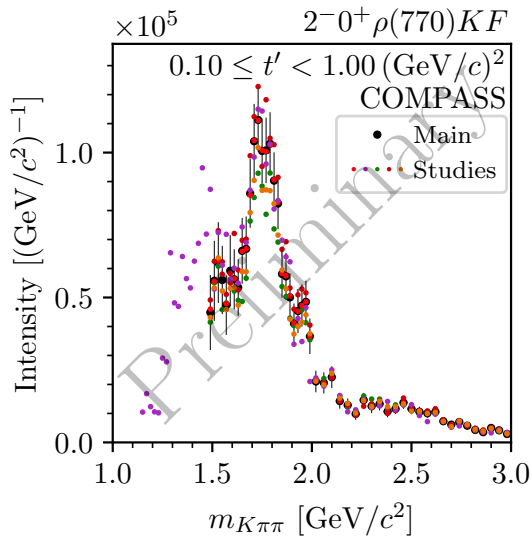


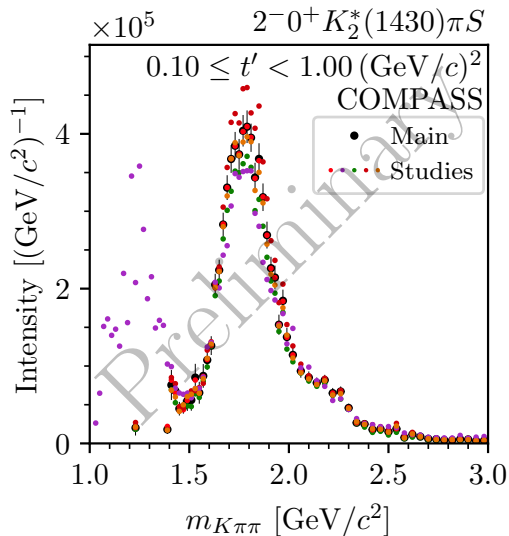


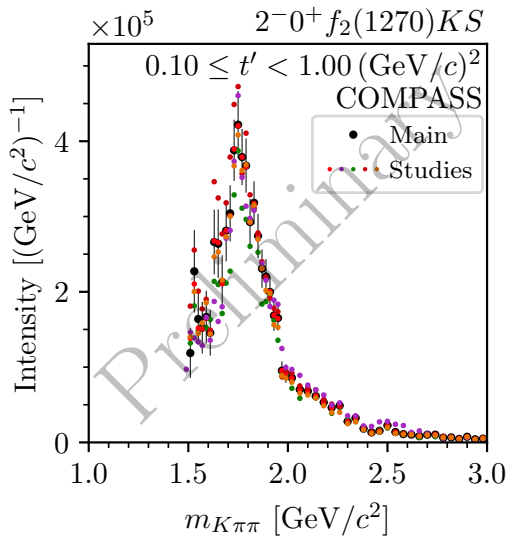


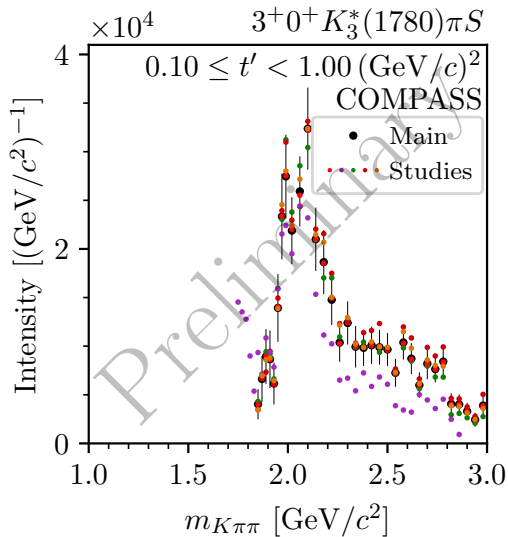


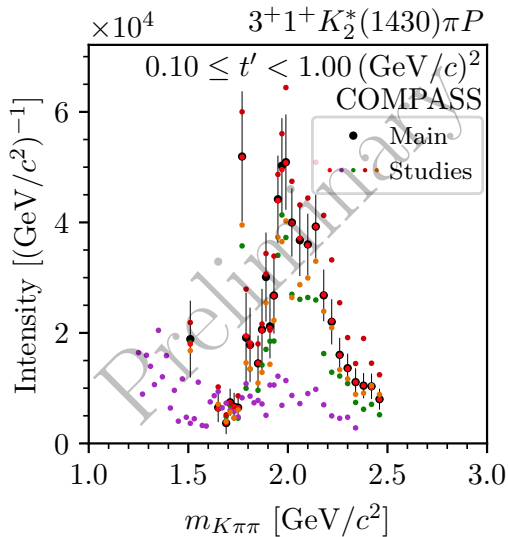


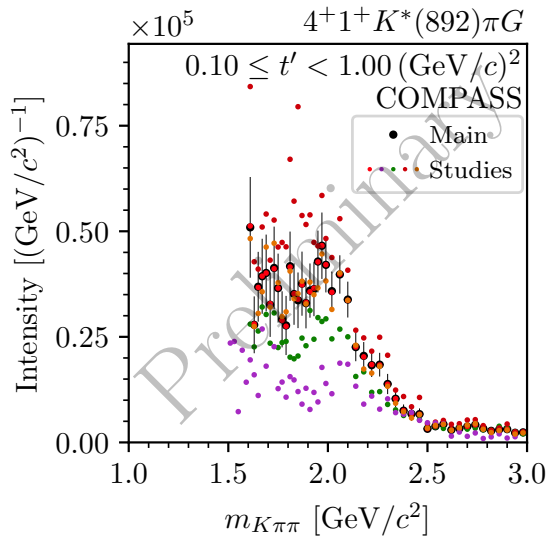


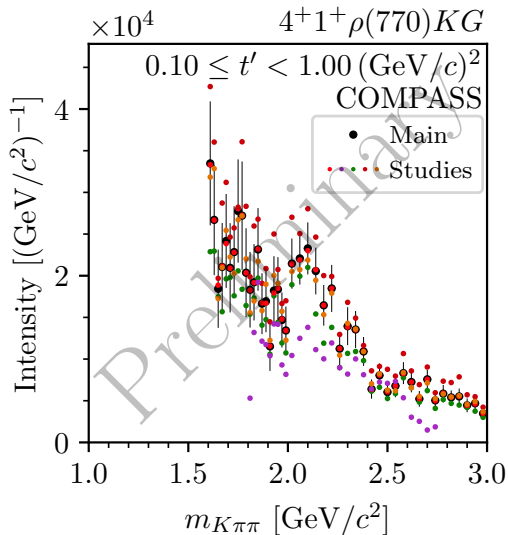


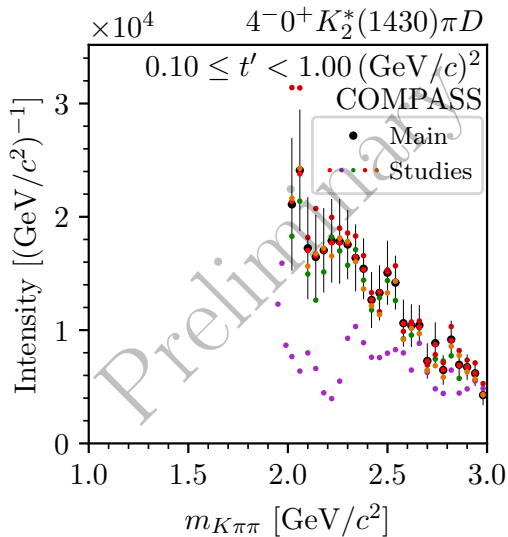


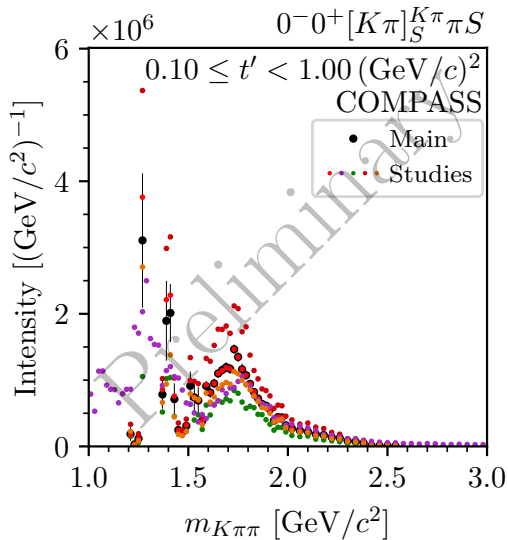


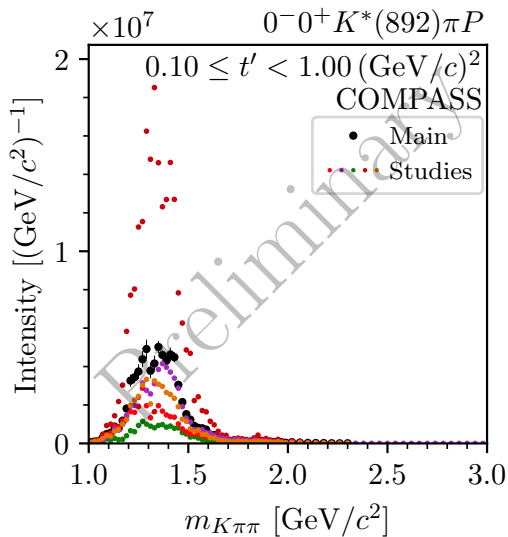


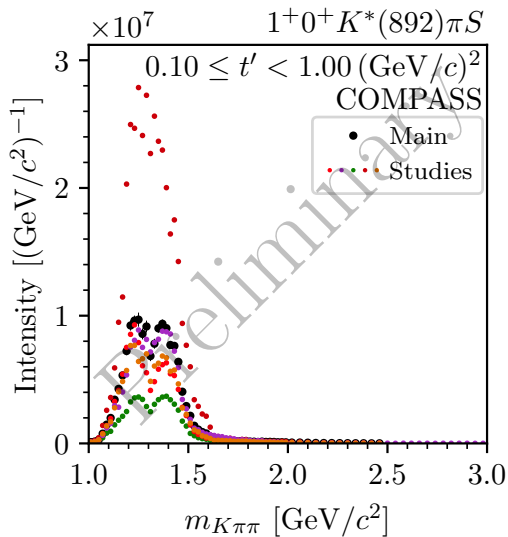


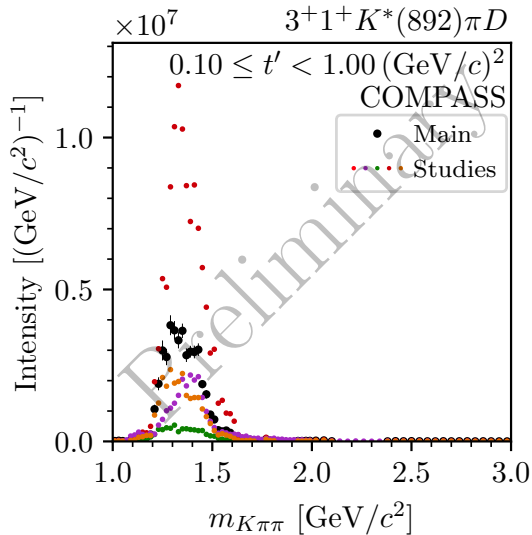




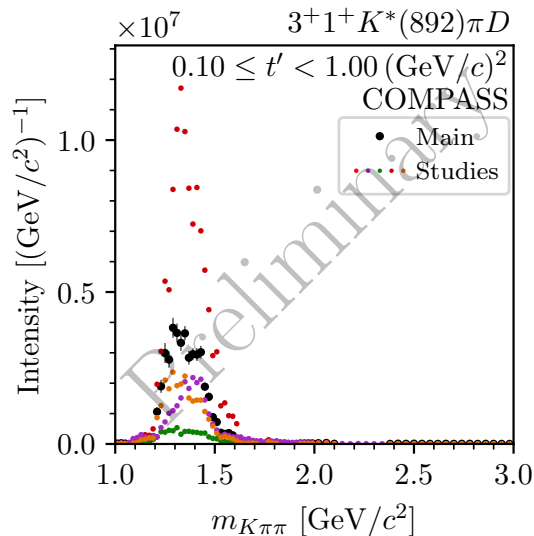




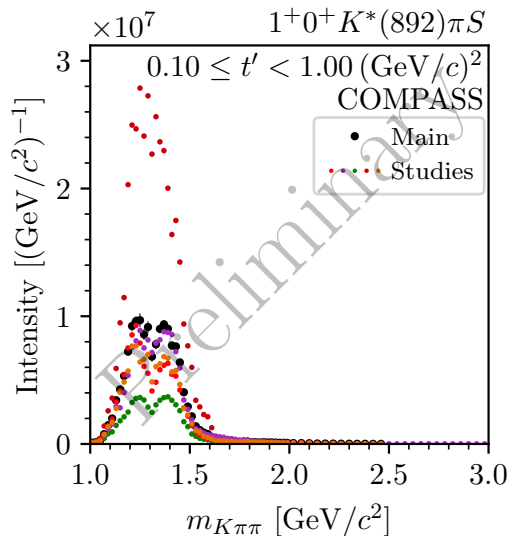




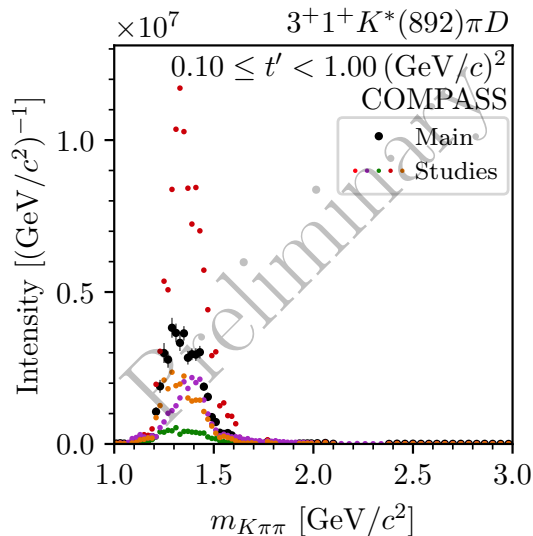
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- ▶ Similar to dominant 1^+ wave
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- ▶ Decay amplitudes of different J^P are orthogonal
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- ▶ Loss of orthogonality taking acceptance into account
 - ▶ Reduced differentiability of certain partial waves
- ▶ Only a sub-set of partial waves affected



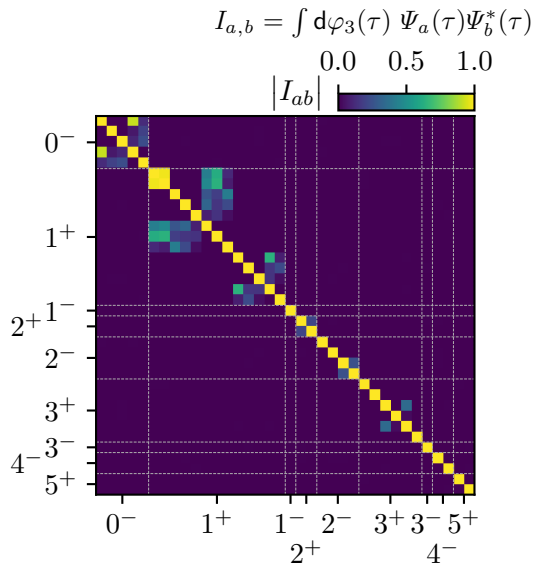
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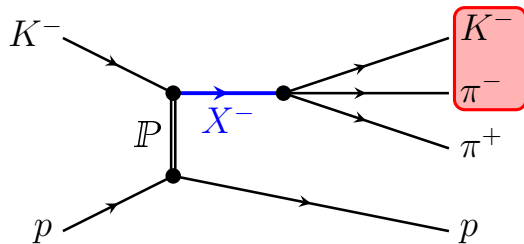
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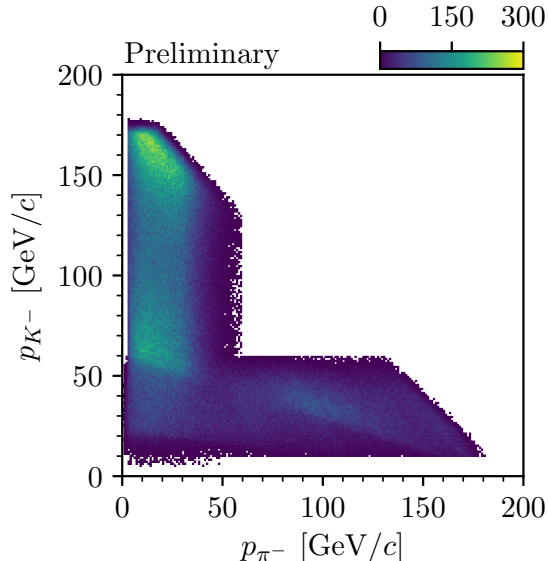
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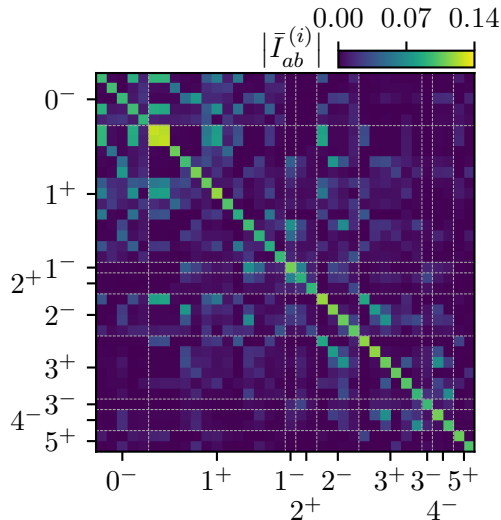


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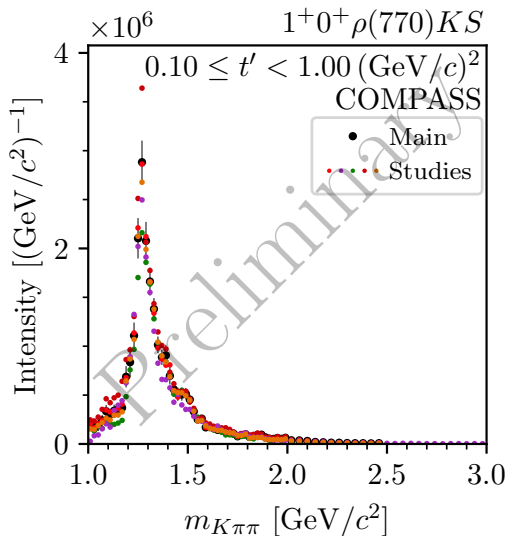


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$$\bar{I}_{a,b} = \int d\varphi_3(\tau) \, \eta(\tau) \, \Psi_a(\tau) \Psi_b^*(\tau)$$

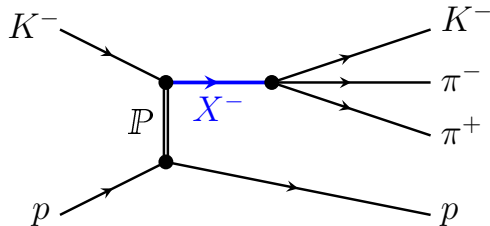


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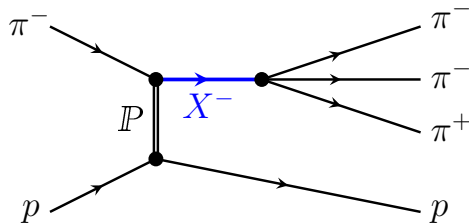


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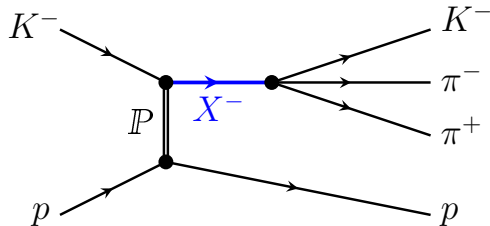
- ▶ $K^-\pi^-\pi^+$ and $\pi^-\pi^-\pi^+$ similar experimental footprint
- ▶ Distinguishable only by
 - ▶ Beam particle identification
 - ▶ Final-state particle identification
- ▶ Excellent beam PID:
 - ▶ Expect small contamination from beam π^-
- ▶ Final-state PID does not suppress $\pi^-\pi^-\pi^+$ background
 - ➡ Non-negligible $\pi^-\pi^-\pi^+$ background in $K^-\pi^-\pi^+$ sample of about 7 %
 - ➡ Dominant background in $K^-\pi^-\pi^+$ sample



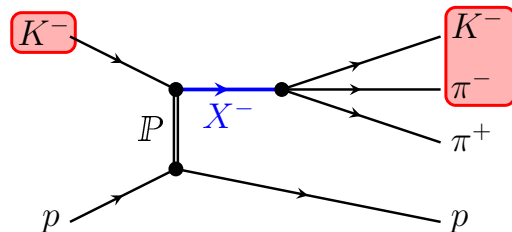
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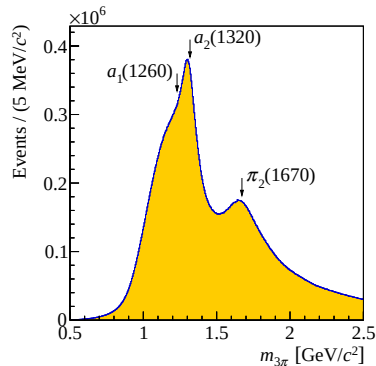
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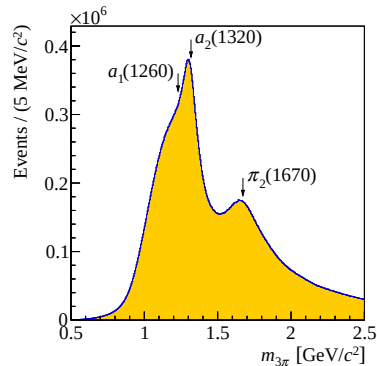
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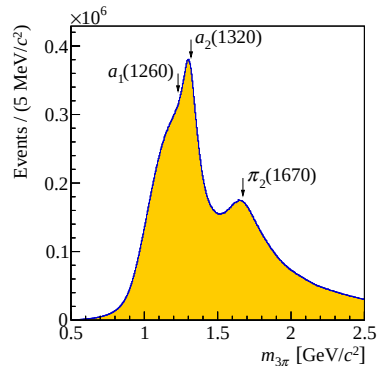
- ▶ Well established model for $\pi^- + p \rightarrow \pi^-\pi^-\pi^+ + p$
 - ▶ From very same data set
 - ▶ Measured with high precision
 - ▶ Acceptance corrected
- ▶ Generate $\pi^-\pi^-\pi^+$ Monte Carlo sample
- ▶ Mis-interpret $\pi^-\pi^-\pi^+$ Monte Carlo events as $K^-\pi^-\pi^+$
 - ▶ Apply wrong mass assumption
 - ▶ Same event reconstruction and selection as for $K^-\pi^-\pi^+$
- ▶ Perform partial-wave decomposition of mis-interpreted $\pi^-\pi^-\pi^+$ Monte Carlo sample
 - ▶ Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample



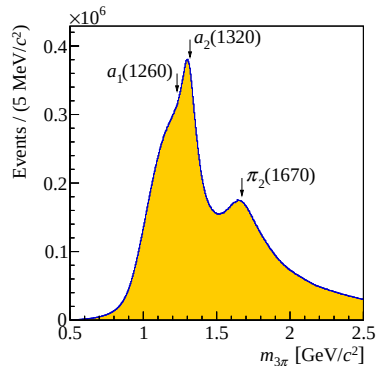
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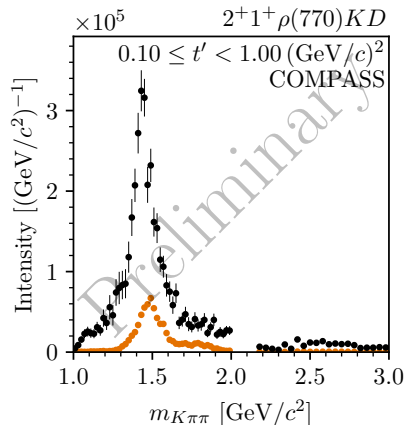
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 - ▶ Using the same PWA model as for measured $K^-\pi^-\pi^+$ sample
- ➡ Study $\pi^-\pi^-\pi^+$ background in individual $K^-\pi^-\pi^+$ partial waves

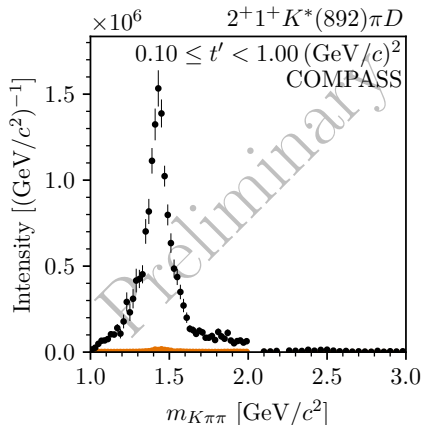


- ▶ Significant contribution to waves with $\rho(770)$ isobar
- ▶ $\pi^-\pi^-\pi^+$ produces peaking structures
- ▶ Largest relative contribution to $2^+ 1^+ \rho(770) K D$ wave
- ▶ Small contribution to waves with $K^*(892)$ isobar
- ▶ Also significant contribution to waves with $f_2(1270)$ and $K_2^*(1430)$ isobars
- ▶ No contribution to flat wave



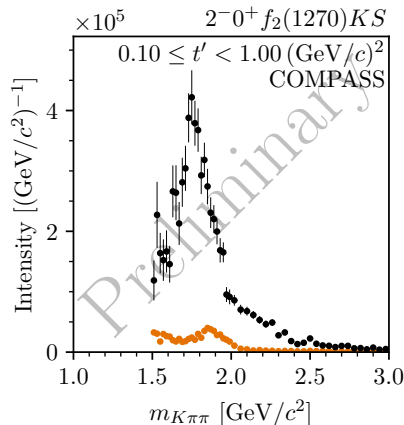
$K^-\pi^-\pi^+$ data, $\pi^-\pi^-\pi^+$ pseudo data

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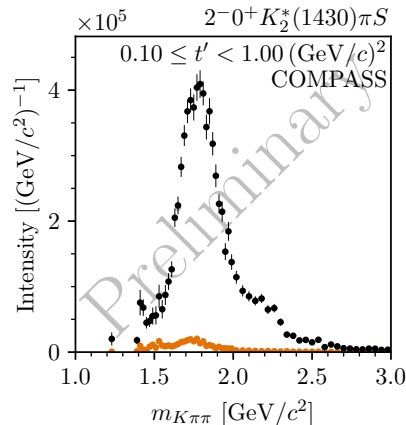
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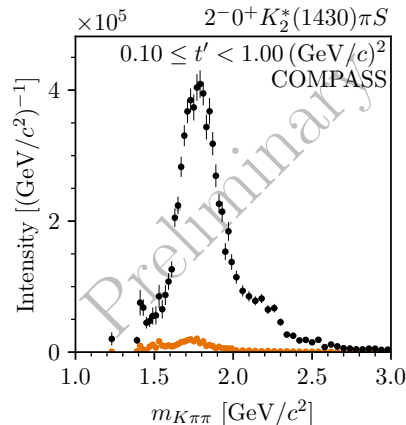
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$K^-\pi^-\pi^+$ data, $\pi^-\pi^-\pi^+$ pseudo data

- 238-wave set can describe main features of $\pi^-\pi^-\pi^+$ pseudodata sufficiently well
- Largest deviation for $K^-\pi^+$ isobar system at thigh $m_{K\pi\pi}$

$\pi^-\pi^-\pi^+$ pseudo data,
prediction (weighted-MC) of $K^-\pi^-\pi^+$ PWD
to $\pi^-\pi^-\pi^+$ pseudo data

