Universität Zürich[UZH]

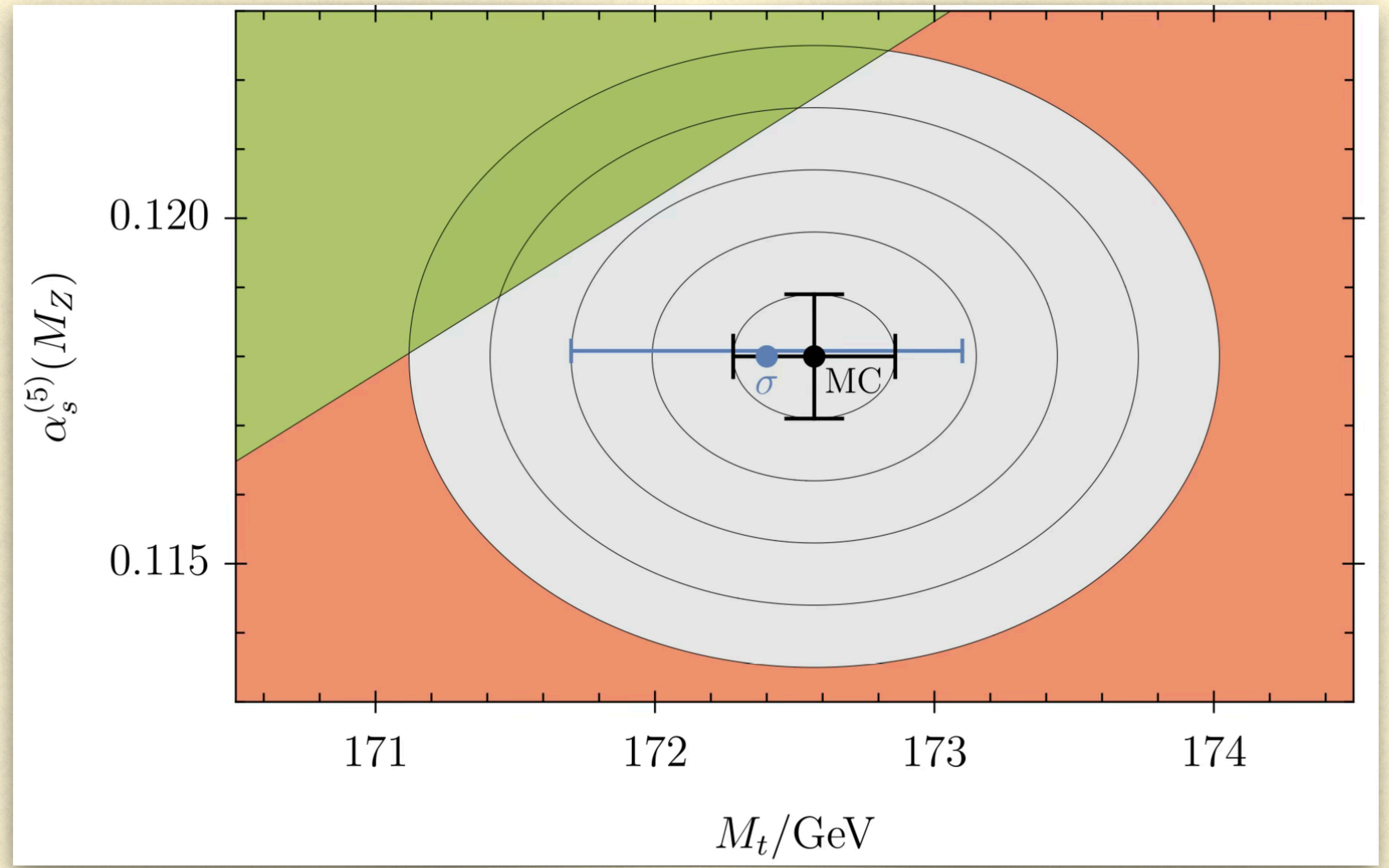
From quarks to hadrons and back again: A journey across branch cuts and energy scales

Florian Herren

Why do we collide particles?

Trying to answer many interesting questions!

- Is the vacuum structure of the universe stable?
- Are there more particles above the electroweak scale?



G. Hiller, T. Höhne, D. F. Litim, T. Steudtner PRD 110 (2024) 11, 115017

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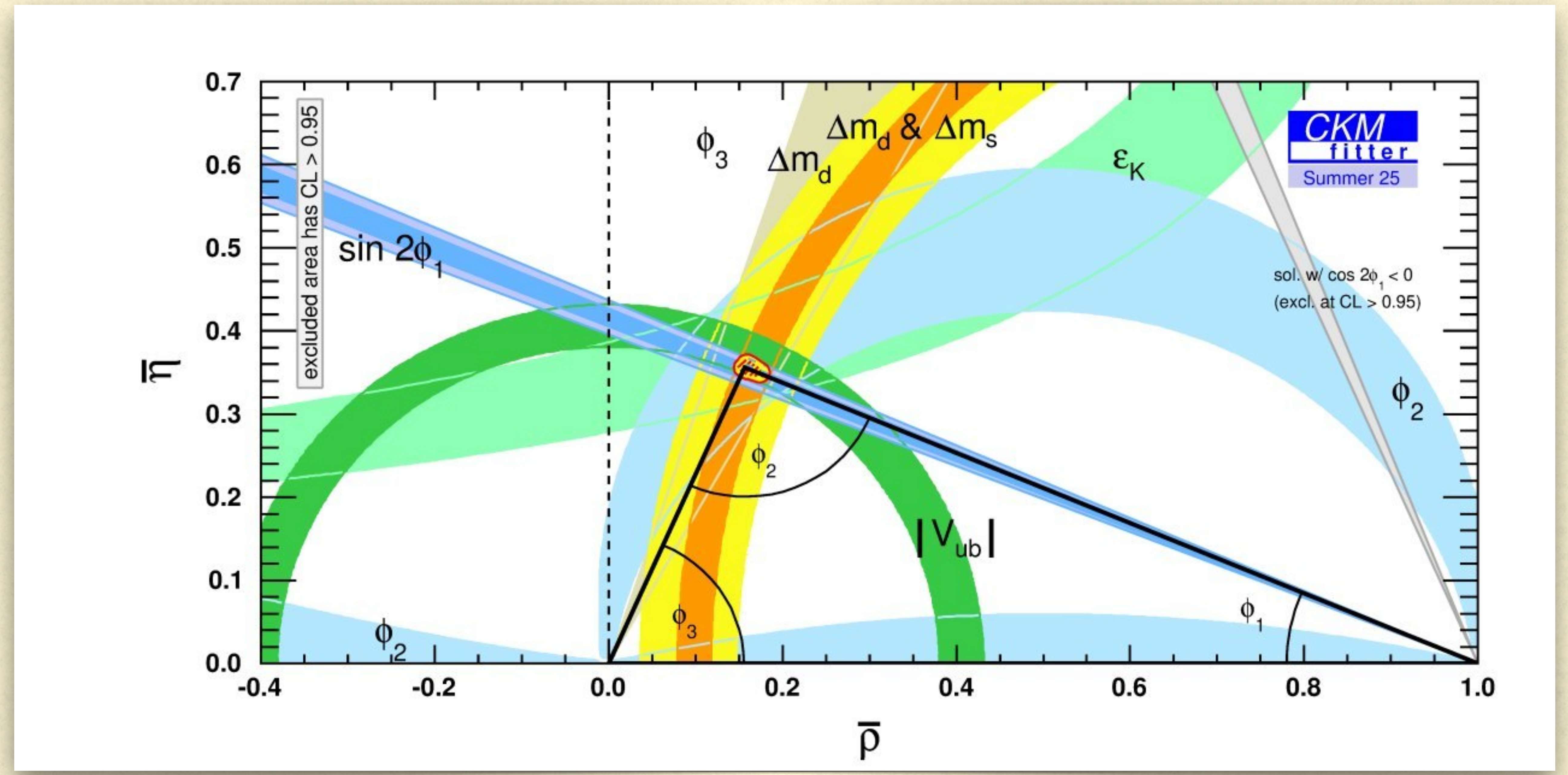
- Why are there three generations?
- Where is all the antimatter?



CKMfitter group, Summer 2025 update

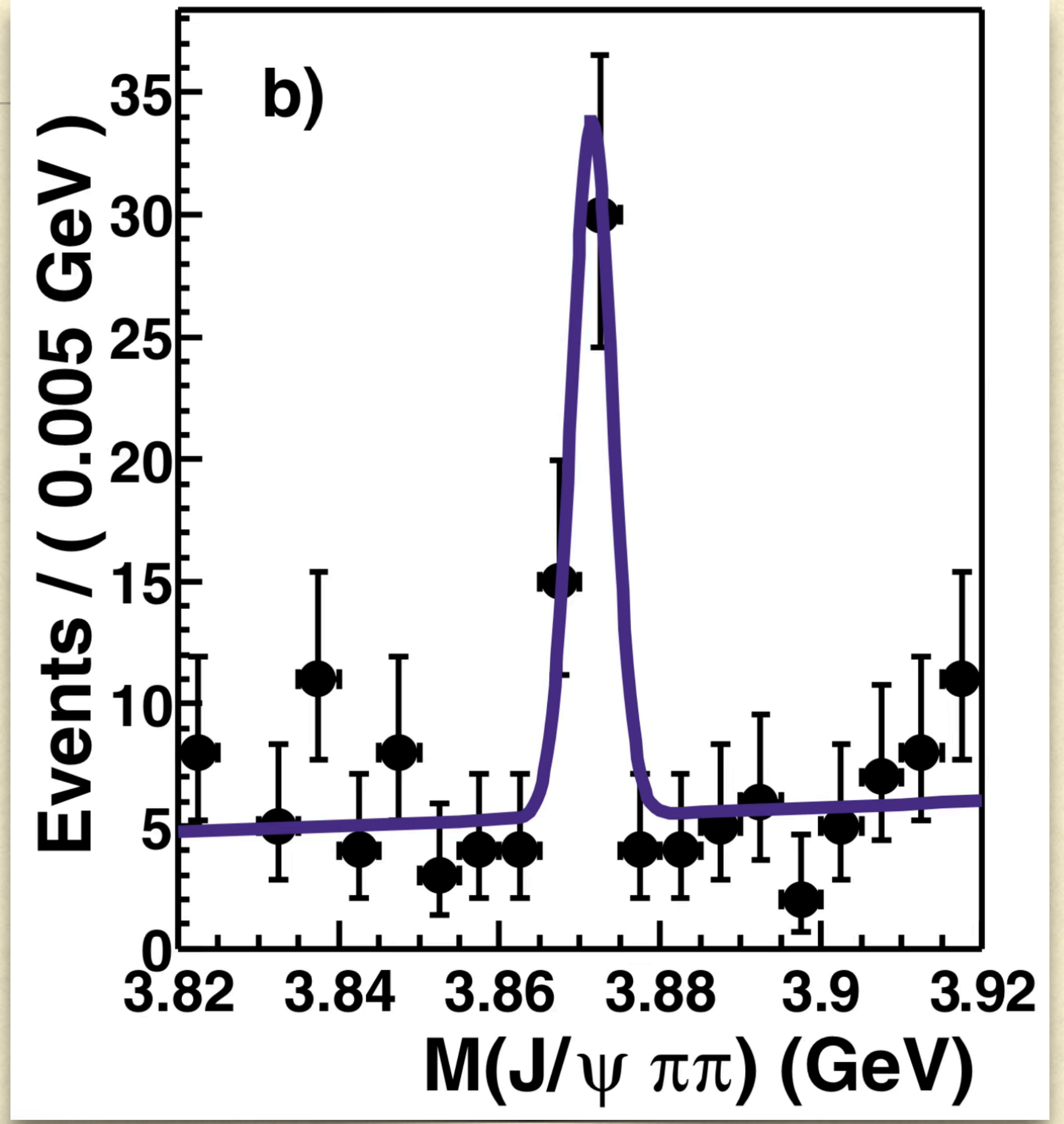
Why do we collide particles?

Trying to answer many interesting questions!

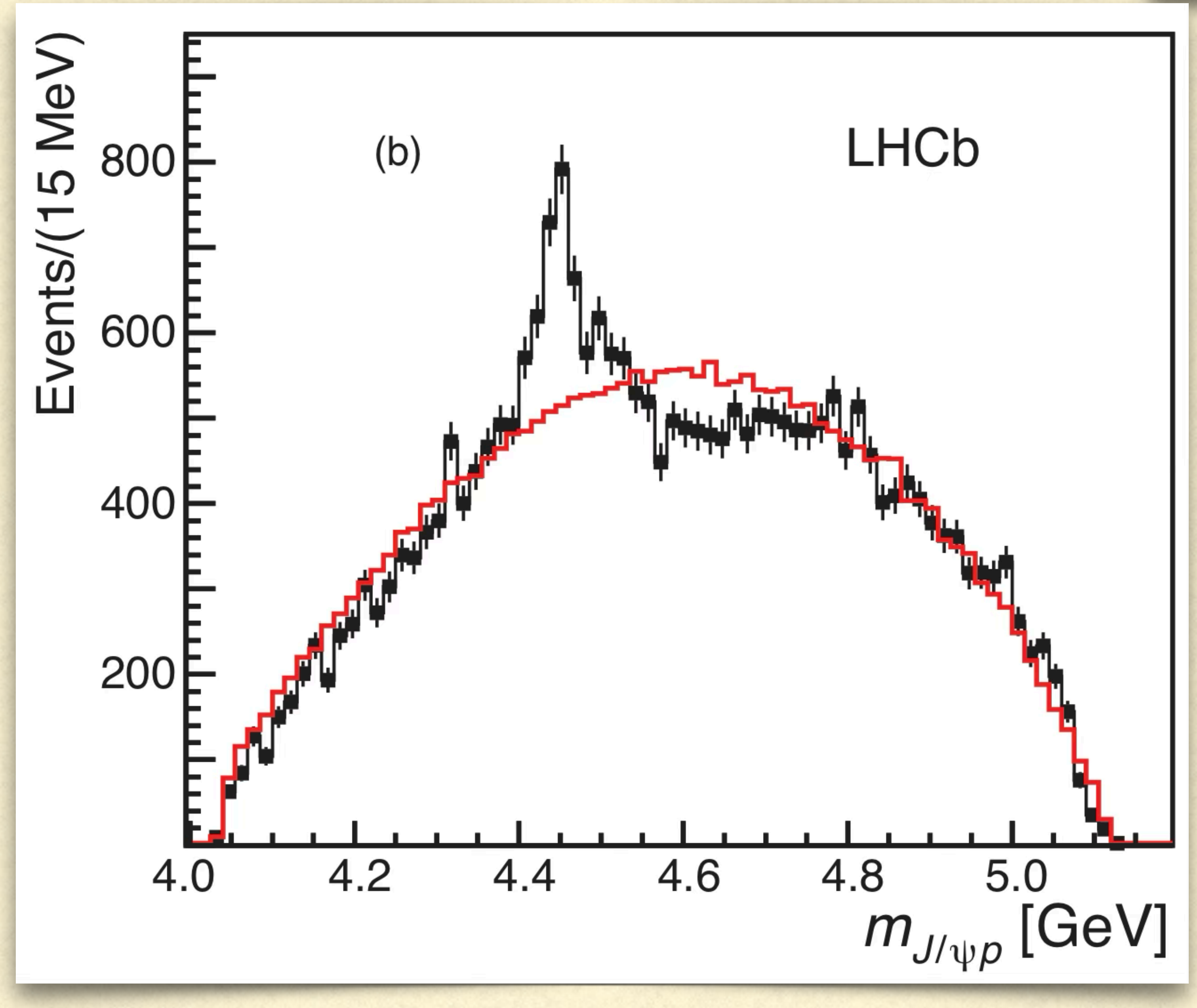
- Is the vacuum structure of the universe stable?
- Are there more particles above the electroweak scale?

- Why are there three generations?
- Where is all the antimatter?

- What exotic structures can be found in the hadron spectrum?
- How does the strong interaction work at energy scales where it is strong?



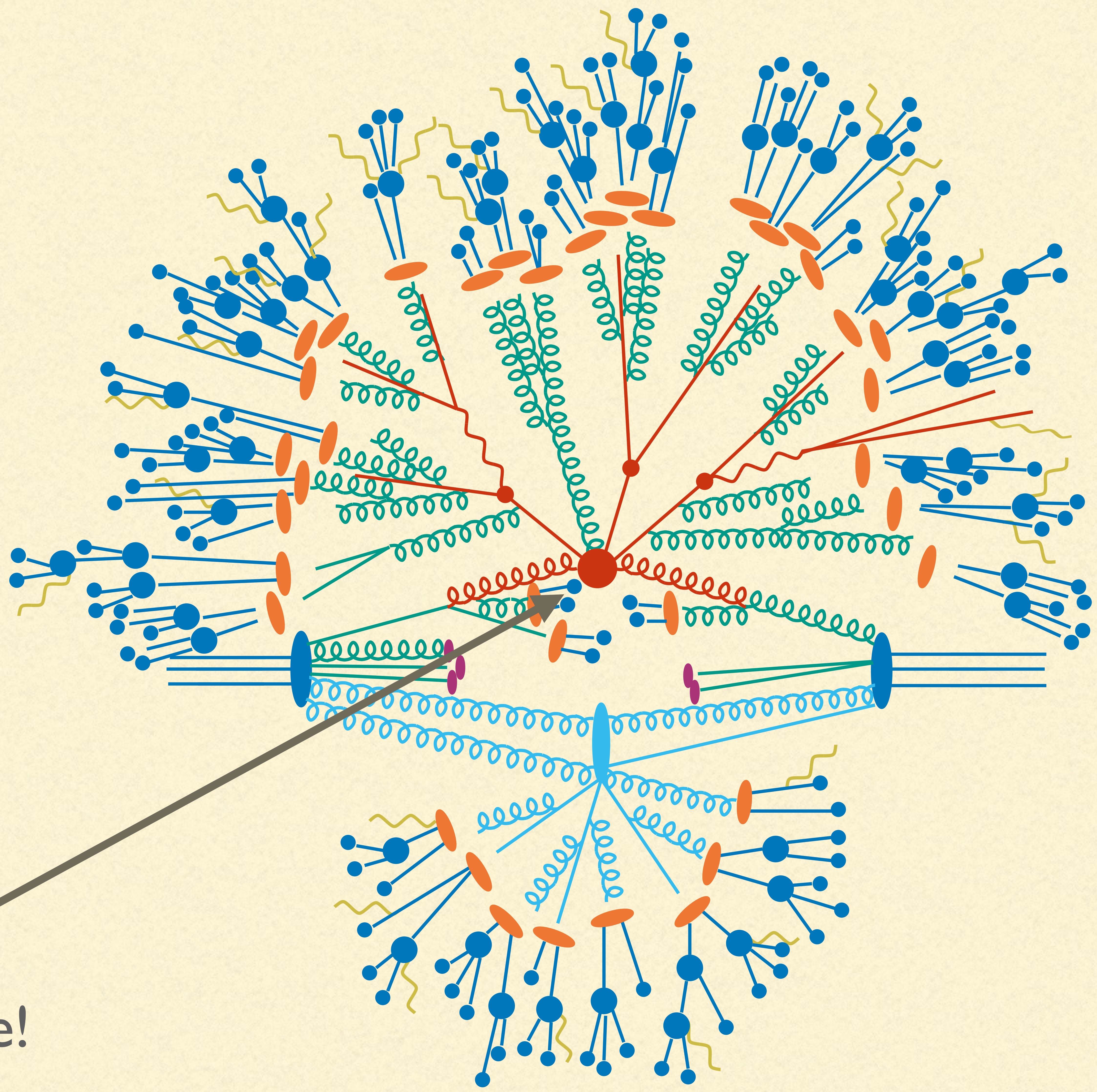
S.K. Choi et al. (Belle), PRL 91 (2003) 262001



LHCb collaboration, PRL 115 (2015) 072001

Today's itinerary

1. Surfing the parton cascade down to the hadronic scale
2. Venturing through a branch cut onto the second Riemann sheet
3. Going on a Hadron decay safari
4. Climbing back up the energy hill



You are here!

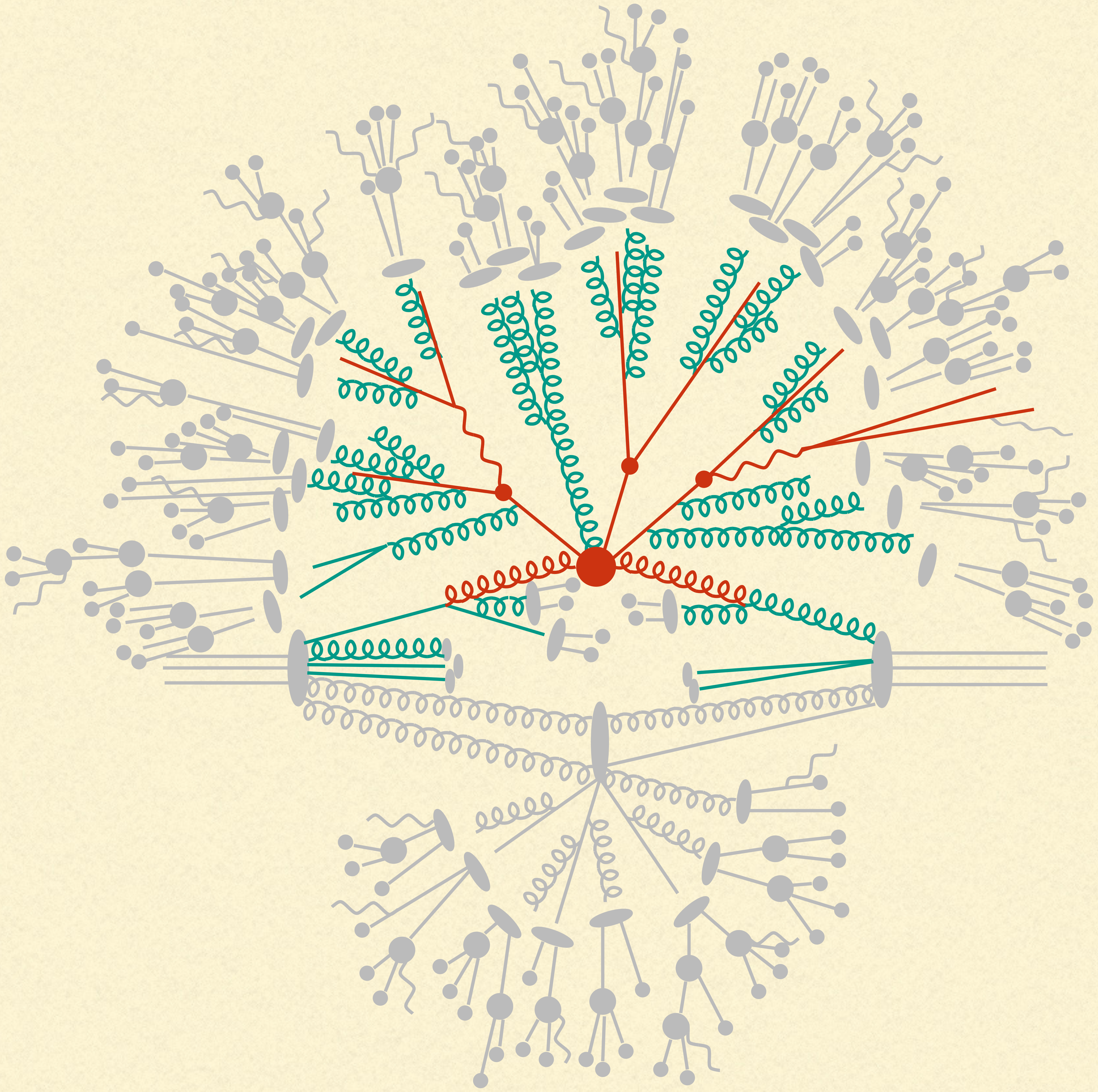
Event display by Sherpa

Surfing the parton cascade

- Fixed order predictions typically done at (N)NLO → one or two real emissions
- Perturbative expansion spoiled by large logarithms

$$d\sigma = (\alpha_s \ln^2(E_1/E_0))^l d\sigma^{LL}$$
$$+(\alpha_s \ln(E_1/E_0))^l d\sigma^{NLL} + \ldots$$

- Dressing of the event and resummation of LL performed by Parton shower algorithms

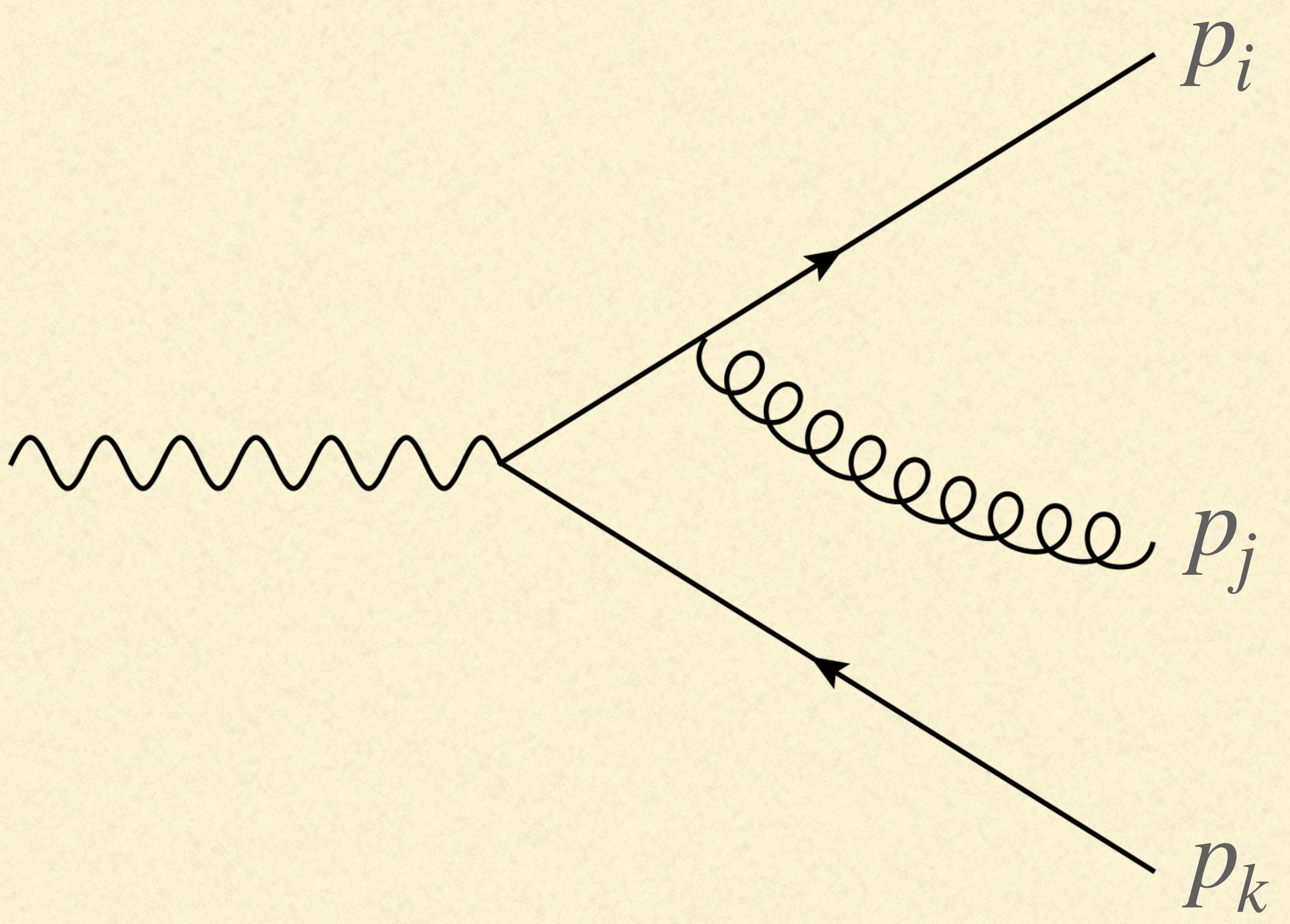


Singular limits

- Soft emission is dipole radiation

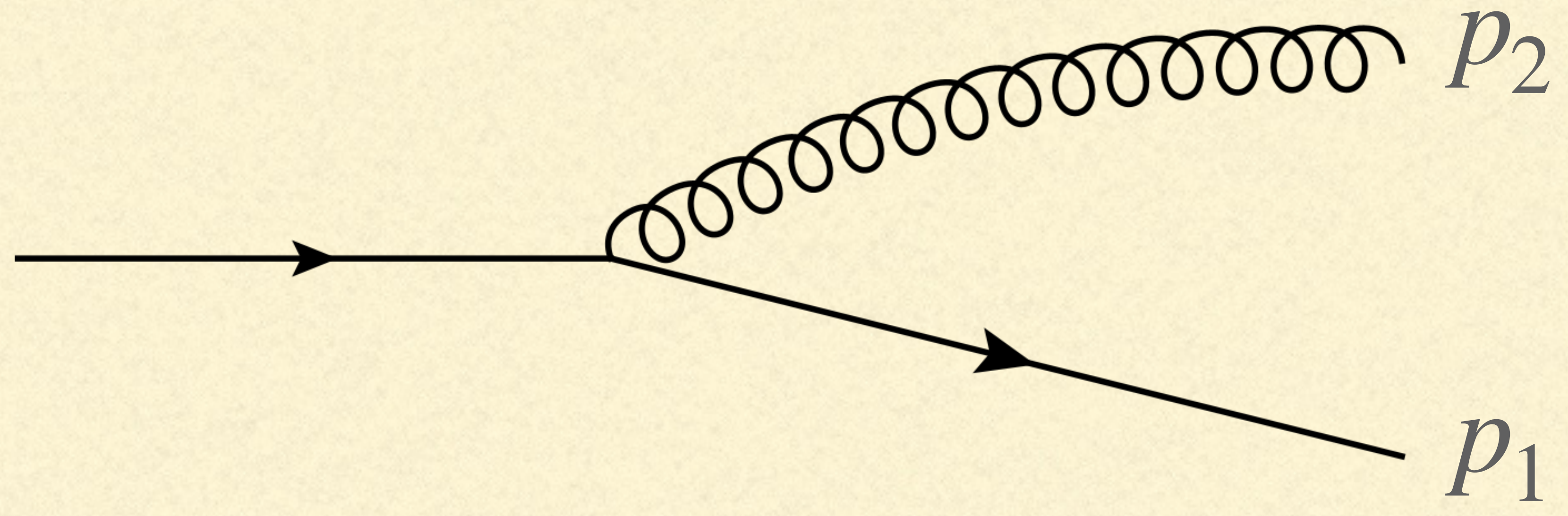
$$\langle 1,\ldots,n \,|\, 1,\ldots,n \rangle = -\,8\pi\alpha_s \sum_{i,k} \left\langle 2,\ldots,n \,|\, T_i T_k w_{ik,2} \,|\, 2,\ldots,n \right\rangle$$

$$w_{ik,j} = \frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2} \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})}$$



- Collinear splittings only depend on one parton

$$\langle 1,\ldots,n \,|\, 1,\ldots,n \rangle = \frac{4\pi\alpha_s}{p_1 p_2} \left\langle (12),\ldots,n \,|\, P_{(12),1}(z) \,|\, (12),\ldots,n \right\rangle$$



- Soft-collinear limit contained in both terms!

Avoiding the double counting

- To define splitting probability, distribute soft Eikonal over emitters, subtract collinear limit

$$E_j^2\, w_{ik,j} = \tilde{W}_{ik,j} + \tilde{W}_{ki,j}$$

- Additive matching:

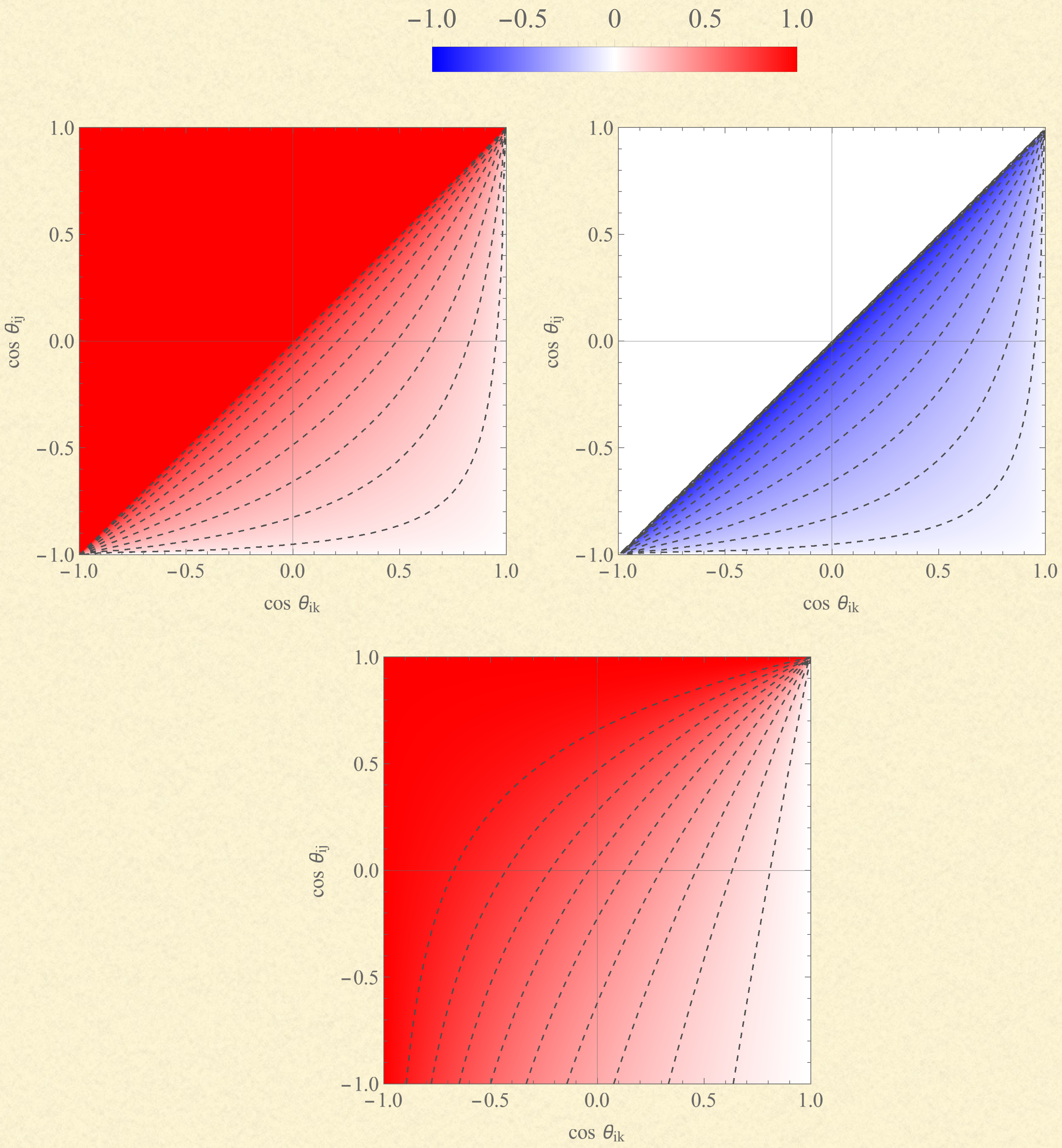
$$\tilde{W}_{ik,j} = \frac{1}{2}\left[\frac{(1-\cos\theta_{ik})}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} - \frac{1}{1-\cos\theta_{ij}} + \frac{1}{1-\cos\theta_{jk}}\right]$$

- Multiplicative matching:

$$\tilde{W}_{ik,j} = \frac{(1-\cos\theta_{ik})}{(1-\cos\theta_{ij})(2-\cos\theta_{ij}-\cos\theta_{jk})}$$

- Strictly positive, absorb in splitting function

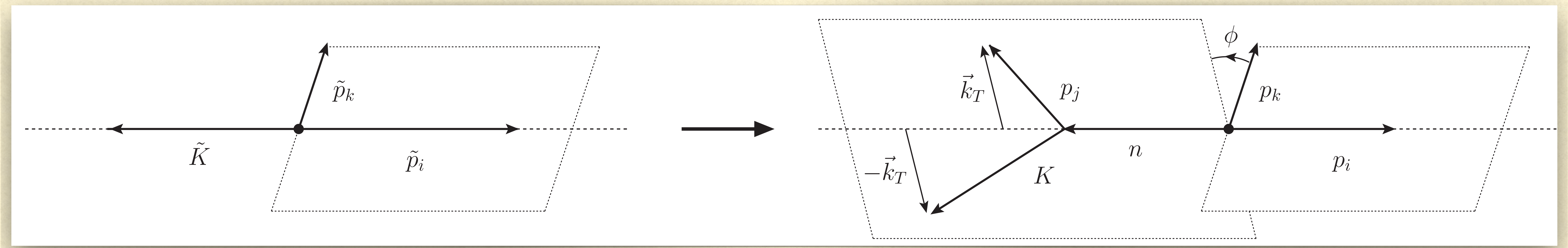
$$\frac{P_{(ij),i}(z)}{2p_ip_j} \to \frac{P_{(ij),i}(z)}{2p_ip_j} + \delta_{(ij)i}T_i^2\left[\frac{\tilde{W}_{ik,j}}{E_j^2} - w_{ik,j}^{(coll)}(z)\right]$$



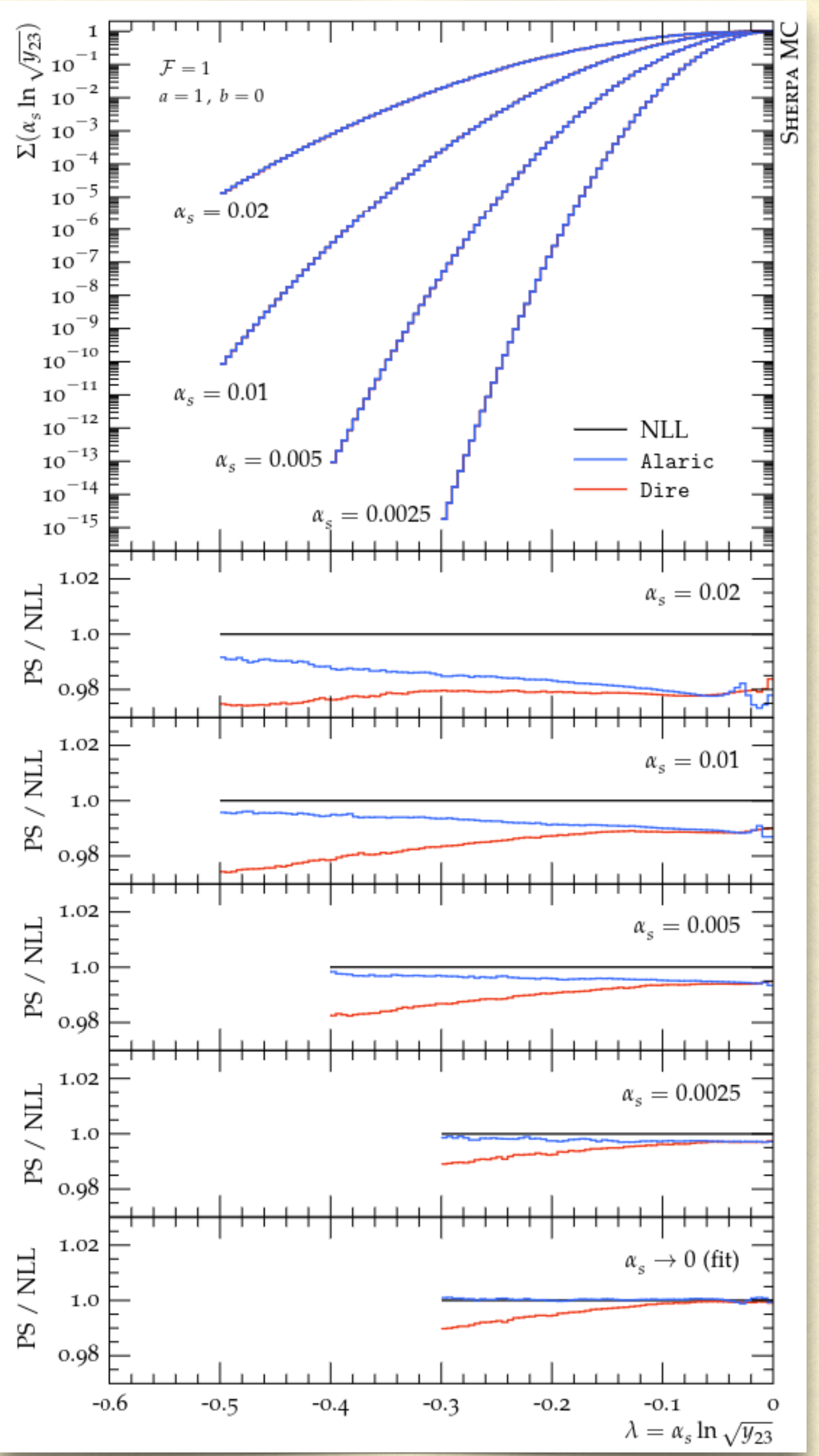
FH, S. Höche, F. Krauss, D. Reichelt, M. Schönherr, JHEP 10 (2023) 091

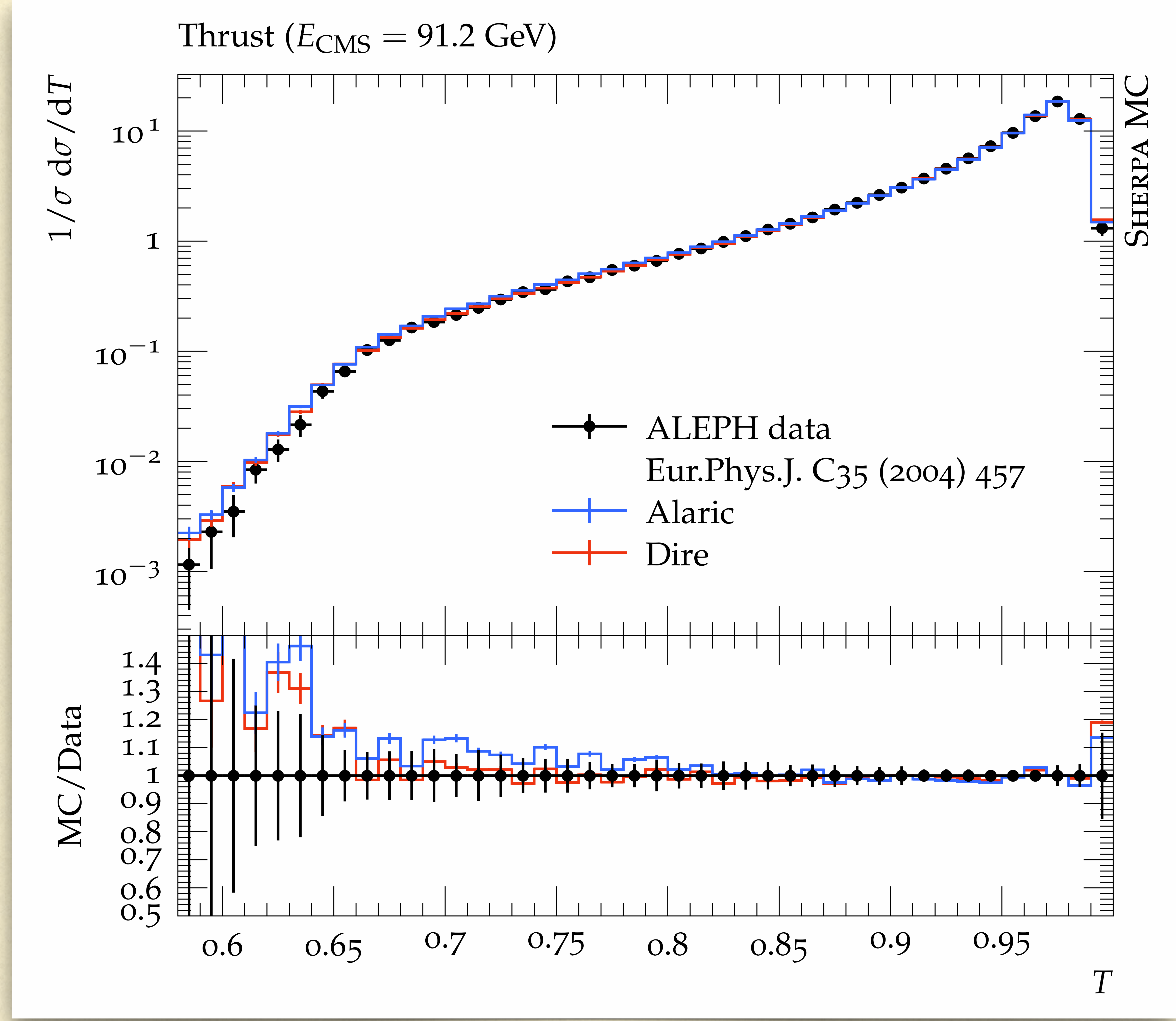
Recoil & Evolution

- Can not split one onshell momentum into two
- Recoil needs to be distributed
- Collinear safety: $p_i \to z p_{(ij)}$ & $p_j \to (1-z) p_{(ij)}$
- Need to preserve hierarchy of emissions for NLL accuracy
- Our choice: keep direction of p_i distribute recoil over the rest of the event



FH, S. Höche, F. Krauss, D. Reichelt, M. Schönherr, JHEP 10 (2023) 091

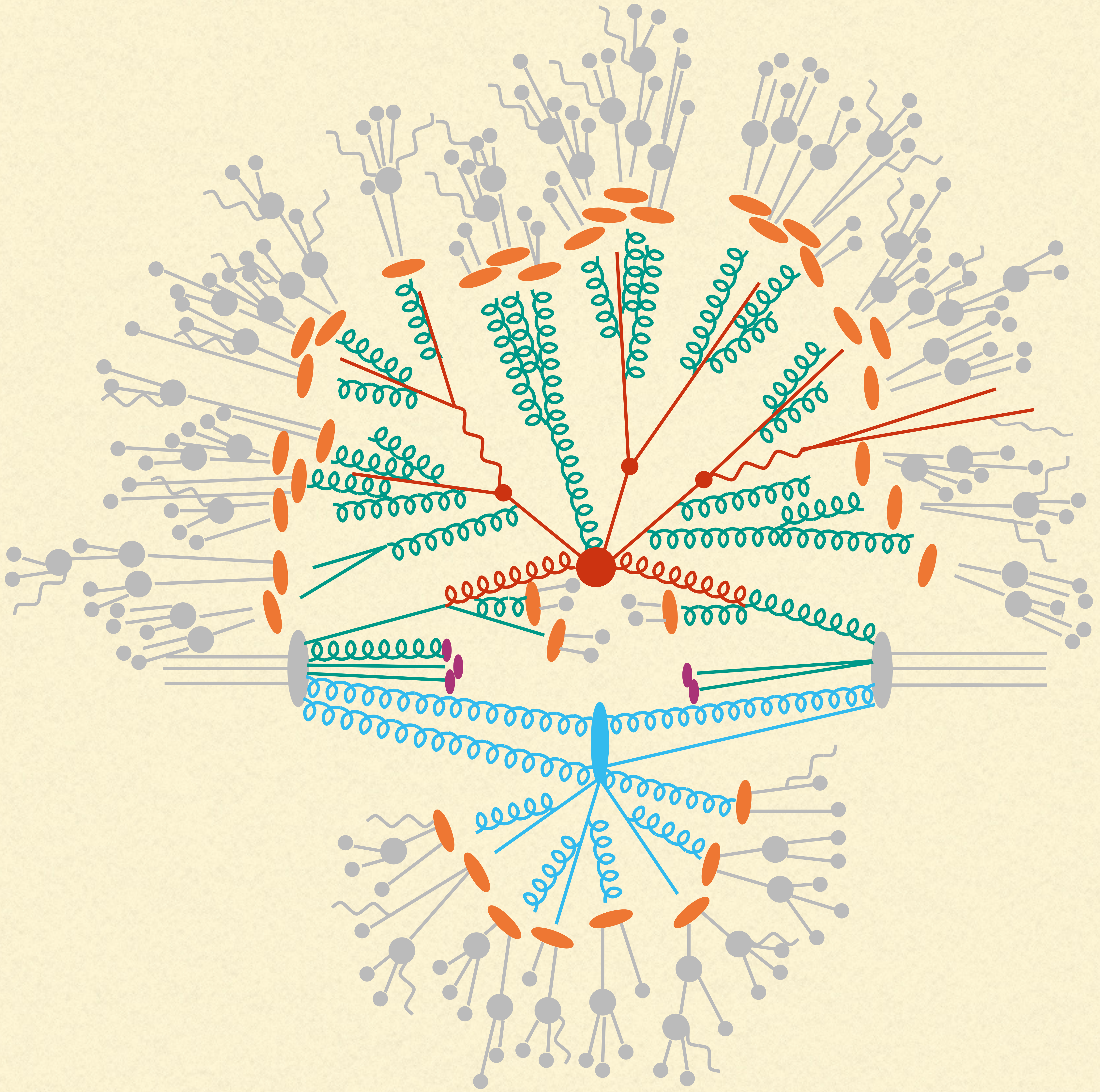


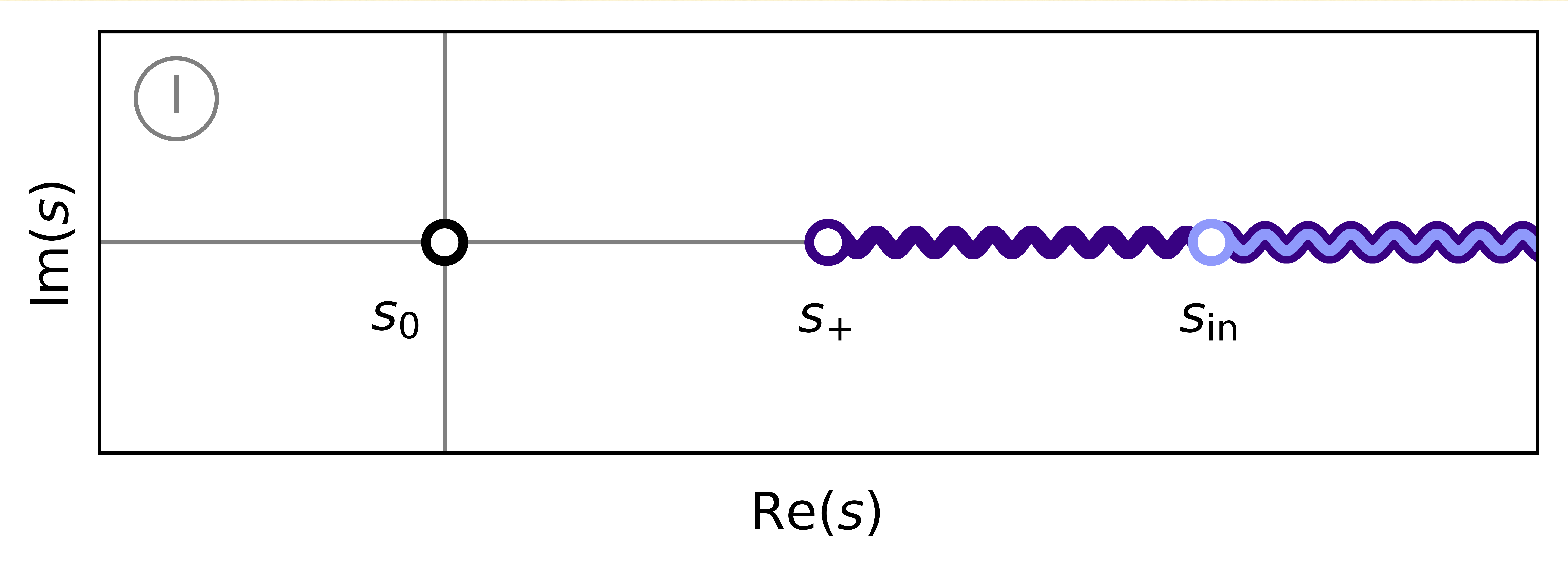


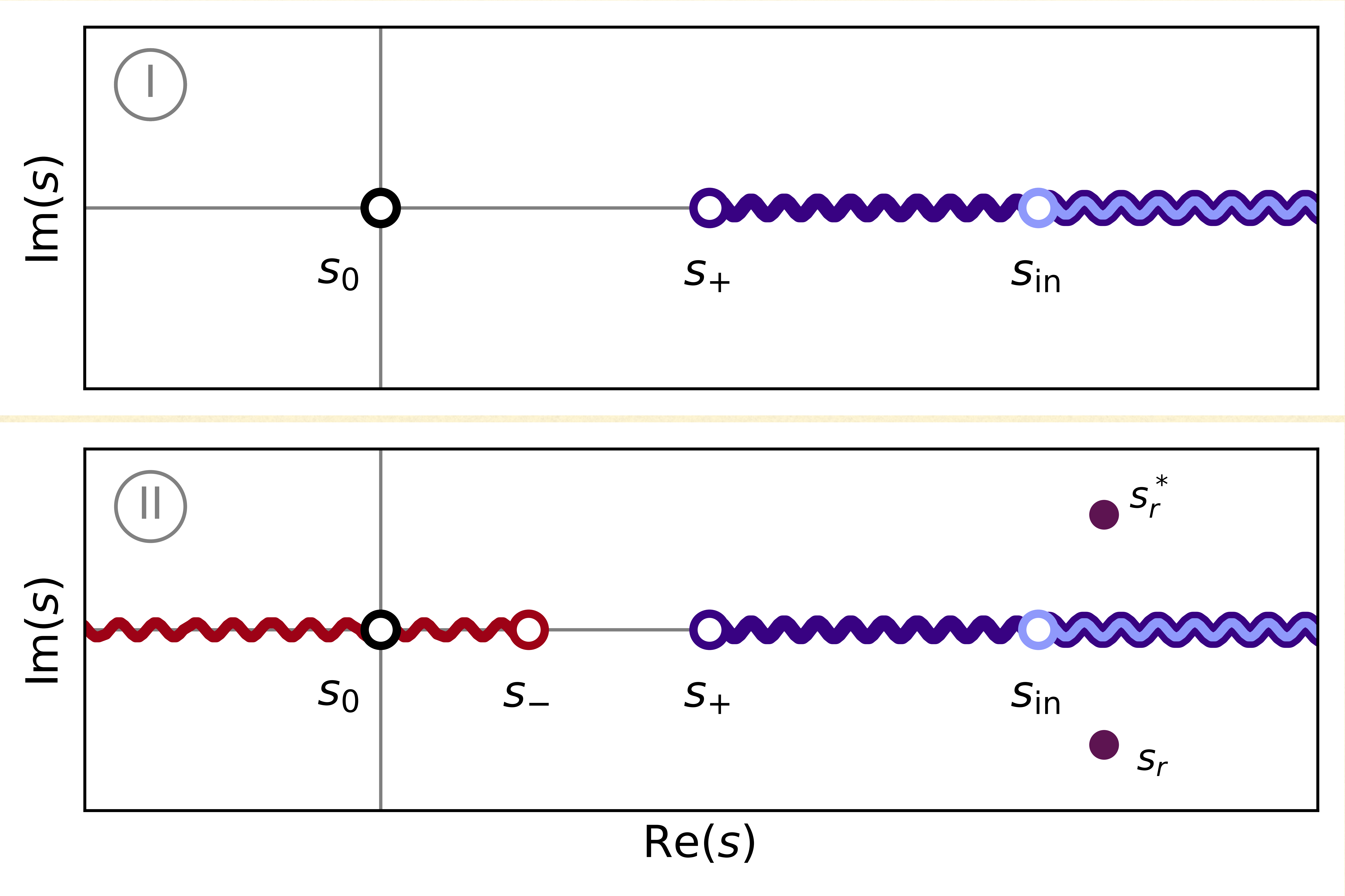
FH, S. Höche, F. Krauss, D. Reichelt, M. Schönherr, JHEP 10 (2023) 091

Sightseeing on the second Riemann sheet

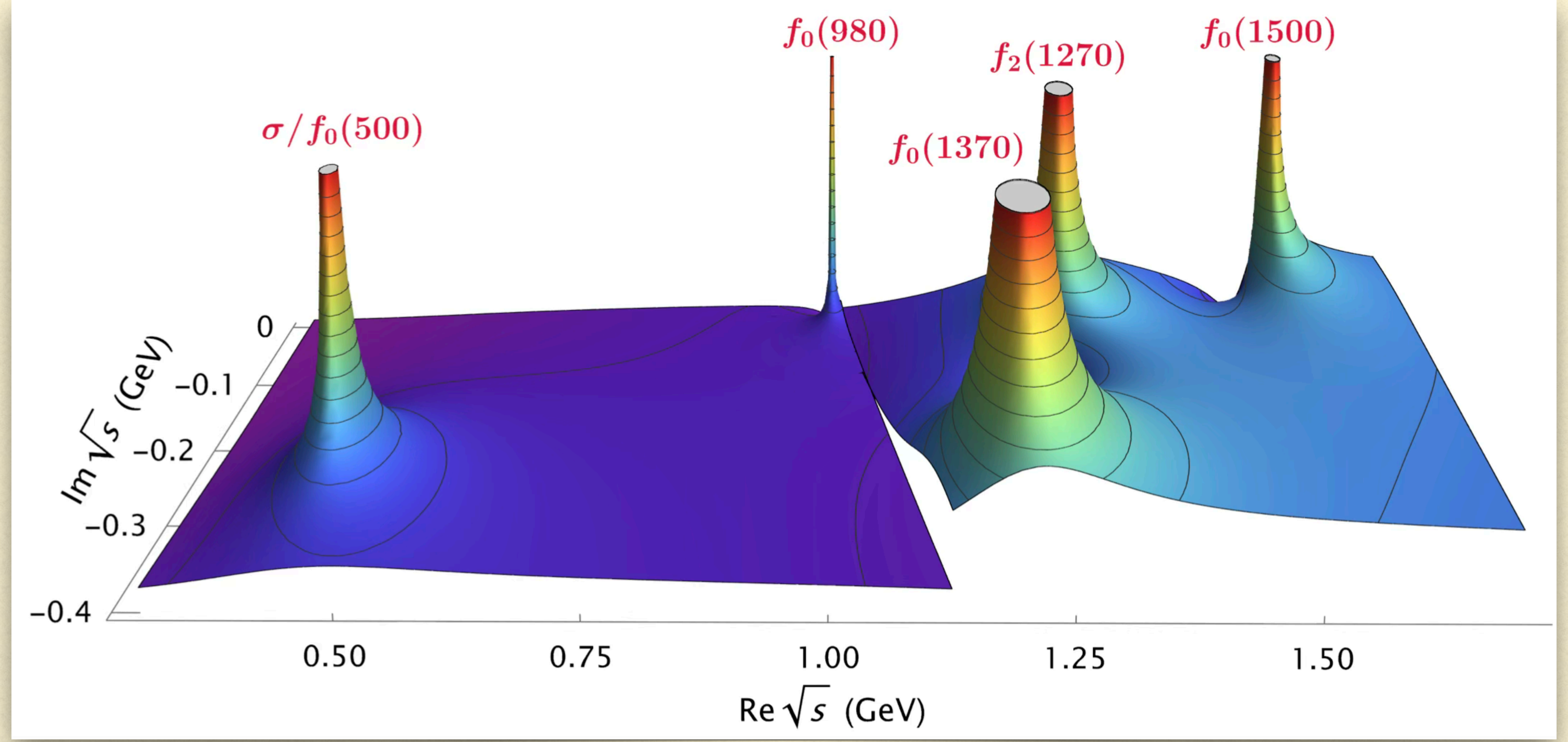
- Production and decay of hadrons described by form factors, i.e. matrix elements of (non-)local currents
- Non-perturbative objects, need to be determined from data or Lattice QCD
- Contain information about resonance poles
- Intricate analytic structure, general model-independent parametrisations complicated



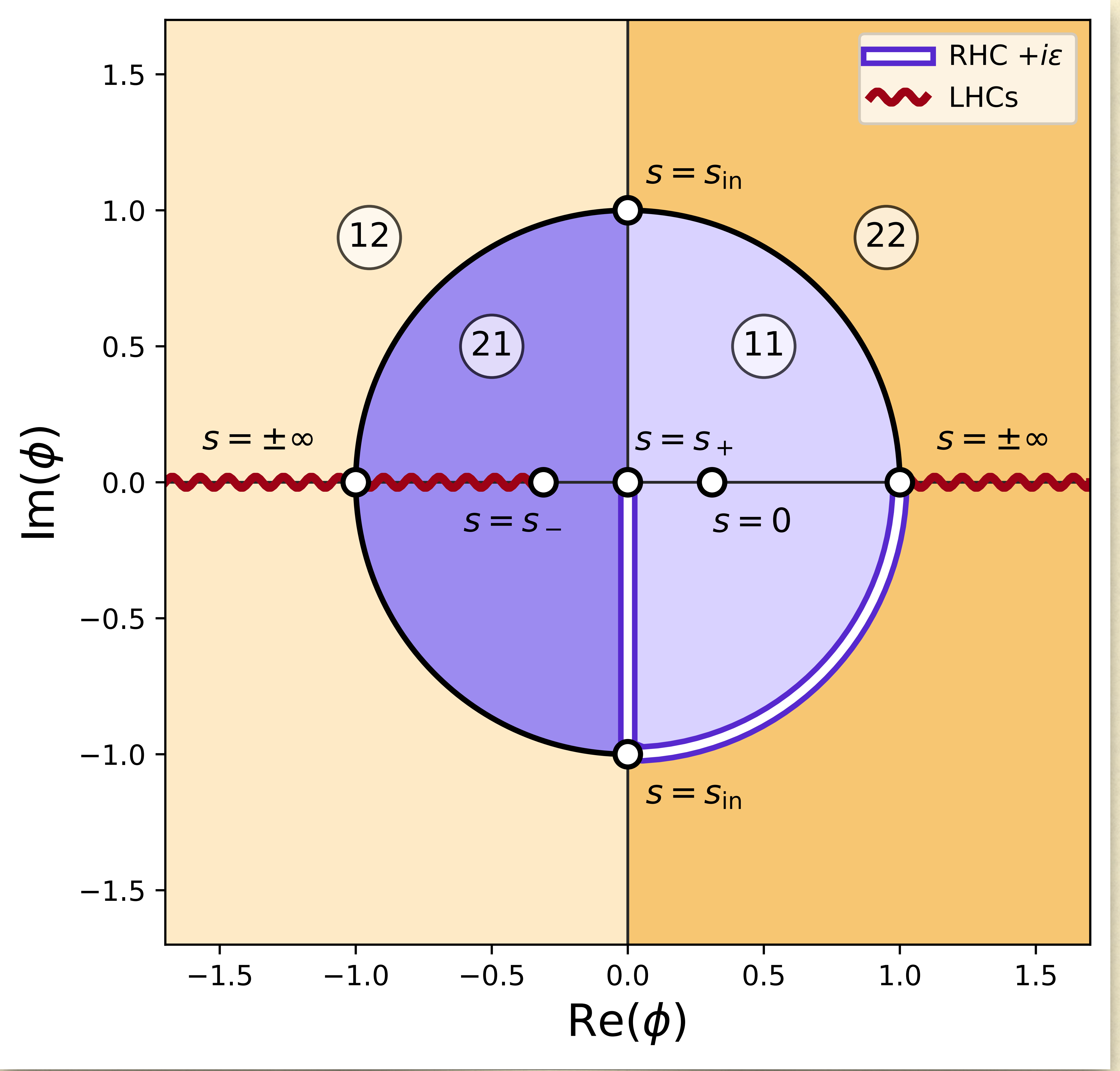

I
Im(s)
s_0
s_+
s_{in}
Re(s)



N. C. Balz, FH, B. Kubis, S. Mutke, M. Reboud, JHEP 01 (2026) 158



Peláez, Rabán, Ruiz de Elvira PRD 113 (2026) 3, 034018



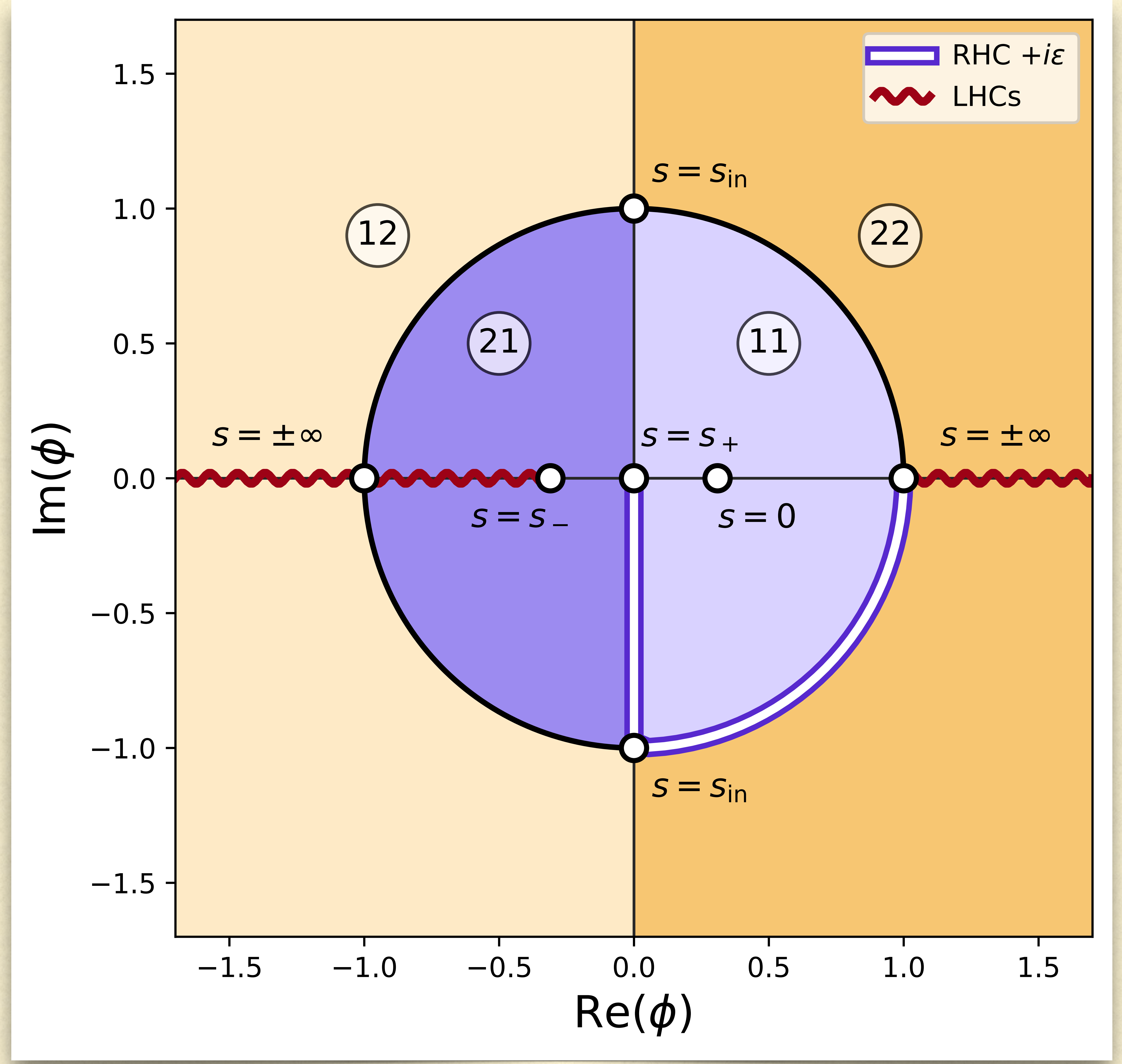
A (not so) clever choice of variables

- Find a variable in which form factors take a simple form: Laurent expansion
- Well known for RS I
- For two threshold problem a rational transformation is possible

$$s(\phi) = \frac{s_+(1+\phi^2)^2 - 4s_{in}\phi^2}{(1-\phi^2)^2}$$

$$F(s) = \prod_i \frac{1}{\phi - \phi_i} \sum_j a_j \phi^j$$

N. C. Balz, FH, B. Kubis, S. Mutke, M. Reboud, JHEP 01 (2026) 158



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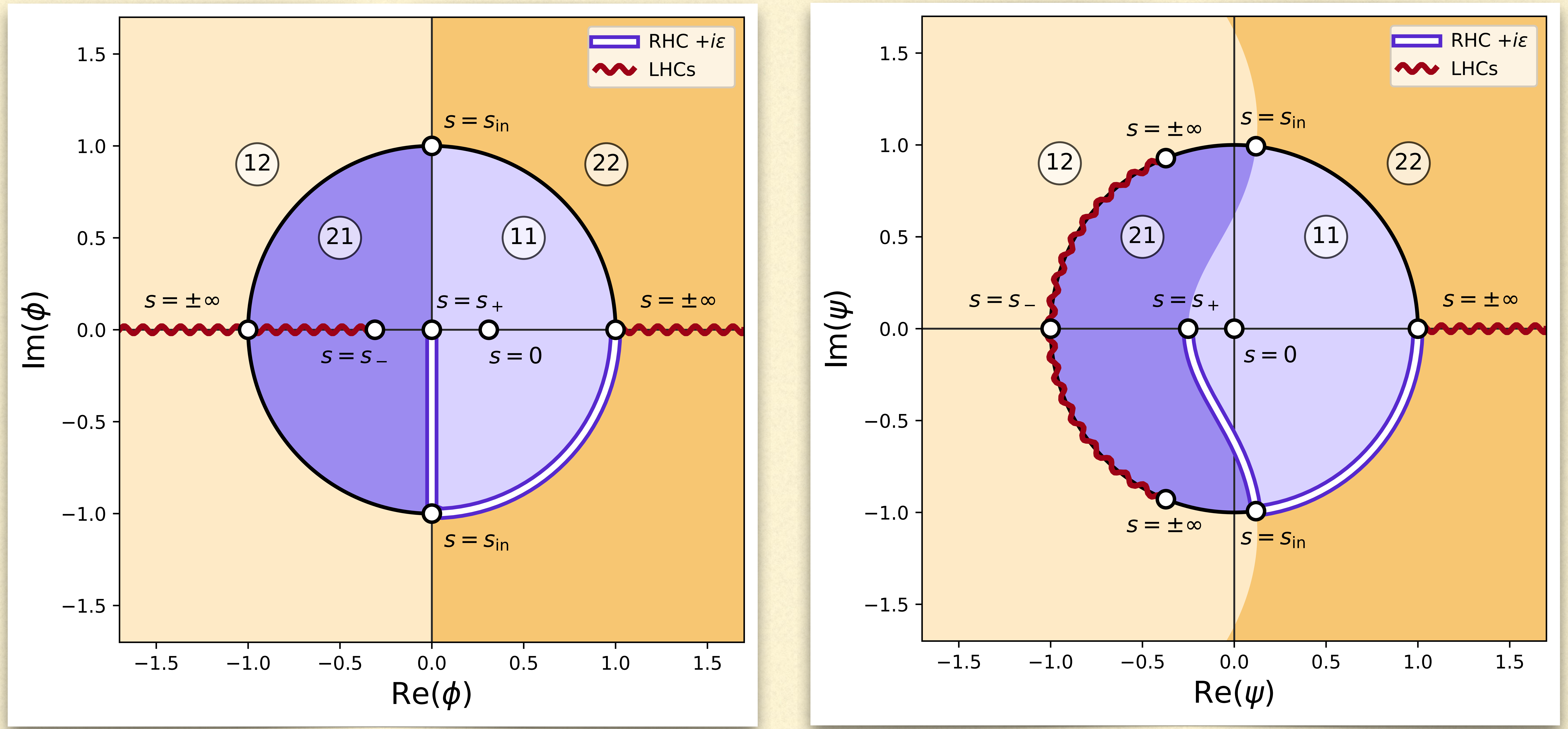
- Left-hand cut strikes back…
- Can we do better?

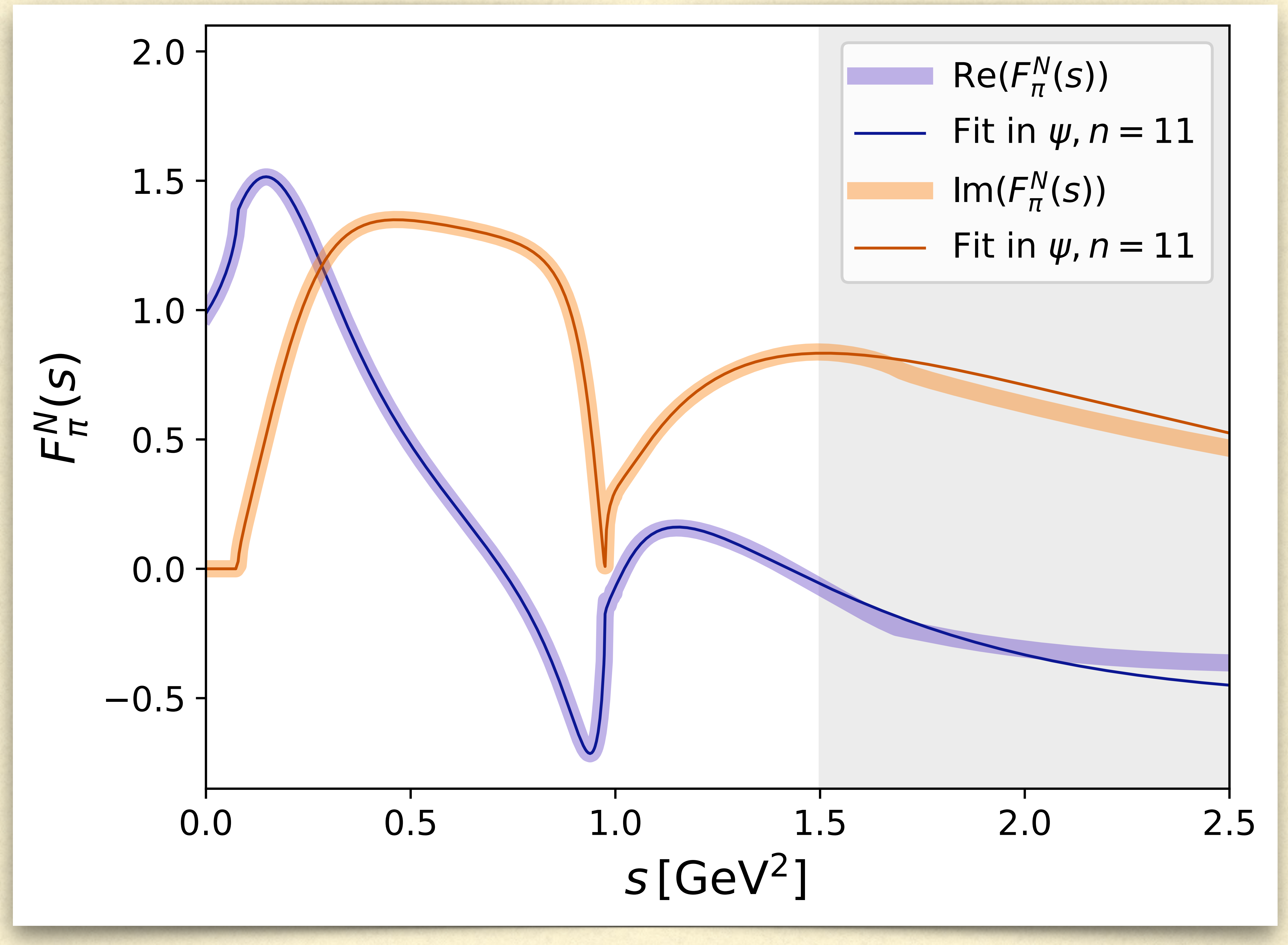
N. C. Balz, FH, B. Kubis, S. Mutke, M. Reboud, JHEP 01 (2026) 158



N. C. Balz, FH, B. Kubis, S. Mutke, M. Reboud, JHEP 01 (2026) 158

Does it work?

- Tests on non-trivial form factors promising
- Reproduce resonance properties of $\rho, f_0(500)$ and $f_0(980)$ on theoretical input
- Can disentangle resonance couplings to final state and current
- Fits to data in progress



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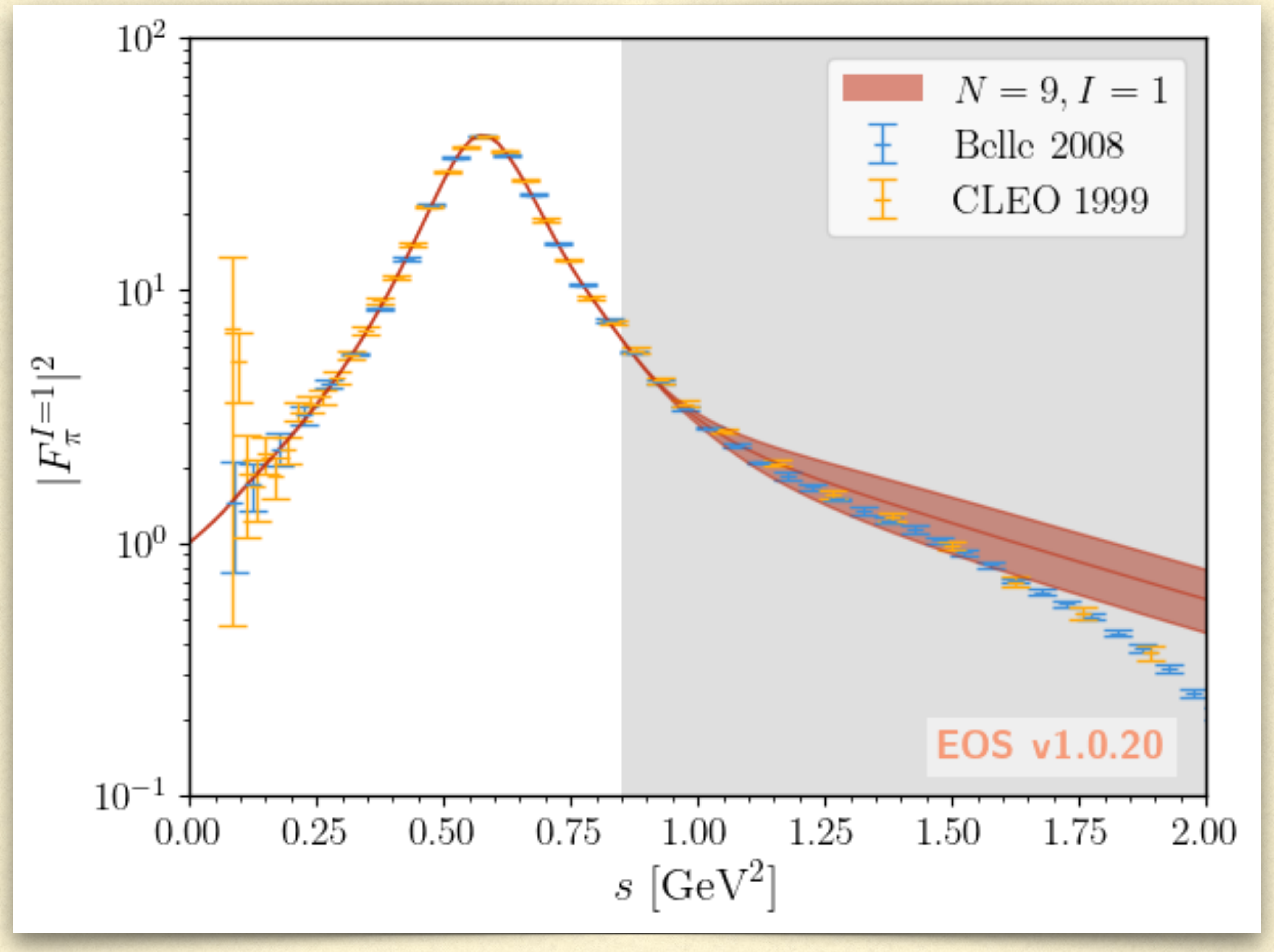
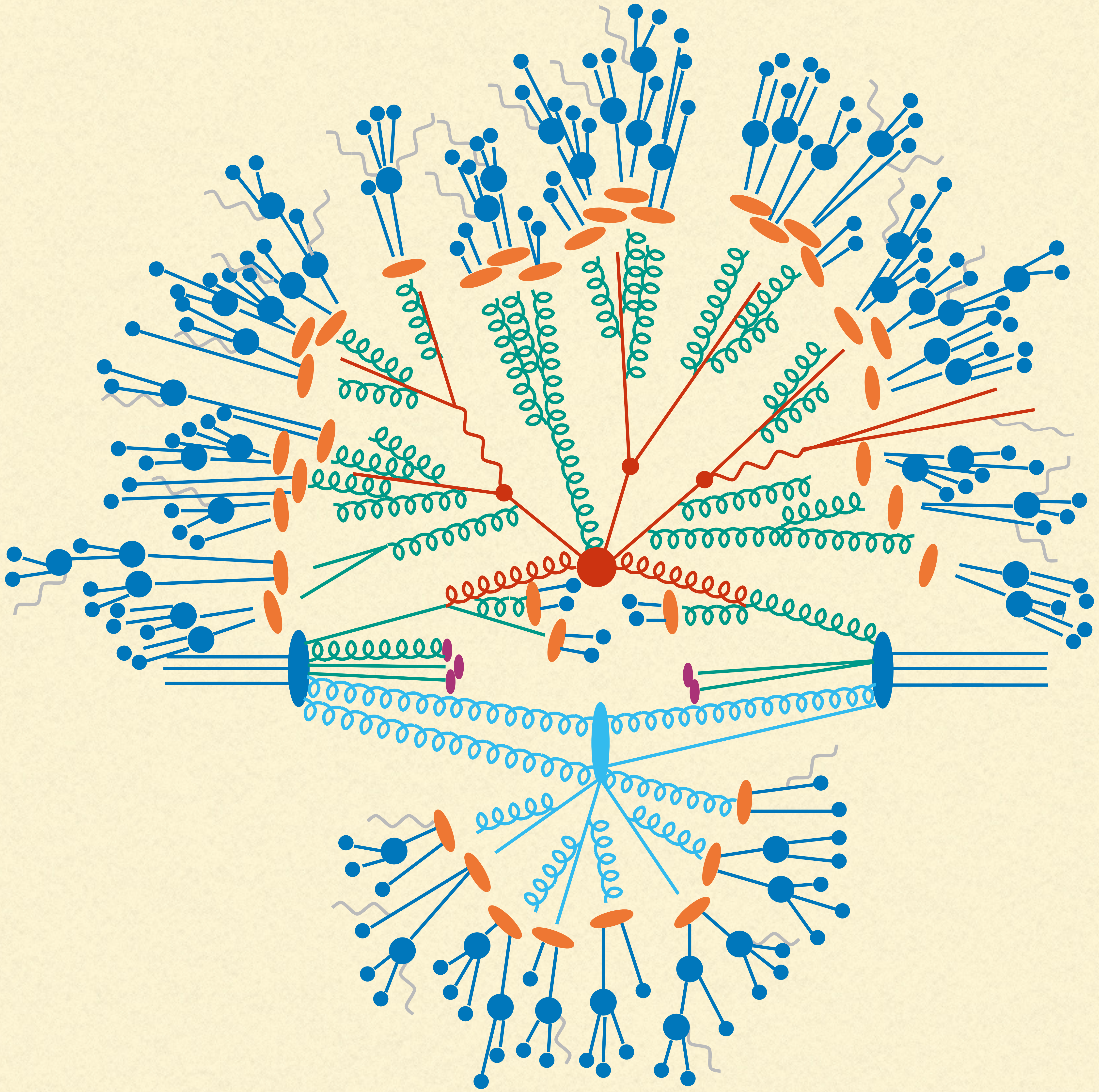
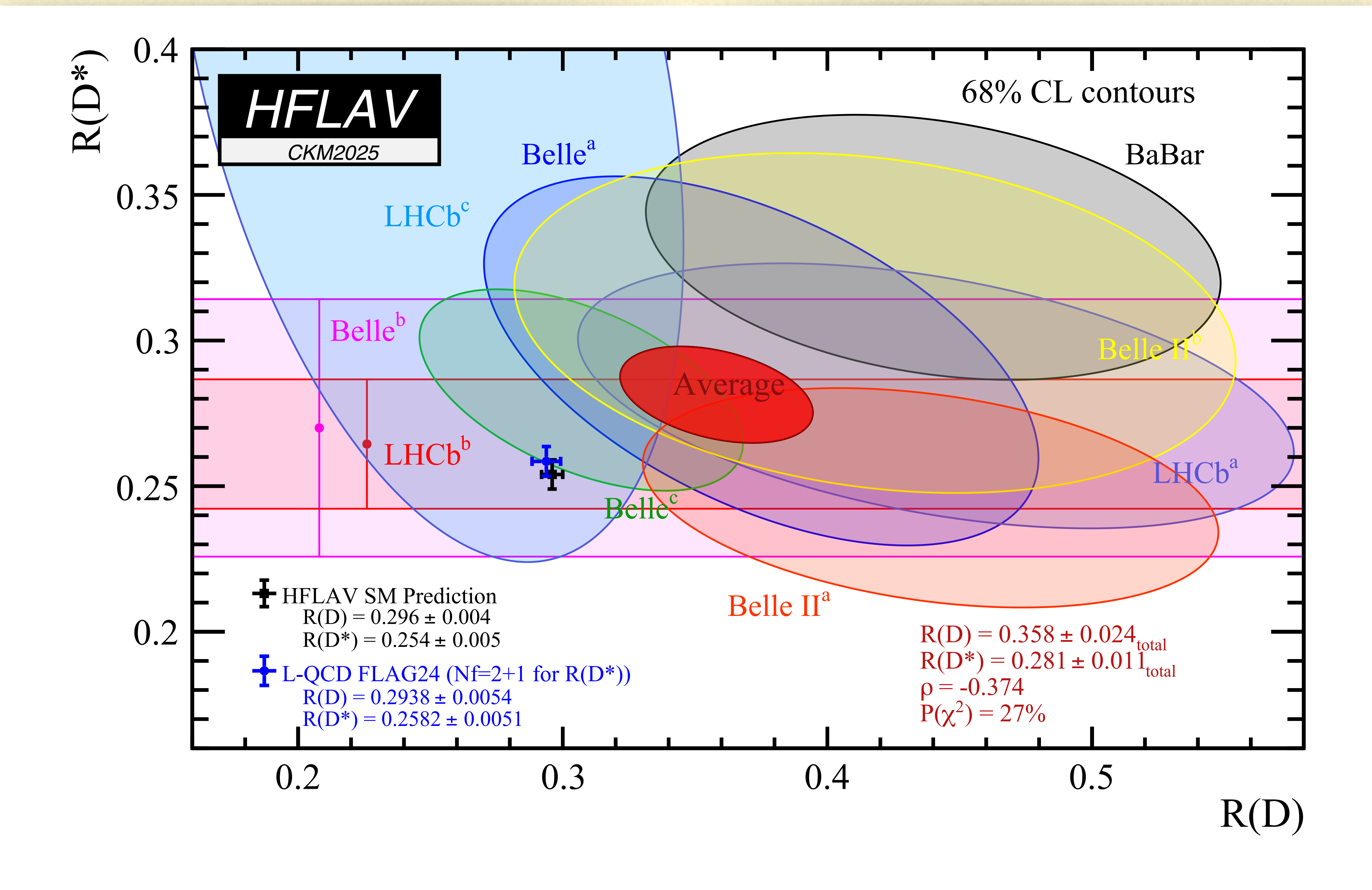


N. C. Balz, FH, B. Kubis, F. Nouri, S. Mutke, M. Reboud, WIP

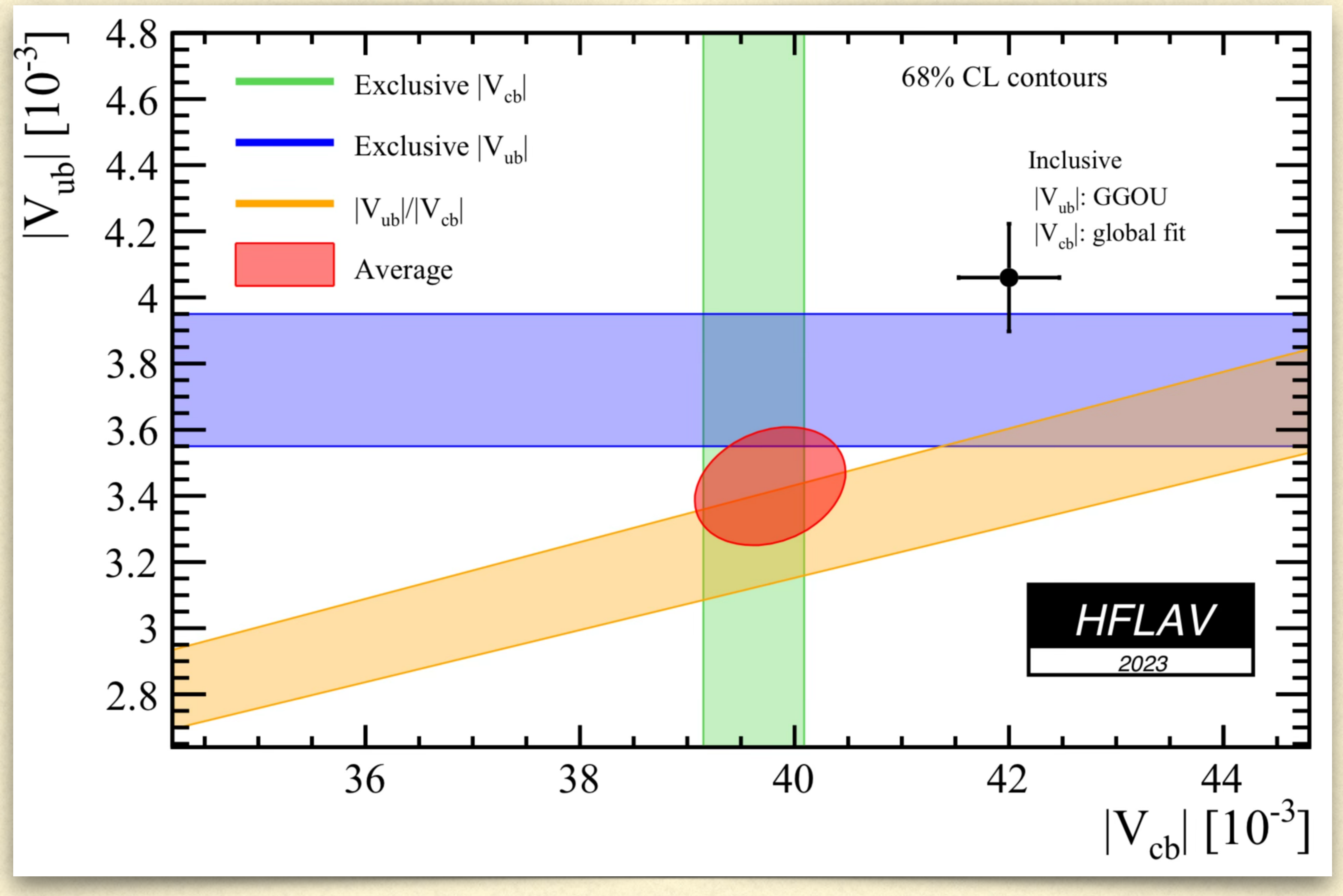
Hadron decay safari

- Decays of heavy hadrons important tools to study weak interaction, CP violation, hadron spectrum
- Today's focus: Semileptonic B decays
- Good understanding of hadronic form factors and hadron spectrum required for precision measurements and SM tests
- Exclusive & inclusive decays provide orthogonal access to similar quantities





HFLAV, CKM 2025 update



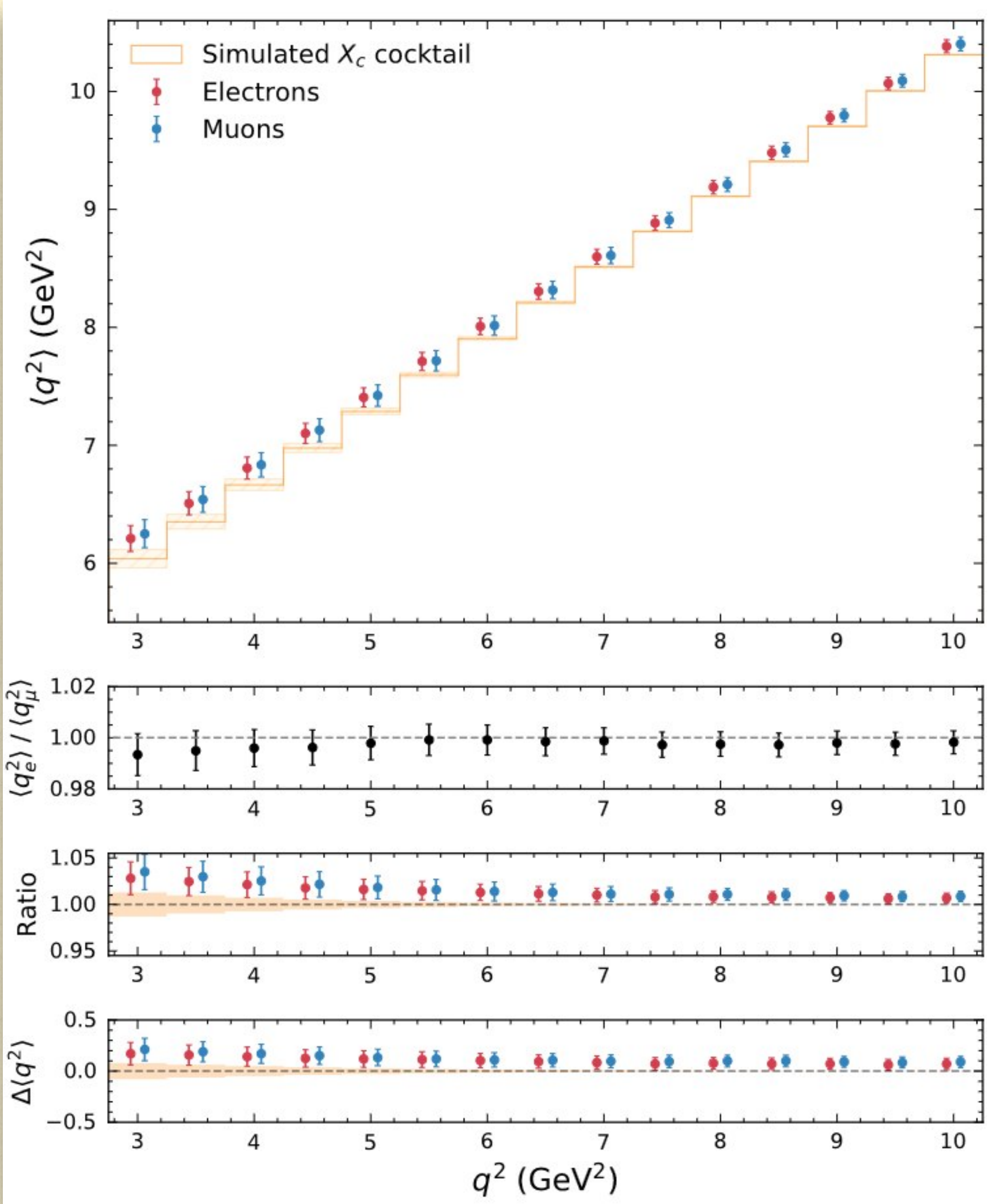
HFLAV, PRD 113 (2026) 012008

Precision for inclusive decays

- Basis: operator product expansion of hadronic tensor

$$W^{\mu\nu} = \frac{(2\pi)^3}{2M_B}\sum_X \langle B|J^{\dagger\mu}|X\rangle\langle X|J^\nu|B\rangle$$

- Leading order: free quark decay
- Nonperturbative matrix elements at higher orders
- NNLO corrections known for most observables
- Quark hadron duality: observables need to be sufficiently smeared
- Observables: $E_l^B, M_X^2, q^2, \mathscr{A}_{FB}$

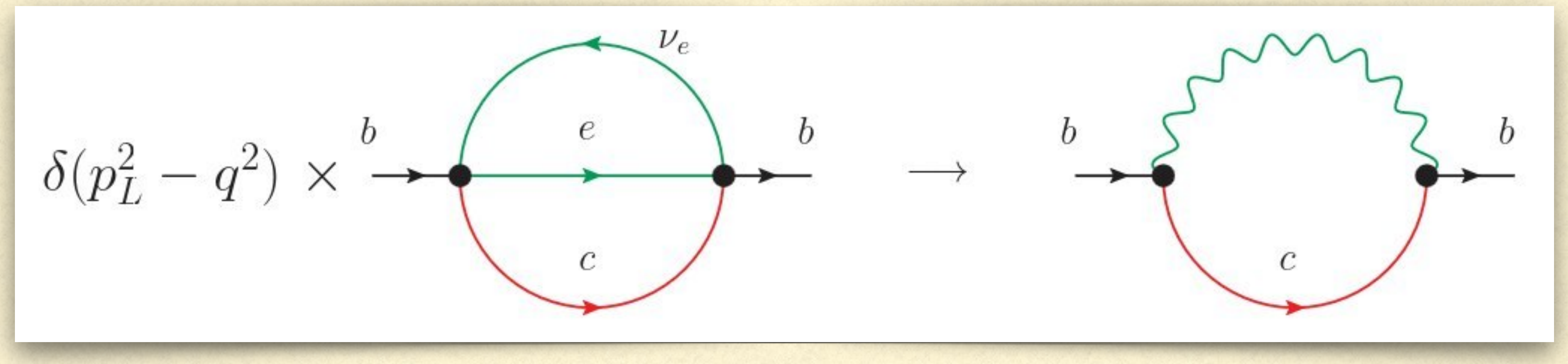


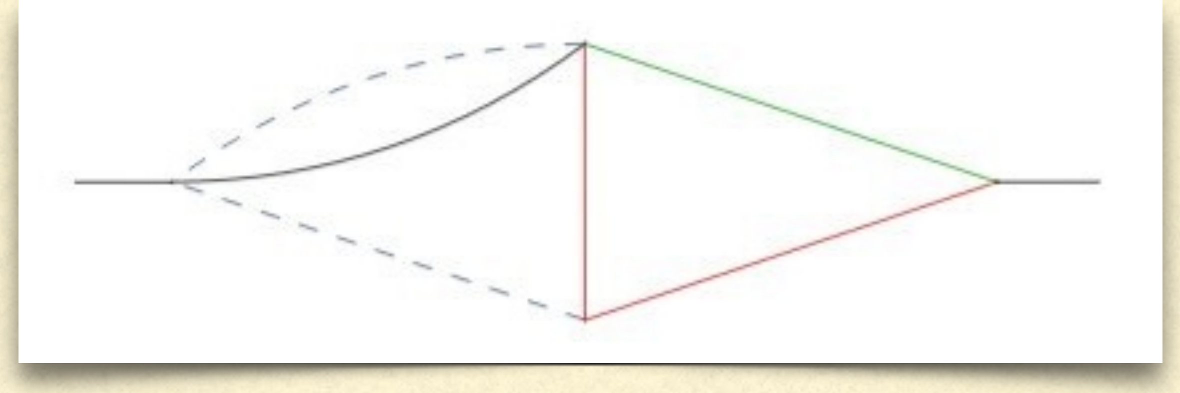
R. van Tonder et al. (Belle), PRD 104 (2021) 11, 112011

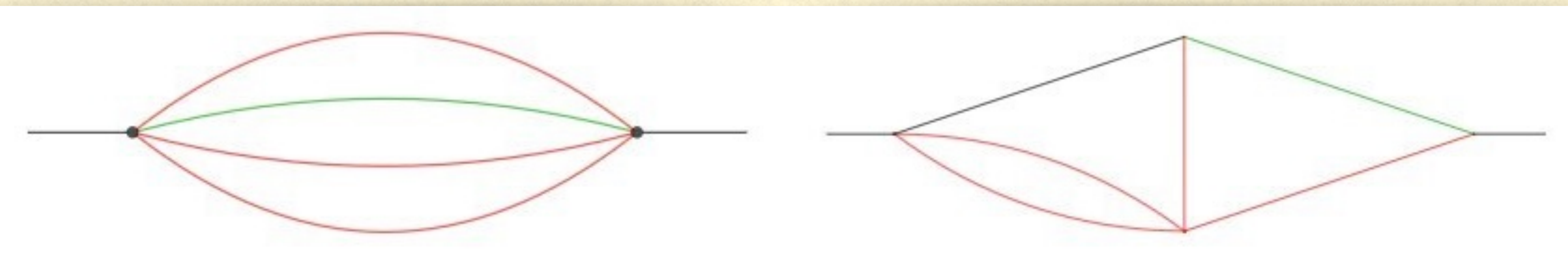
M. Gerlach, FH, M. Lang, CPC 282 (2023) 108544

More loops, more fun

- Missing piece: q^2-spectrum at NNLO
- Re-write phase-space integrals via reverse unitarity
- Integrate out leptons
- Three-loop calculation involving 3 internal masses
- Automated calculation of amplitude with Tapir & FORM, integration by parts reduction with Kira
- 87 Master integrals







M. Fael, FH, JHEP 05 (2024) 287

More loops, more fun

- Derive differential equations for MIs in $\rho = m_c/m_b$ and $\hat{q}^2 = q^2/m_b^2$

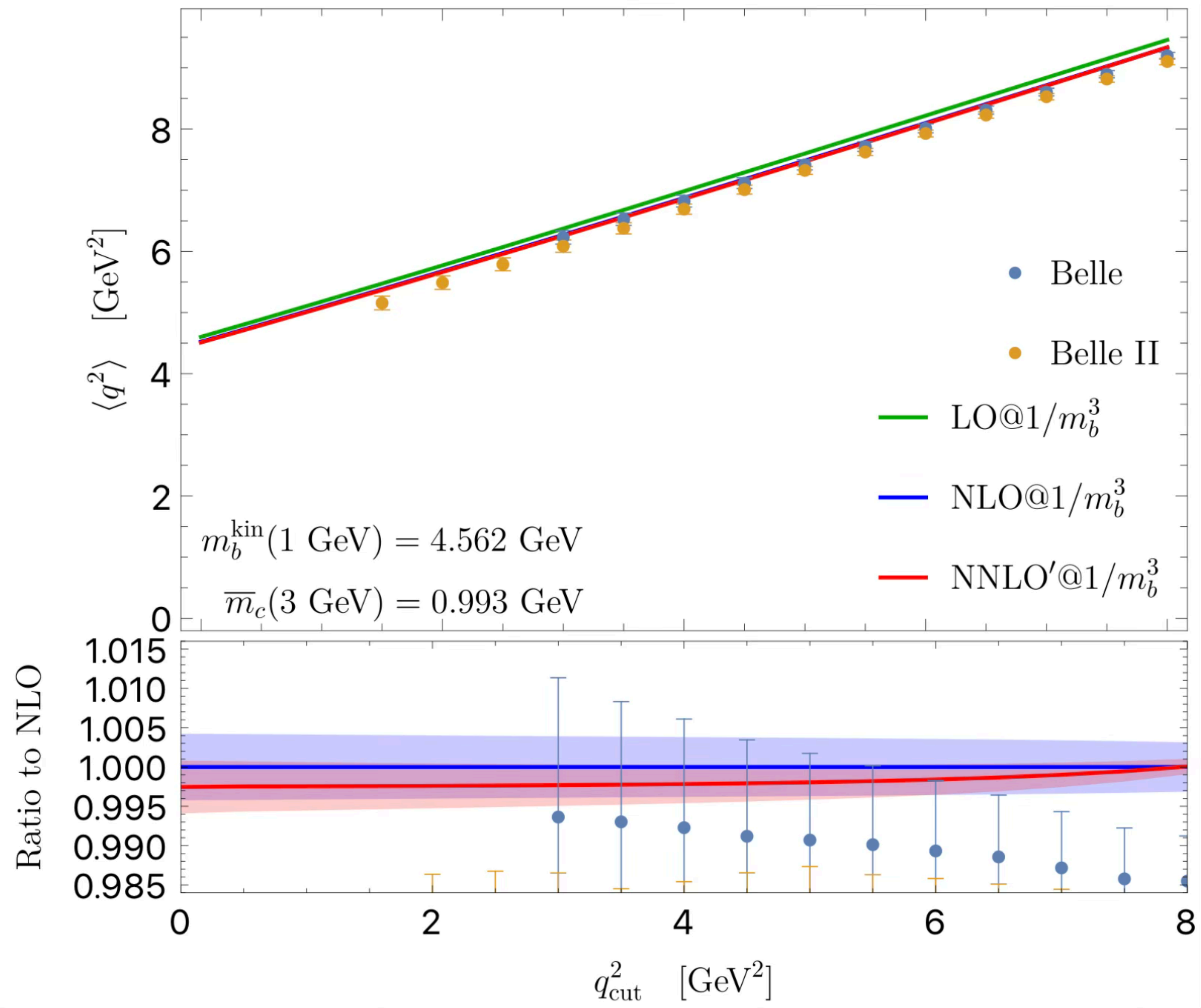
$$\frac{\partial I}{\partial \rho} = M_\rho(\rho, \hat{q}^2, \epsilon) I \qquad \frac{\partial I}{\partial \hat{q}^2} = M_{\hat{q}^2}(\rho, \hat{q}^2, \epsilon) I$$

- Change of variables and balance transformations

$$\rho = v \qquad \hat{q}^2 = \frac{u - v - u^2 v + u v^2}{u}$$

$$\frac{\partial \tilde{I}}{\partial v} = \epsilon M_v(v, u) \tilde{I} \qquad \frac{\partial \tilde{I}}{\partial u} = \epsilon M_u(v, u) \tilde{I}$$

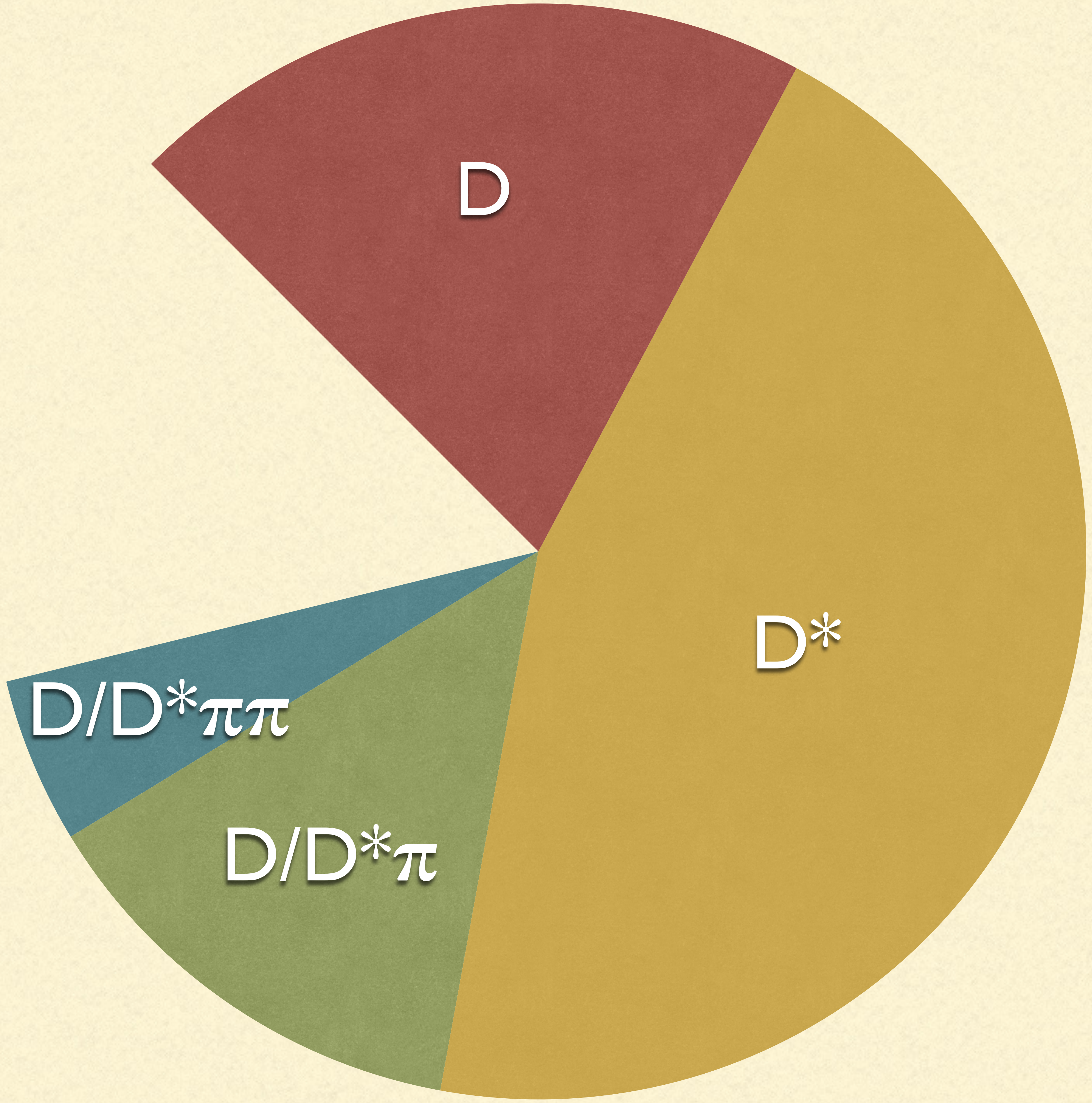
- Express results in terms of Generalized Polylogarithms



M. Fael, FH, JHEP 05 (2024) 287

A closer look

- Overall: 10.63% per light lepton mode
- Large part of the inclusive rate saturated by (quasi-)three-body decays
- Multi-pion modes measured by Belle and BaBar, two narrow resonances, several broad ones
- Remainder, approximately 1.8%, known as SL gap
- Belle II fills gap with $B \to D^{(*)}\eta\ell\nu$, LHCb with more $B \to D^{(*)}\pi\pi\ell\nu$
- Large systematic uncertainty in inclusive measurements and $R(D^{(*)})$



Following projects together with Raynette van Tonder (Belle/Belle II)

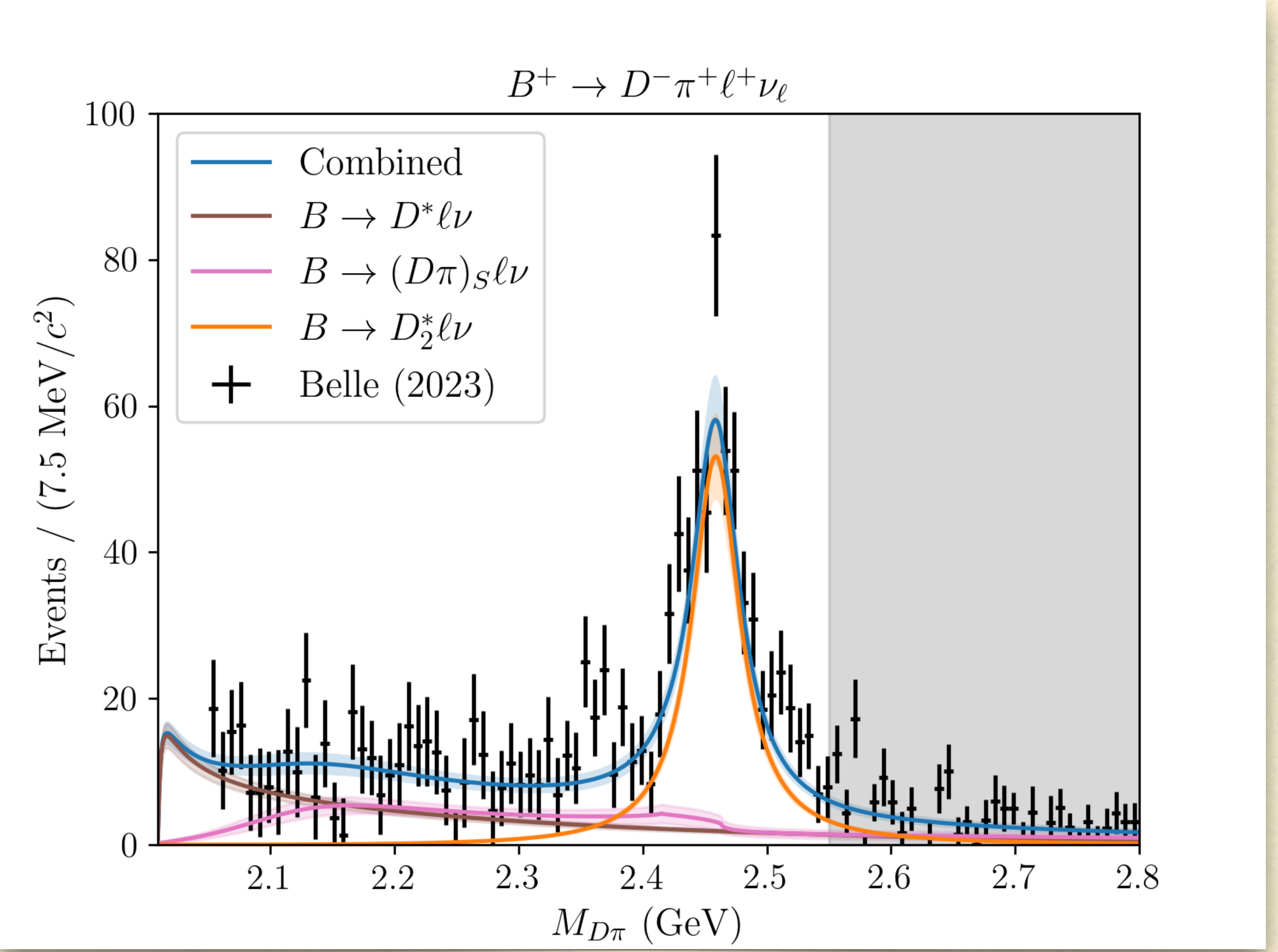
UχPT potential from: L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, U.-G. Meissner, PRD 87 (2013) 1, 014508

The "easy" one: $D\pi$

- Prominent resonance in P-wave, D^*, and D-wave $D_2^*(2460)$
- Treatment in literature: single $D_0^*(2300)$
- Belle data for invariant mass spectrum not compatible with previous modelling
- Intricate S-wave, likely two-pole structure, $D\pi/D\eta/D_sK$ scattering matrix from UχPT

$$f_i(q^2) = T_{ij}(M^2)P_j(q^2)$$

- Relative normalisation fixed by $SU(3)$ symmetry

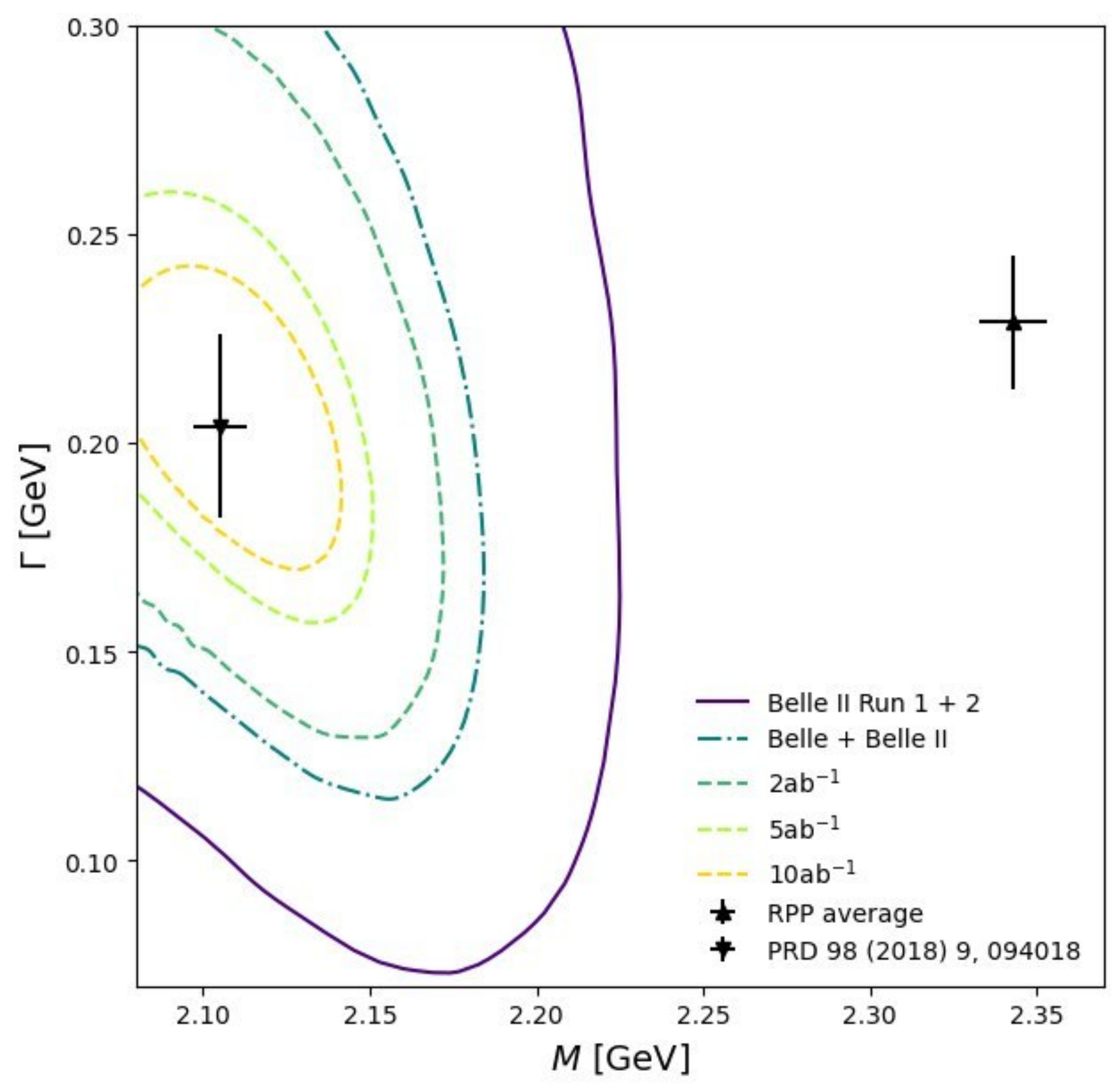


Data from F. Meier et al. (Belle) PRD 107 (2023) 9, 092003

For $B \to D\pi\pi$ see: M.-L. Du, F.-K. Guo, C. Hanhart, B. Kubis, U.-G. Meissner, PRL 126 (2021) 19, 192001

Where is the $D_0^*(2100)$?

- Direct extraction of $D_0^*(2100)$ pole location from $B \to D\pi\pi$ decays complicated by three-body rescattering
- Semileptonic decays offer direct access to differences of scattering phases (example: $K \to \pi\pi\ell\nu_\ell$)
- Ratios of angular asymmetries provide $\tan(\delta_0 - \delta_1)$
- Reference phase: P-wave, nearly constant
- Sensitivity study promising!



M.-L. Du, F.-K. Guo, C. Hanhart, FH, B. Kubis, R. van Tonder, EPJ C 85 (2025) 11, 1289

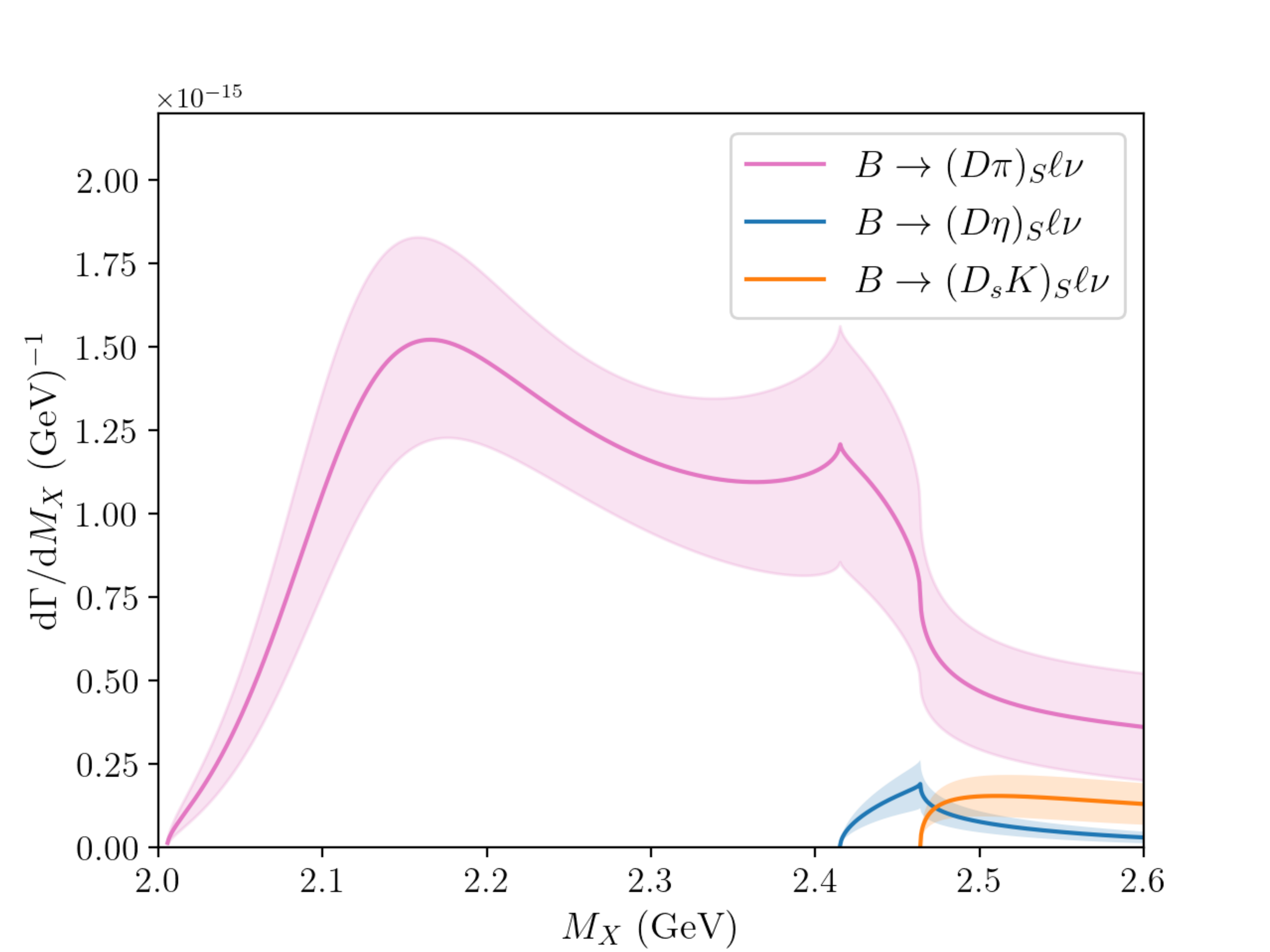
No place for η

- Advantage of the coupled-channel description: predictions for the other two channels

$$\mathscr{B}(B^+ \to (D_s^- K^+)_S \ell^+ \nu_\ell) = (1.58 \pm 0.78) \times 10^{-4}$$

$$\mathscr{B}(B^+ \to (D\eta)_S \ell^+ \nu_\ell) = (0.56 \pm 0.26) \times 10^{-4}$$

- Saturating half of the measured $B^+ \to D_s^- K^+ \ell^+ \nu_\ell$ rate
- $B \to D\eta\ell\nu_\ell$ small component, but interesting once enough data available to resolve lineshape
- Same pattern repeating in the $D^*\pi/D^*\eta/D_s^*K$ system



FH, R. van Tonder, arXiv:2605.03021

So, what about the gap?

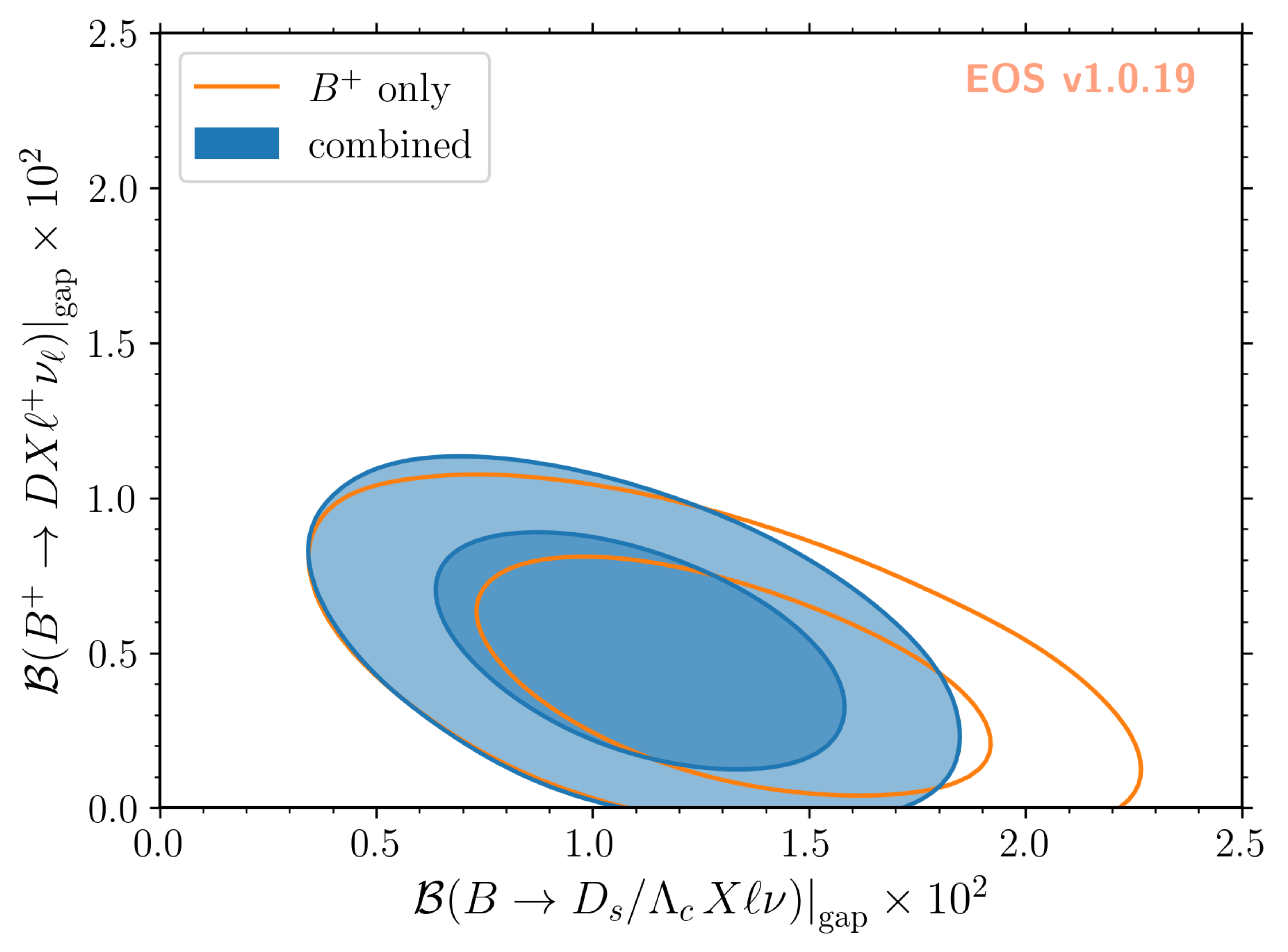
- Fit to all available branching fraction measurements, of particular importance $\mathcal{B}(B \to D^{(*)}\ell\nu)/\mathcal{B}(B \to DX\ell\nu)$:

$$\mathcal{B}(B \to DX\ell\nu) = (9.47 \pm 0.27)\,\%$$

$$\mathcal{B}(B \to D_s/\Lambda_c X\ell\nu)\,|_{gap} = (1.10 \pm 0.29)\,\%$$

- Analysis could be extended with input on inclusive measurements, but depends on modelling of poorly understood $D^{(*)}\pi(\pi)$ modes

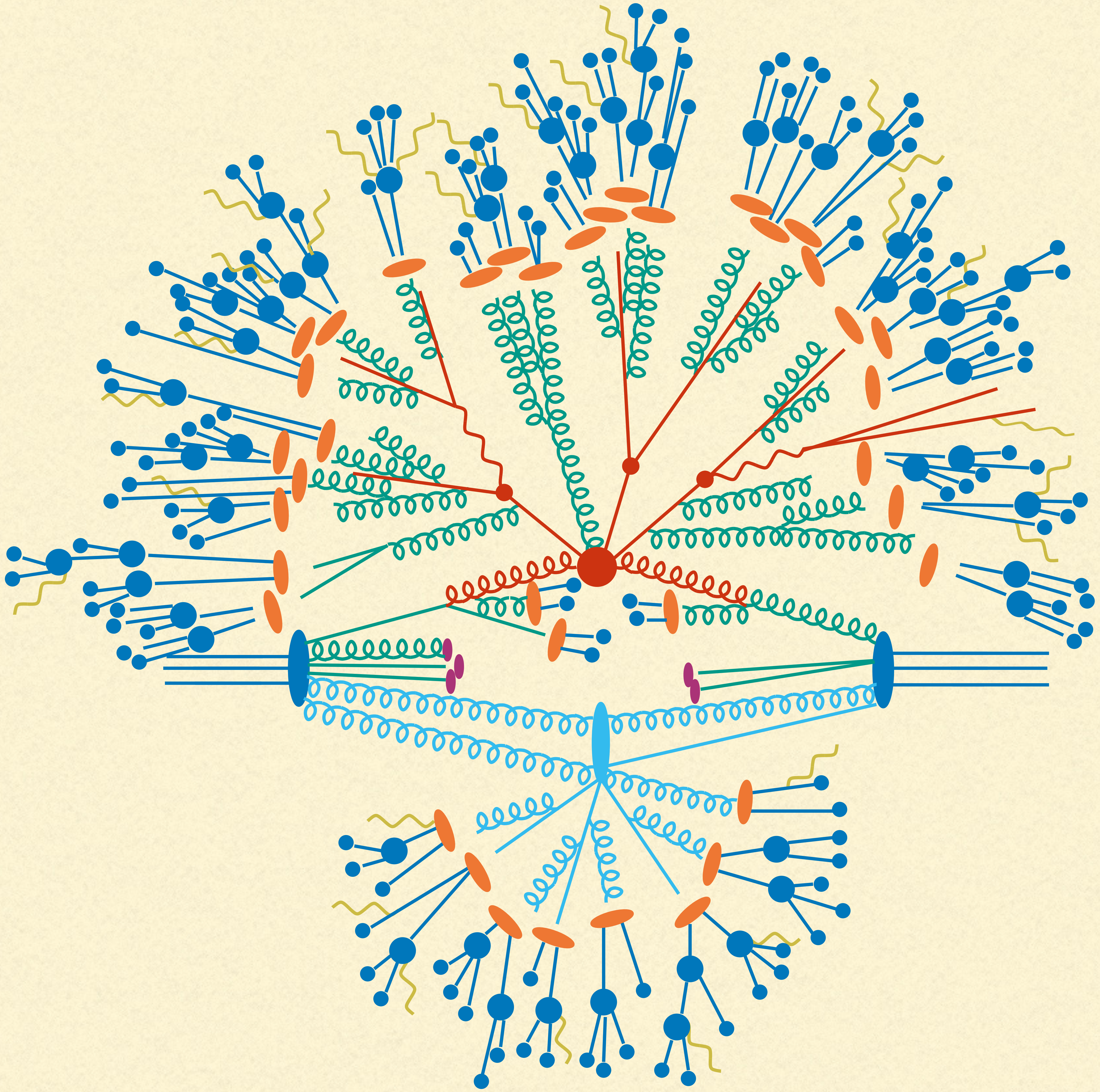
- Need semi-inclusive measurements and direct searches for potential candidates!



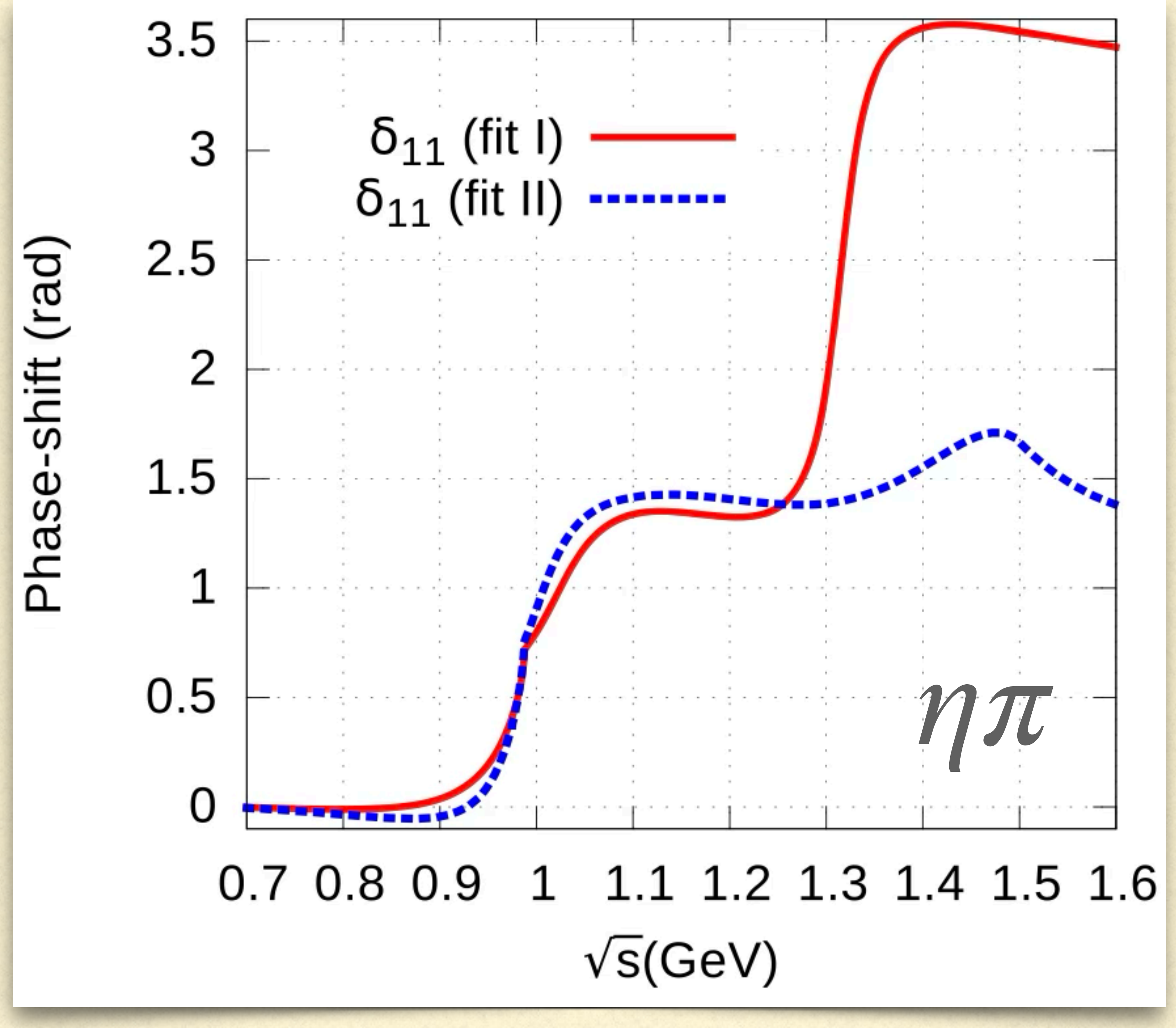
FH, R. van Tonder, arXiv:2605.03021

Back up the energy hill

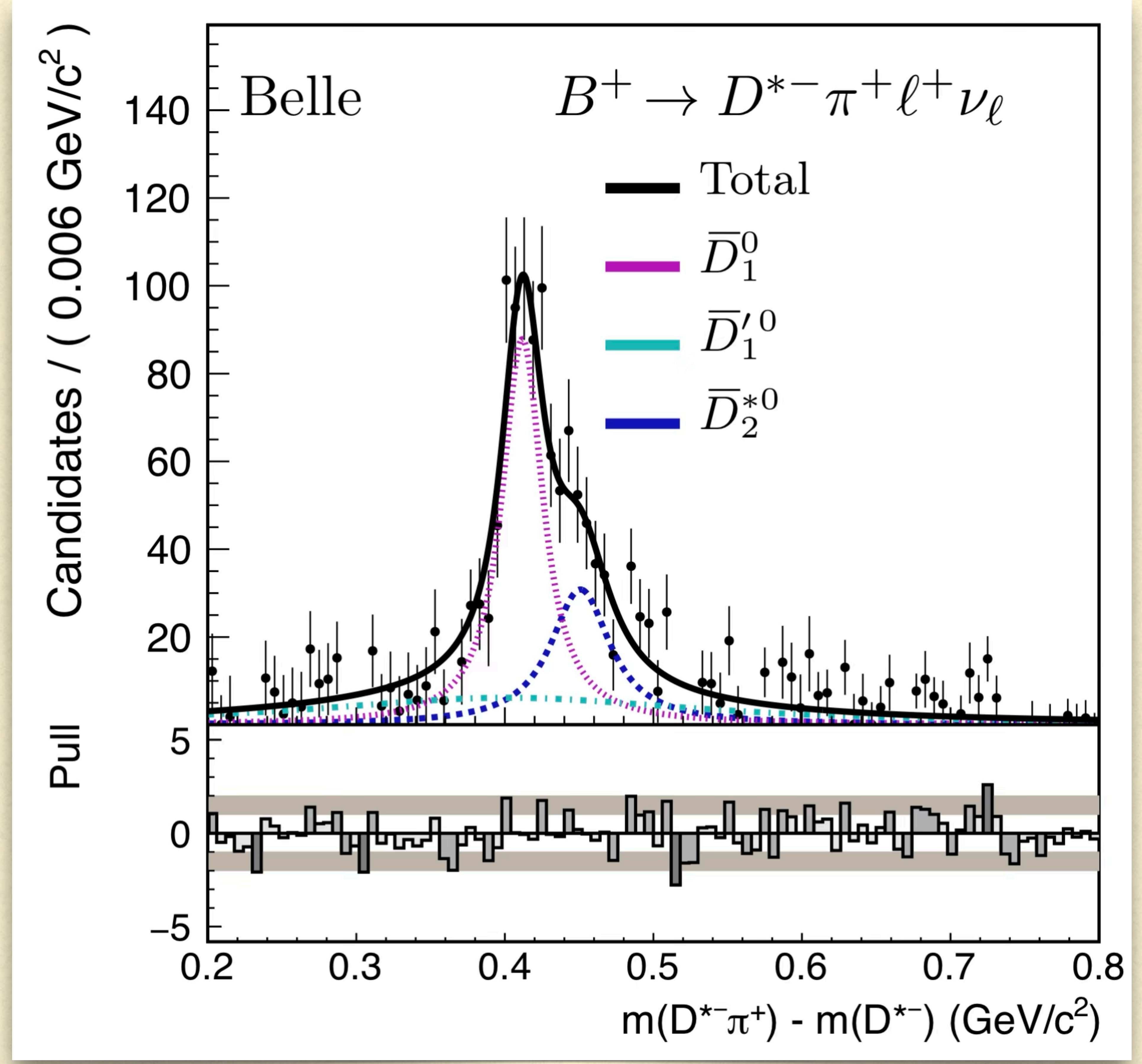
- QCD is interesting across all energy scales
- Interplay of many different approaches
- More theoretical & phenomenological aspects covered in the talks by Philipp and Chiara
- While we climb back up, let me give a brief outlook



Resonances & Semileptonic decays

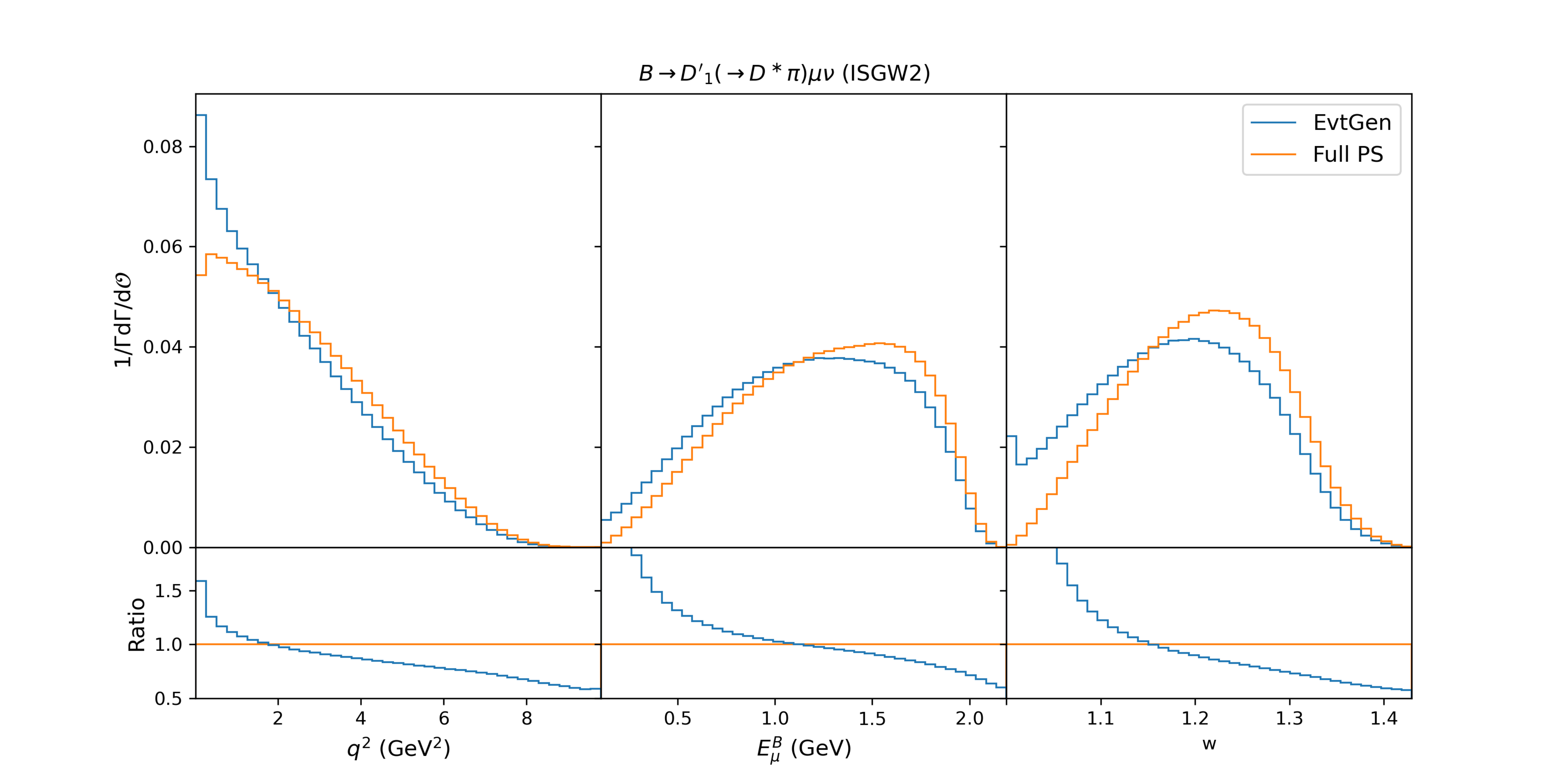


Lu, Moussallam EPJC 80 (2020) 5, 436



F. Meier et al. PRD 107 (2023) 9, 092003

$|V_{xb}|$ & the top quark

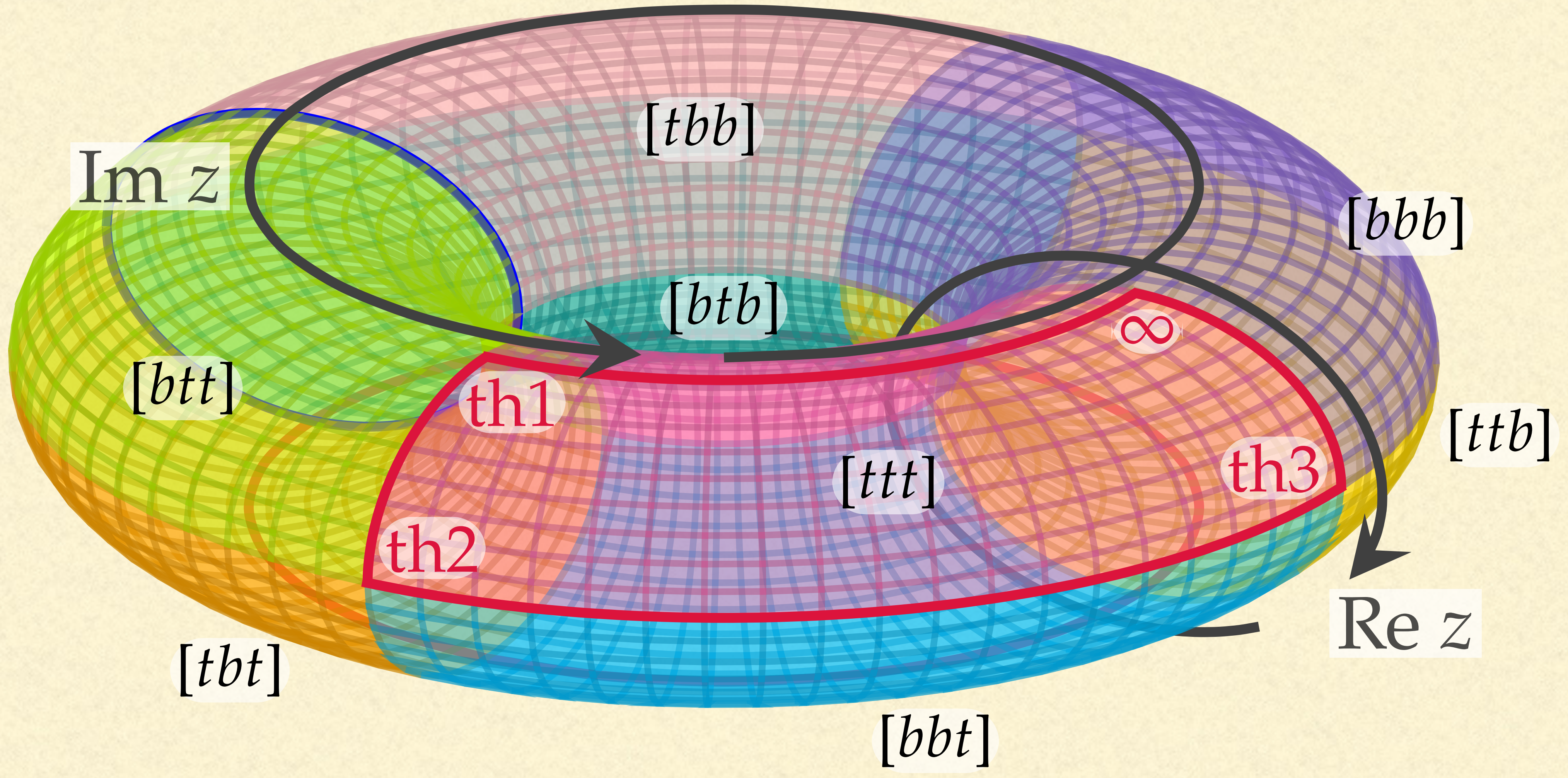


FH, R. van Tonder, SciPost Phys. 20 (2026) 161

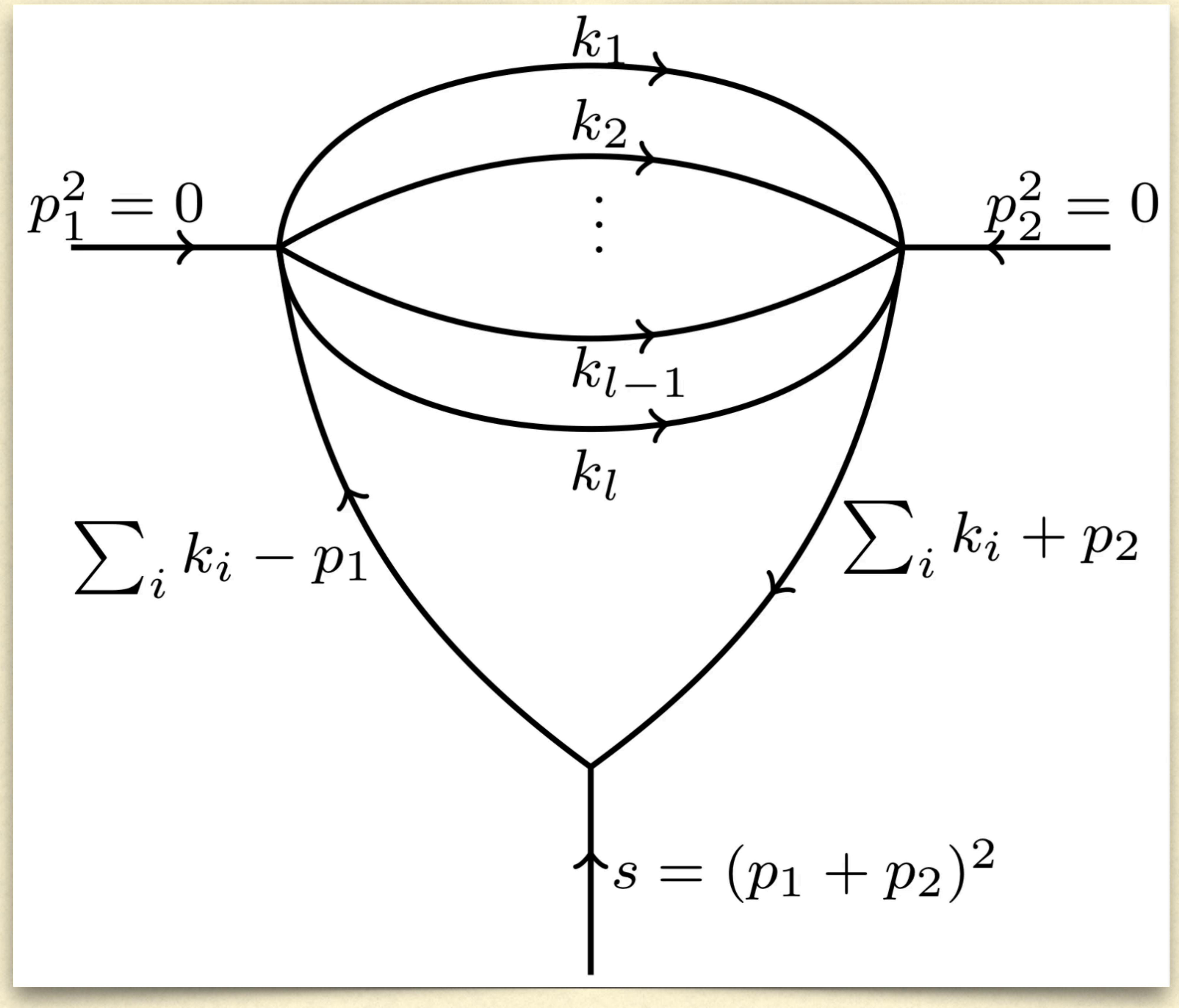
Source	Unc. on m_t [GeV]	Stat. precision [GeV]
Statistical and datasets		
Data statistics	0.39	
Signal and background model statistics	0.17	
Luminosity	< 0.01	± 0.01
Pile-up	0.07	± 0.03
Modelling of signal processes		
Monte Carlo event generator	0.04	± 0.06
b, c-hadron production fractions	0.11	± 0.01
b, c-hadron decay BRs	0.40	± 0.01
b-quark fragmentation r_b	0.19	± 0.06
Parton shower α_S^{FSR}	0.07	± 0.04
Parton shower and hadronisation model	0.06	± 0.07
Initial-state QCD radiation	0.23	± 0.08
Colour reconnection	< 0.01	± 0.02
Choice of PDFs	0.07	± 0.01
Modelling of background processes		
Soft muon fake	0.16	± 0.03
Multijet	0.07	± 0.02
Single top	0.01	± 0.01
W/Z+jets	0.17	± 0.01
Detector response		
Leptons	0.12	± 0.01
Jet energy scale	0.13	± 0.02
Soft muon jet p_{T} calibration	< 0.01	± 0.01
Jet energy resolution	0.08	± 0.07
b-tagging	0.10	± 0.01
Missing transverse momentum	0.15	± 0.01
Total stat. and syst. uncertainties (excluding recoil)	0.77	± 0.03
Recoil uncertainty	0.25	
Total uncertainty	0.81	

ATLAS collaboration, JHEP 06 (2023) 019

Donuts & ice cream!



Yamada, Morimatsu, Sato, PRL 129 (2022), 192001



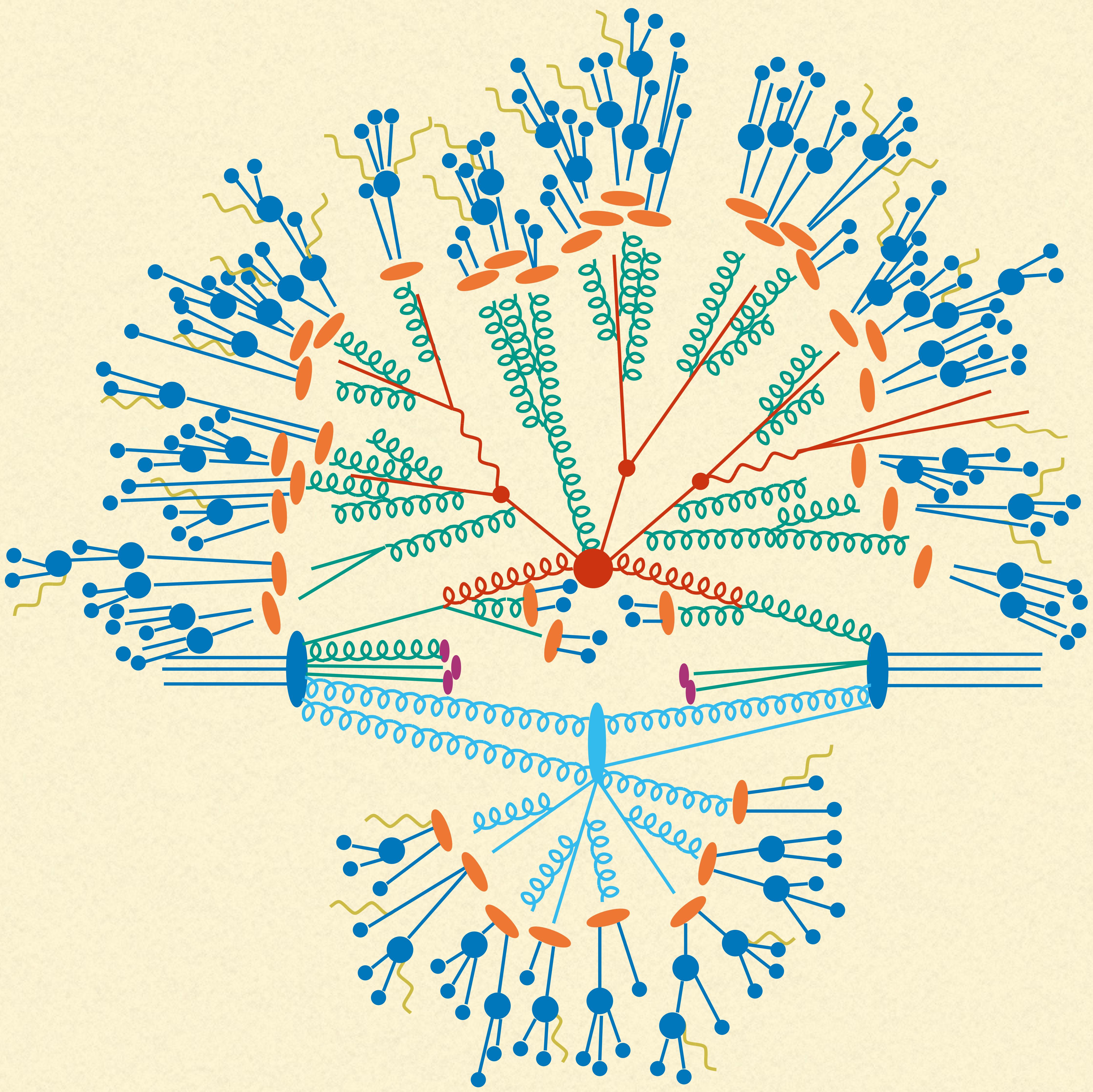
Duhr, Klemm, Nega, Tancredi, JHEP 02 (2023) 228

The end of the journey

There are many synergies between:

- High- and low-energy QCD
- Spectroscopy and precision Flavour physics
- Perturbative and non-perturbative methods

CmF is uniquely suited to exploit these synergies!



Backup

Ingredient 1: $2 \to 2$ scattering

$$\left\langle p_3 p_4; b \,\middle|\, \mathcal{S} - 1 \,\middle|\, p_1 p_2; a \right\rangle = i(2\pi)^4 \delta^{(4)}\left(\sum p_i\right) \mathcal{M}_{ba}(\{p_i\})$$

$$\mathcal{M}_{ab} - \mathcal{M}^*_{ba} = i(2\pi)^4 \sum_c \int \mathrm{d}\Phi_c \mathcal{M}_{ca} \mathcal{M}^*_{cb}$$

$$\mathcal{A}_a - \mathcal{A}^*_a = i(2\pi)^4 \sum_c \int \mathrm{d}\Phi_c \mathcal{M}^*_{ca} \mathcal{A}_c$$

- Simplest scattering process with nontrivial kinematic dependence
- Described by unitary operator $\mathcal{S}$
- Scattering amplitude $\mathcal{M}$ depends on 2 independent Mandelstam variables
- $\mathcal{M}$ real below lowest threshold, imaginary part constrained by Unitarity above
- Two-particle production amplitude $\mathcal{A}$ shares phase with $\mathcal{M}$, e.g. pion production in lepton collisions

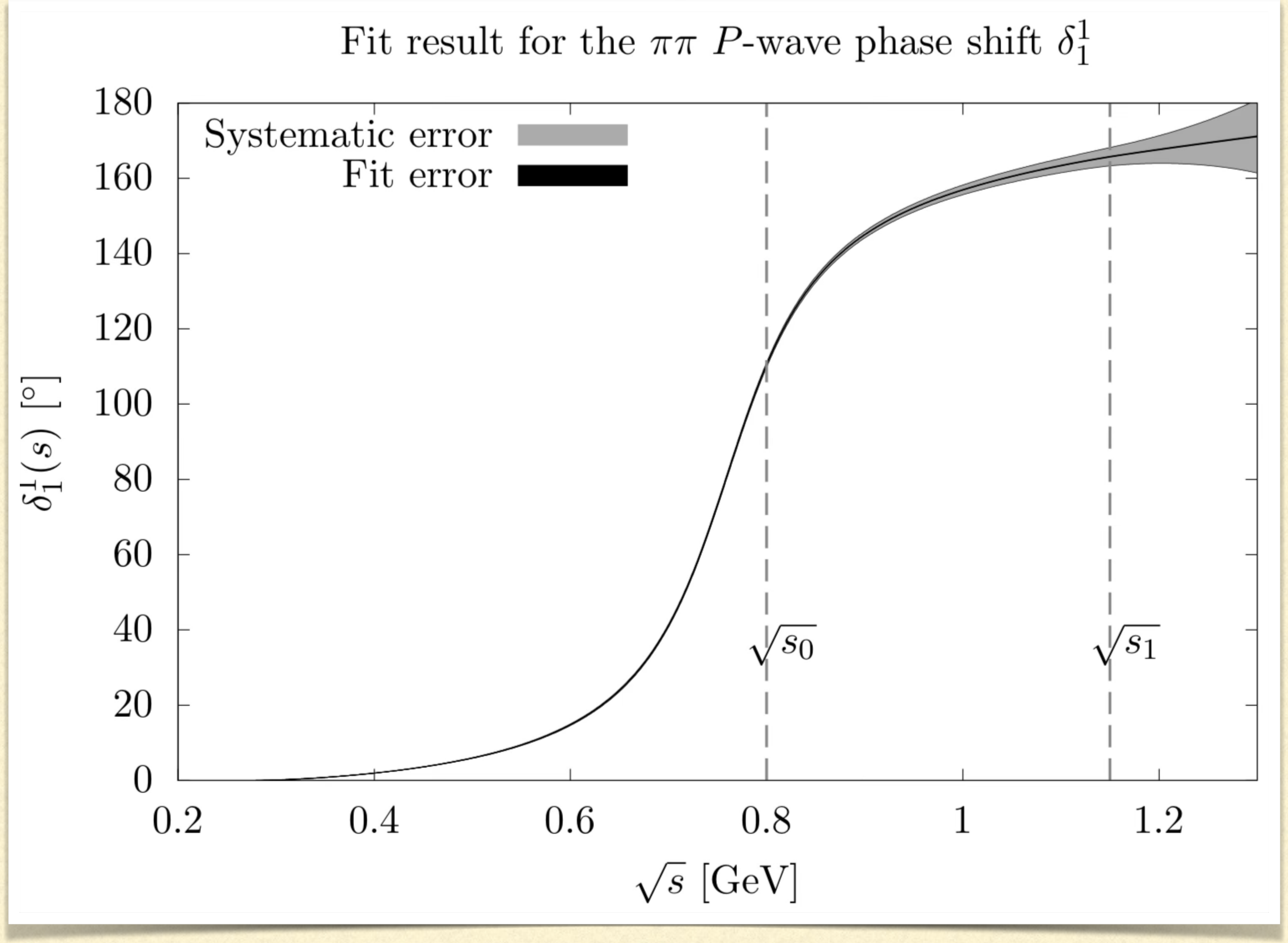
Ingredient 1: Partial waves

$$\mathscr{M}_{ba}(s,t) = \sum_l P_l(\cos\theta)\sqrt{\rho_b}^{-1} f_{ba}(s)\sqrt{\rho_a}^{-1}$$

$$f^l_{aa}(s) = \frac{\eta_l(s)e^{2i\delta_l(s)} - 1}{2i}$$

- Resonances have well-defined spin, their poles only occur in a specific partial wave of $\mathscr{M}$
- Partial-wave expansion conveniently separates different resonances, e.g. in pion scattering: $\rho, f_0(500), f_0(980), f_2(1270)$
- Diagonal elements can be expressed through scattering phase δ_l and inelasticity η_l

Ingredient 1: Partial waves



Colangelo, Hoferichter, Stoffer JHEP 02 (2019) 006

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Ingredient 1: Phase shifts

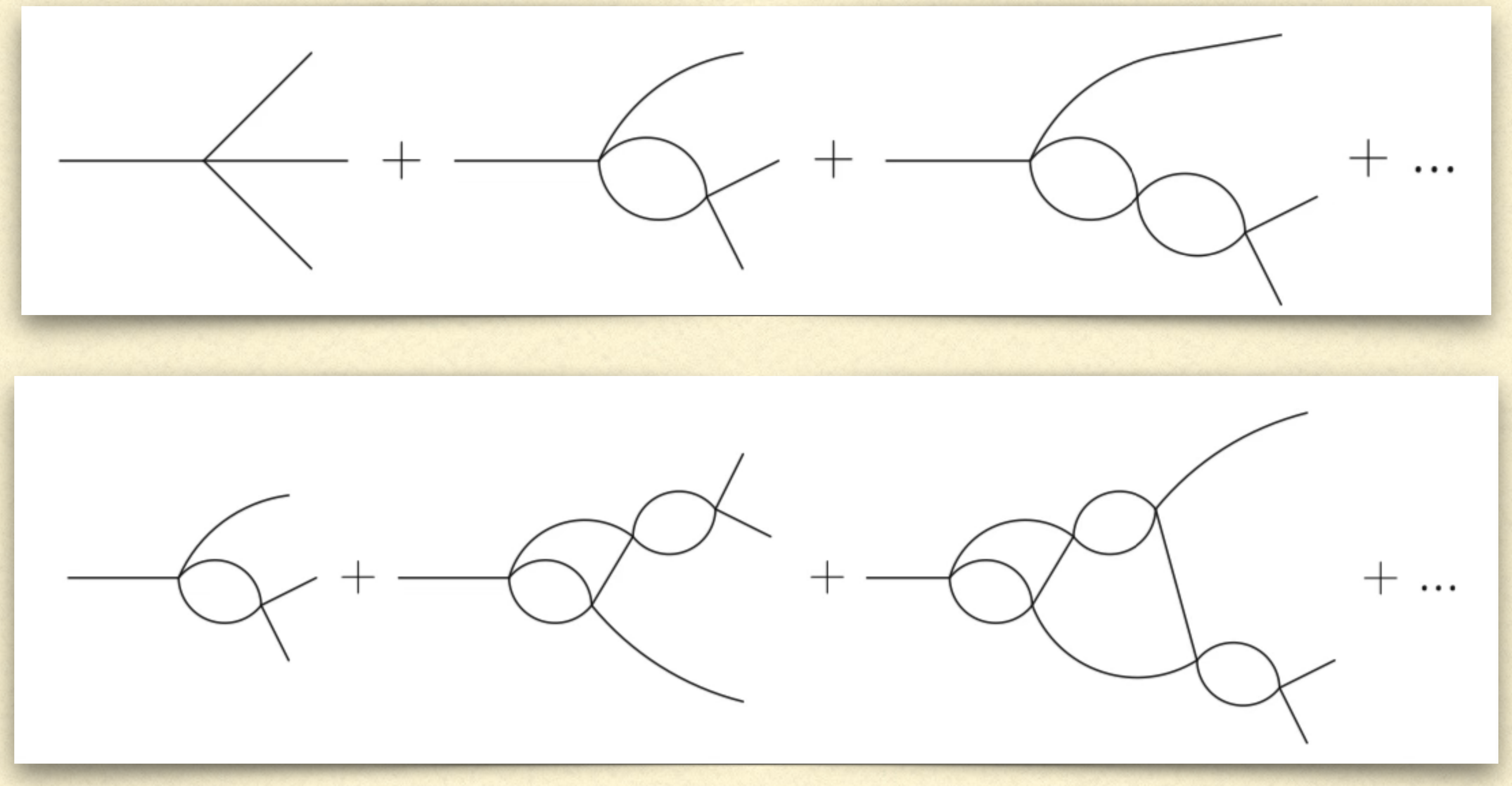
$$\ln \Omega_l(s) = \frac{s}{\pi} \int \mathrm{d}s' \frac{\delta_l(s')}{s'(s'-s)}$$

$$F(s) = \Omega(s) \sum a_i z(s, s_{in})^i$$

$$\mathrm{Im}\Omega(s+i\epsilon) = \frac{1}{\pi} \int_{s_{\mathrm{thr}}}^{\infty} \frac{T^*(s')\Sigma(s')\Omega(s')}{s'-s-i\epsilon} \mathrm{d}s'$$

- Watson's theorem: Below the first inelastic threshold, the elastic scattering phase is universal
- Omnès function is a model-independent way to transport this information
- Common treatment of lineshapes in $e^+e^- \to \pi^+\pi^-, \tau \to \pi^-\pi^0\nu_\tau, K \to \pi\pi\ell\nu,$ $B_{(s)} \to J/\Psi\pi^+\pi^-, \ldots$
- Works best for light mesons, $\pi\pi$, $K\pi$, but also S-wave $D\pi$ (see Raynette's talk tomorrow)
- Extensions beyond first inelastic threshold clear

Ingredient 2: Three-body decays



Taken from: EPJC 83 (2023) 6, 510

$$F^{(l)}_{(s)}(s) = \Omega^{(l)}_{(s)}(s)\left(Q^{(l)}_{(s)}(s) + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n}\frac{\sin\delta^{(l)}_{(s)}(s)\hat{F}^{(l)}_{(s)}(x)}{|\Omega^{(l)}_{(s)}(x)|(x-s)}\right)$$

- Amplitudes relevant for Unitarity bounds are $1 \to n$ amplitudes of particle with mass q^2
- Khuri-Treiman formalism allows to reconstruct three-body rescattering (PR 119 1115-1121 (1960))
- Write decay amplitude as sum of 3 partial-wave expanded amplitudes
- Fixed s, t & u dispersion-relations lead to coupled system of integral equations
- The two other channels enter via hat functions (B^* exchange)

Ingredient 2: Three-body decays

$$\mathcal{F}(s,t,u) = \sum_{x\in\{s,t,u\}} \sum_{l} F^{(l)}_{(x)}(x,q^2) P_l(\cos\theta_x)$$

- Amplitudes implicitly depend on mass
- s-dependence not polynomial above inelastic thresholds
- The hat functions depend on $B^* \to \pi$ FFs

$$F^{(l)}_{(s)}(s,q^2) = \Omega^{(l)}_{(s)}(s)\left(f^{(l)}_{(s)}(s,q^2) + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n} \frac{\sin\delta^{(l)}_{(s)}(s)\hat{F}^{(l)}_{(s)}(x,q^2)}{|\Omega^{(s)}_l(x)|(x-s)} \right)$$

Ingredient 3: Unitarity bounds

$$\Pi^{L/T}_{(J)}(q) \equiv i \int d^4x\, e^{iq\cdot x}\, \langle 0 \big|\, J^{L/T}(x)\, J^{L/T}(0) \big| 0 \rangle$$

$$\chi^{L}_{(J)}(Q^2) \equiv \frac{\partial \Pi^{L}_{(J)}}{\partial q^2}\bigg|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty \mathrm{d}q^2 \frac{\mathrm{Im}\Pi^{L}_{(J)}(q^2)}{(q^2-Q^2)^2}$$

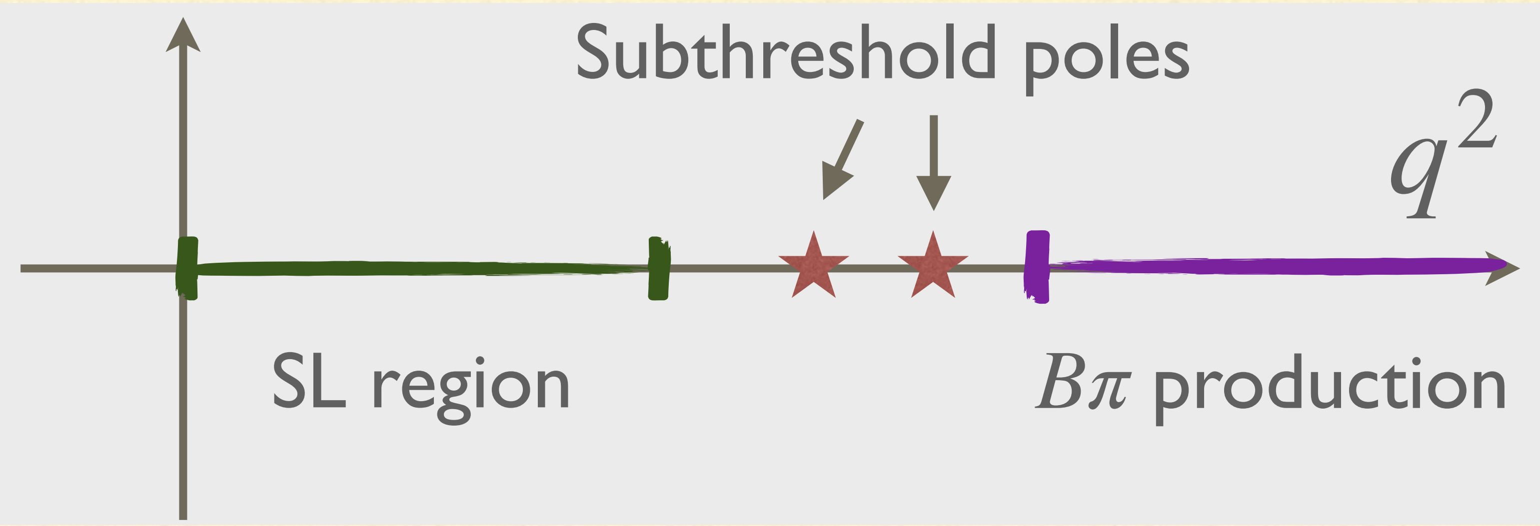
$$\chi^{T}_{(J)}(Q^2) \equiv \frac{1}{2}\frac{\partial^2 \Pi^{T}_{(J)}}{\partial (q^2)^2}\bigg|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty \mathrm{d}q^2 \frac{\mathrm{Im}\Pi^{T}_{(J)}(q^2)}{(q^2-Q^2)^3}$$

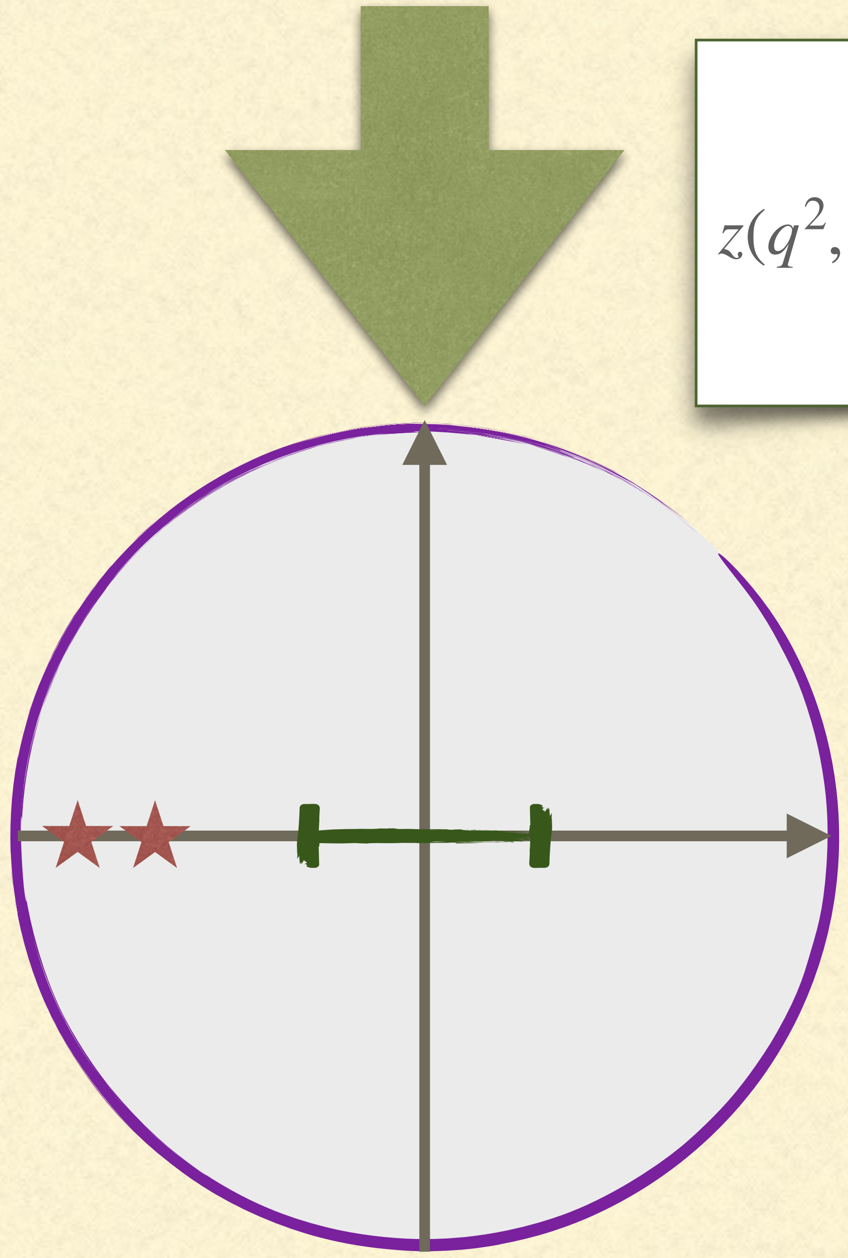
$$\mathrm{Im}\Pi^{T/L}_{(J)} = \frac{1}{2}\sum_X \int \mathrm{dPS}\, P^{\mu\nu}_{T/L} \left\langle 0 \left| J_\mu \right| X \right\rangle \left\langle X \left| J_\nu \right| 0 \right\rangle \delta^{(4)}(q-p_X)$$

$$\mathrm{Im}\Pi^{T}_{(V)}\,|_{BD} = K(q^2)\,|f_+(q^2)|^2$$

- Starting point: once and twice subtracted dispersion relations [Boyd, Grinstein, Lebed; Caprini; …]
- Susceptibilities perturbatively computable for large space-like Q^2 or at $Q^2=0$ if heavy quarks involved; also on the Lattice! (Martinelli, Simula, Vittorio; Harrison)
- Optical theorem allows to write the imaginary part as sum over all possible final states
- Neglecting a final state leads to an inequality

Ingredient 3: Unitarity bounds





$$z(q^2, q_0^2) = \frac{\sqrt{q_+^2 - q^2} - \sqrt{q_+^2 - q_0^2}}{\sqrt{q_+^2 - q^2} + \sqrt{q_+^2 - q_0^2}}$$

$$1 \geq \frac{1}{2\pi i} \oint \frac{\mathrm{d}z}{z} \left| B(z)\Phi(z)f(z) \right|^2$$

$$f(z) = \frac{1}{\Phi(z)B(z)} \sum_{i=0}^{\infty} a_i z^i \qquad 1 \geq \sum_{i=0}^{\infty} |a_i|^2$$

- Mapping q^2 to the dimensionless variable z transforms integration region to unit circle
- Insert Blaschke products to get rid of subthreshold poles and zeroes in kinematic factors
- Series expansion (or orthogonal polynomials)
- Semileptonic region: $|z| < 1$

Ingredient 3: Double-expansion

$$f(s,q^2) = \frac{1}{B(q^2)\phi(q^2;s)} \sum_i a_i(s) p_i(z(q^2), q_+^2(s))$$

$$\chi \geq \frac{1}{\pi} \sum_i \int_{s_+}^{\infty} \mathrm{d}s \hat{K}(s) \,|\, a_i(s) \,|^2$$

$$a_i(s) = \frac{1}{\tilde{B}(s)\tilde{\phi}(s)} \sum_j b_{ij} y^j$$

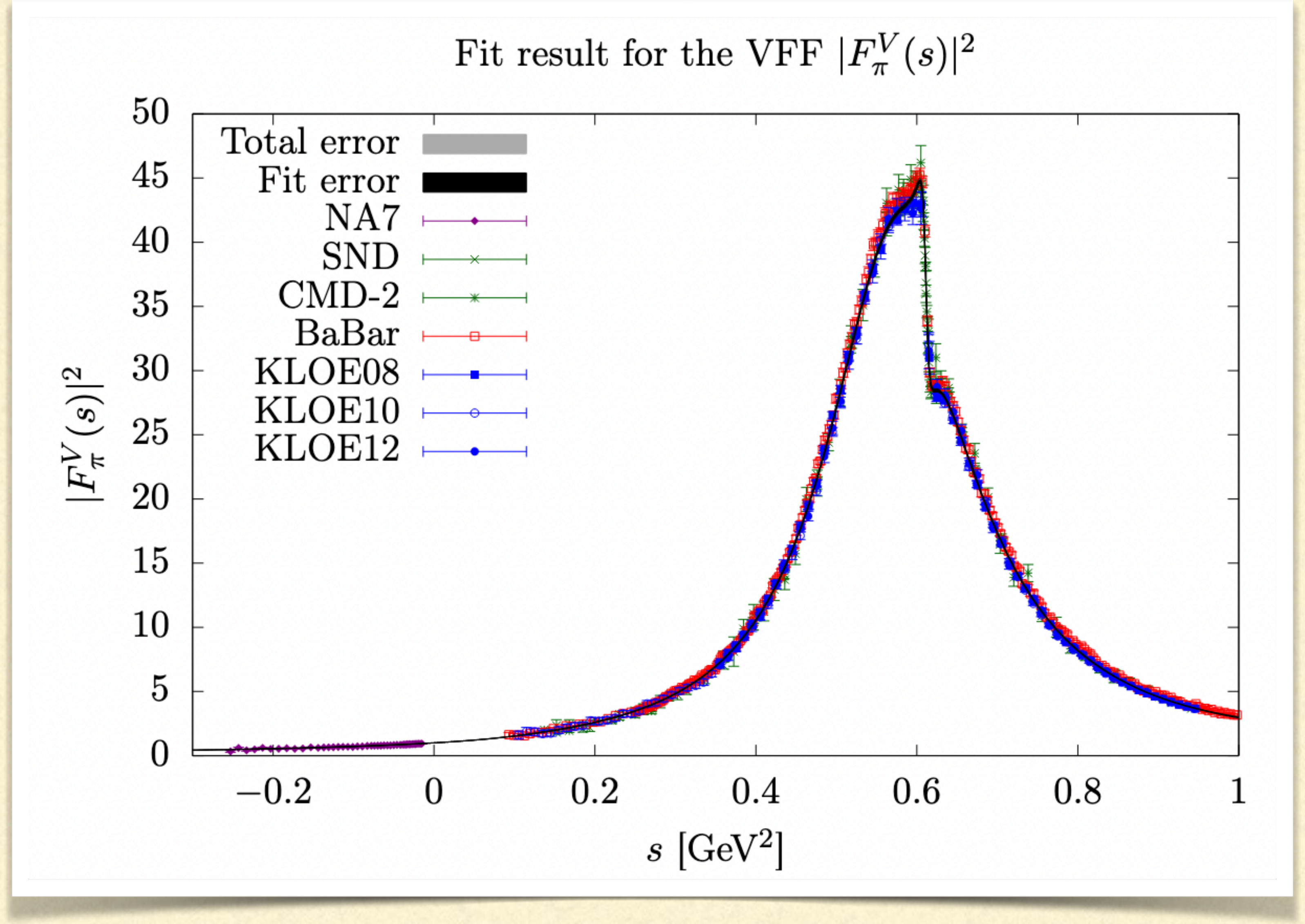
- q^2-integration as in standard BGL
- If $q_+^2(s)$ larger than lowest two-body threshold: use of orthogonal polynomials
- Now we can treat every a_i as an s-dependent FF
- Follow Caprini's treatment of pion VFF, (EPJ C 13 471-484 (2000))
- Alternative: BCL-like expansion, taking into account behaviour at inelastic threshold

$$y = \frac{\sqrt{s_{in} - s} - \sqrt{s_{in}}}{\sqrt{s_{in} - s} + \sqrt{s_{in}}}$$

Form factors, again…

$$F_{(s)}^{(l)}(s,q^2) = \Omega_{(s)}^{(l)}(s)\left(\frac{1}{B_{(s)}(q^2)\tilde{B}_{(s)}^{(l)}(s)\phi_{(s)}^{(l)}(q^2)\tilde{\phi}_{(s)}^{(l)}(s)}\sum_{i,j} b_{ij,(s)}^{(l)} z_{(s)}^{i} y_{(s)}^{j} + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n}\frac{\sin\delta_{(s)}^{(l)}(s)\hat{F}_{(s)}^{(l)}(x,q^2)}{|\,\Omega_l^{(s)}(x)\,|\,(x-s)}\right)$$

Form factors, again…



Colangelo, Hoferichter, Stoffer JHEP 02 (2019) 006

Ingredient 1: Process-independent lineshapes

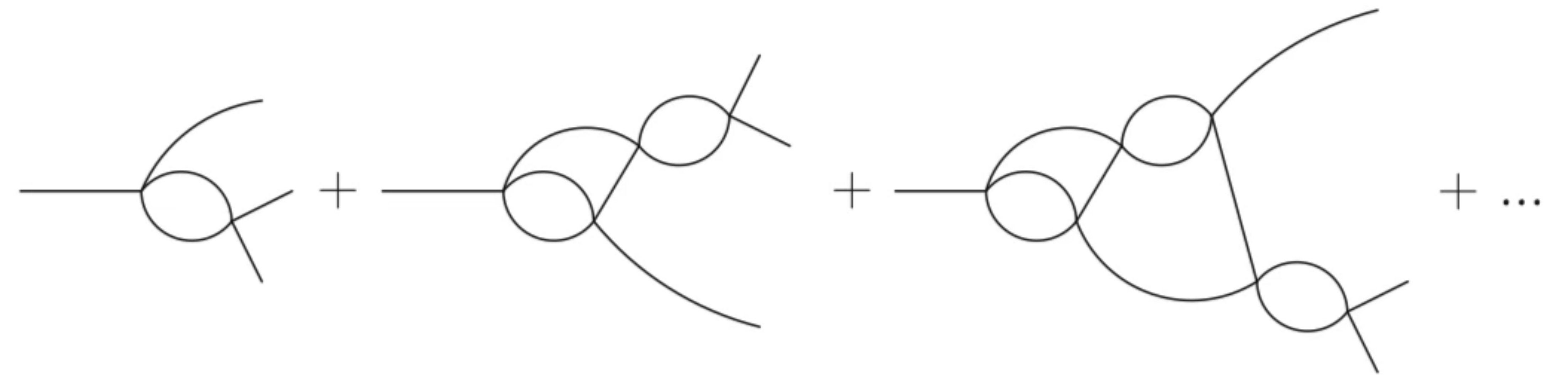
$$F_{(s)}^{(l)}(s,q^2) = \Omega_{(s)}^{(l)}(s)\left(\frac{1}{B_{(s)}(q^2)\tilde{B}_{(s)}^{(l)}(s)\phi_{(s)}^{(l)}(q^2)\tilde{\phi}_{(s)}^{(l)}(s)}\sum_{i,j} b_{ij,(s)}^{(l)} z_{(s)}^i y_{(s)}^j + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n}\frac{\sin\delta_{(s)}^{(l)}(s)\hat{F}_{(s)}^{(l)}(x,q^2)}{|\Omega_l^{(s)}(x)|(x-s)}\right)$$

Form factors, again…

Ingredient 1: Process-independent lineshapes

Ingredient 2: Three-body rescattering

$$F^{(l)}_{(s)}(s,q^2) = \Omega^{(l)}_{(s)}(s)\left(\frac{1}{B_{(s)}(q^2)\tilde{B}^{(l)}_{(s)}(s)\phi^{(l)}_{(s)}(q^2)\tilde{\phi}^{(l)}_{(s)}(s)}\sum_{i,j} b^{(l)}_{ij,(s)} z^i_{(s)} y^j_{(s)} + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n}\frac{\sin\delta^{(l)}_{(s)}(s)\hat{F}^{(l)}_{(s)}(x,q^2)}{|\Omega^{(s)}_l(x)|(x-s)}\right)$$



Form factors, again…

Ingredient 1: Process-independent lineshapes

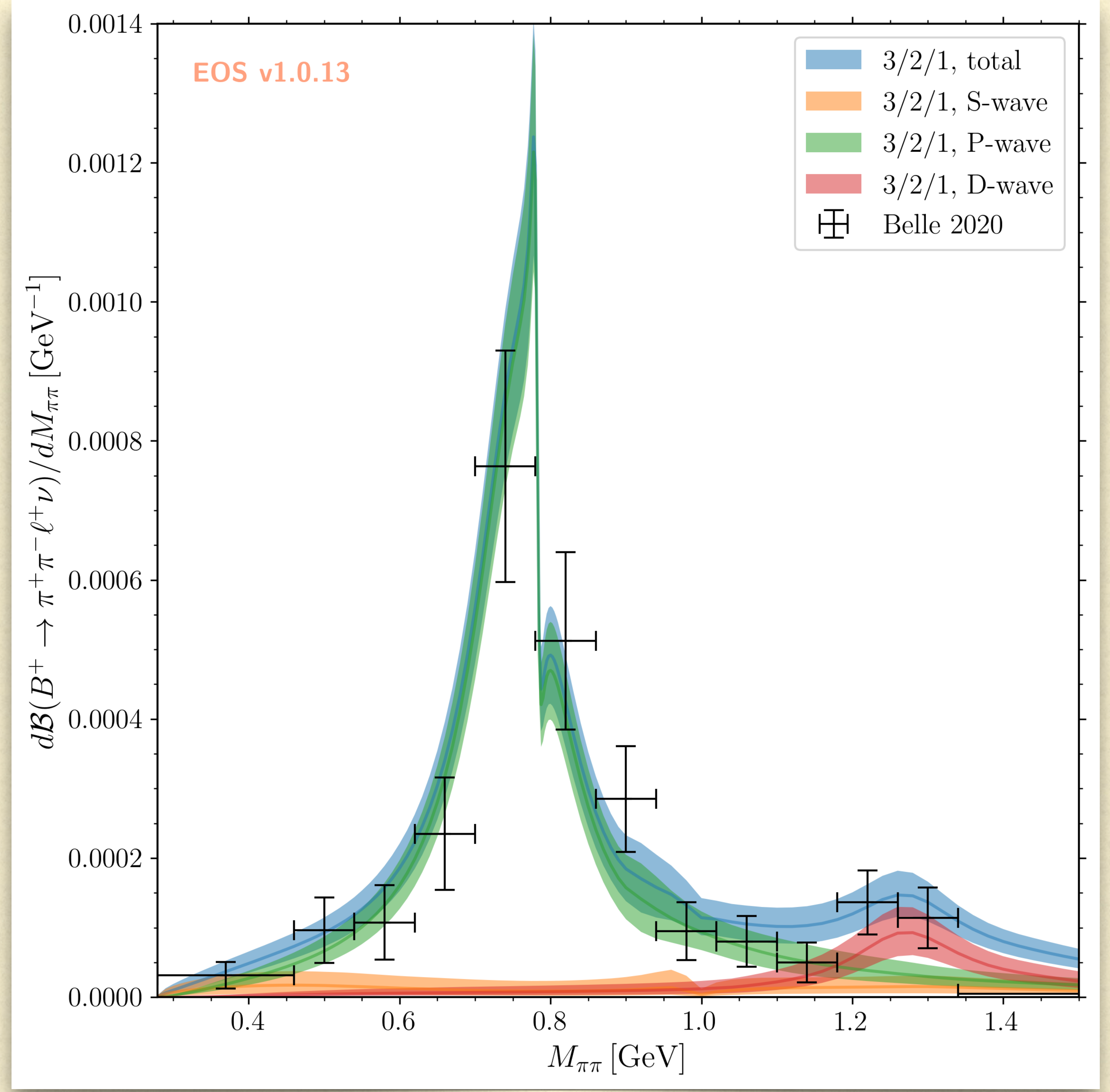
Ingredient 2: Three-body rescattering

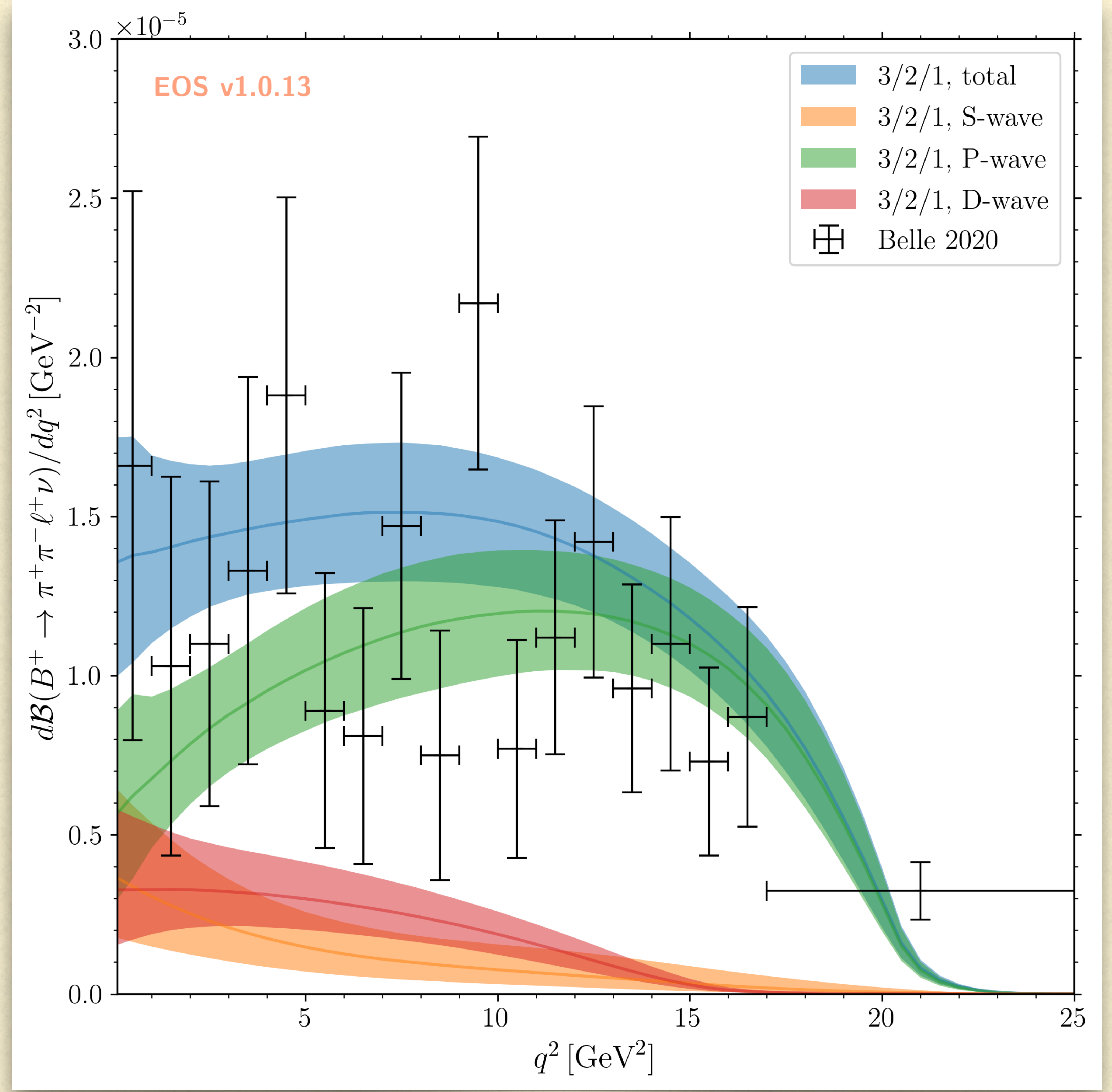
$$F^{(l)}_{(s)}(s,q^2) = \Omega^{(l)}_{(s)}(s)\left(\frac{1}{B_{(s)}(q^2)\tilde{B}^{(l)}_{(s)}(s)\phi^{(l)}_{(s)}(q^2)\tilde{\phi}^{(l)}_{(s)}(s)}\sum_{i,j} b^{(l)}_{ij,(s)} z^i_{(s)} y^j_{(s)} + \frac{s^n}{\pi}\int \frac{\mathrm{d}x}{x^n}\frac{\sin\delta^{(l)}_{(s)}(s)\hat{F}^{(l)}_{(s)}(x,q^2)}{|\Omega^{(s)}_l(x)|(x-s)}\right)$$

Ingredient 3: Model-independent double-expansion

$$\sum_{i,j,k,l} M_{i,j,k,l} b_{i,k} b^*_{j,l} < 1$$

Data from C. Beleño et al. (Belle), PRD 103 (2021) 11, 112001

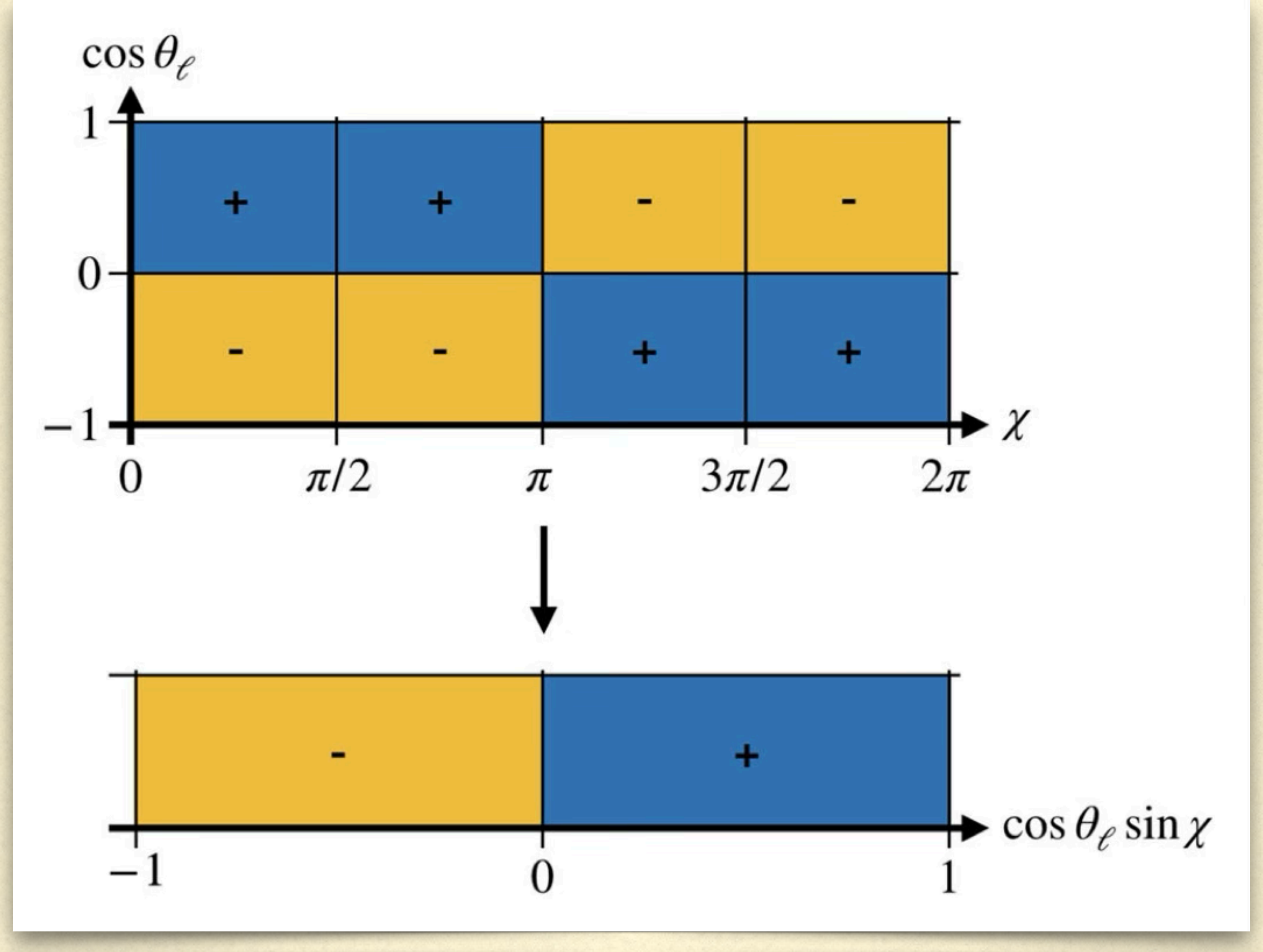




Full description of $B \to \pi\pi\ell\nu$, only need to be combined with LCSR/LQCD calculations

How can we do spectroscopy?

EPJC 85 (2025) 11, 1289 Du et al.



$$\tan(\delta_0 - \delta_1) = \frac{\mathrm{Im}(f_+ f_1^*)}{\mathrm{Re}(f_+ f_1^*)}$$

- The lightest scalar charmed resonances has never unambiguously been observed, PDG lists the $D_0^*(2300)$ but lattice calculations suggest two states: $D_0^*(2105)$ and $D_0^*(2450)$
- Main idea: use narrow Breit-Wigner-like P-wave as reference phase
- Study angular asymmetries projecting out real and imaginary parts of interference terms
- Toy study assuming Belle efficiencies

What is going on here?

2602.18378 FH, Raynette van Tonder

$$\frac{1}{\Gamma}\frac{\mathrm{d}^{n+2}\Gamma}{\mathrm{d}M^2\mathrm{d}q^2\mathrm{d}\Omega_n} = C\,|\,V_{B\to R\ell\nu}(q^2, M^2, \Omega_n)\,P_R(M^2)\,V_{R\to P_1P_2}(M^2)\,|^2$$

$$|\,V_{B\to R\ell\nu}\,|^2 \propto |\vec{p}_l|^2\,|\vec{p}_R\,|^{\max(2J-1,1)}$$

$$|\,V_{R\to P_1P_2}\,|^2 = \left(\frac{F^{L_D}(\,|\vec{p}_1\,|)}{F^{L_D}(\,|\vec{p}_1'\,|)}\right)^2 \left(\frac{|\vec{p}_1\,|}{|\vec{p}_1'\,|}\right)^{2L_D+1}$$

What is happening within EvtGen?

2602.18378 FH, Raynette van Tonder

$$\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}M} = C\,|\,P_R(M^2)\,V_{R\to P_1P_2}(M^2)\,|^2$$

1. Determine invariant mass of the P_1P_2 system
2. Rejection sample $|\,V_{B\to R\ell\nu}\,|^2$ based on this invariant mass
3. Try 10000 times
4. If it fails 10000 times, just take the last point

What is going on here?

2602.18378 FH, Raynette van Tonder

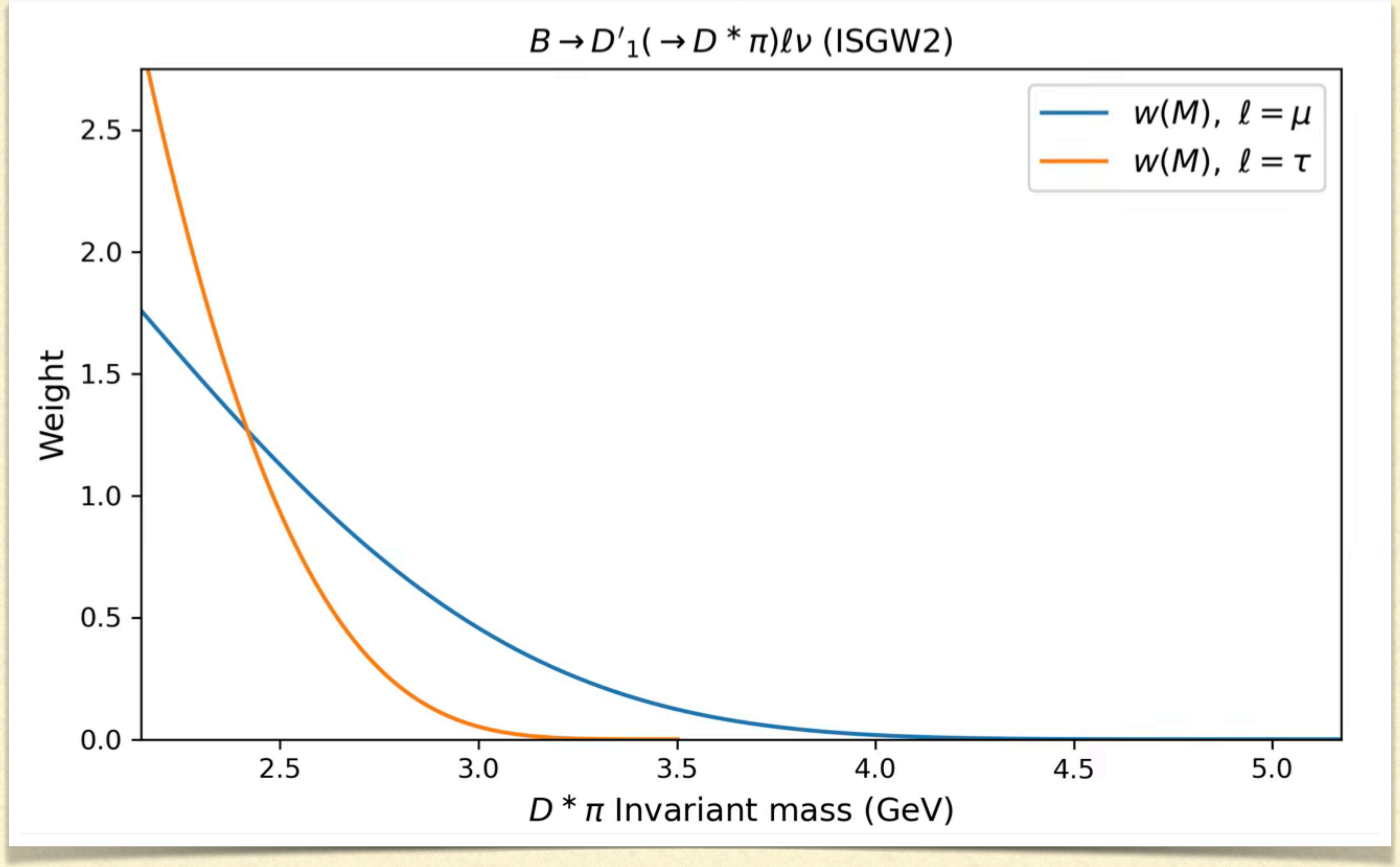
$$\frac{1}{\Gamma}\frac{\mathrm{d}^{n+2}\Gamma}{\mathrm{d}M^2\mathrm{d}q^2\mathrm{d}\Omega_n} = C\,|\,V_{B\to R\ell\nu}(q^2, M^2, \Omega_n)\,P_R(M^2)\,V_{R\to P_1P_2}(M^2)\,|^2$$

$$\Downarrow$$

$$\frac{1}{\Gamma}\frac{\mathrm{d}^{n+2}\Gamma}{\mathrm{d}M^2\mathrm{d}q^2\mathrm{d}\Omega_n} = C'\,\frac{|\,V_{B\to R\ell\nu}(q^2, M^2, \Omega_n)\,P_R(M^2)\,V_{R\to P_1P_2}(M^2)\,|^2}{\int \mathrm{d}q^2 \int \mathrm{d}\Omega_n\,|\,V_{B\to R\ell\nu}(q^2, M^2, \Omega_n)\,|^2}$$

A **temporary** solution

2602.18378 FH, Raynette van Tonder



- Since we know the difference, we can reweight
- Simple, but decay & form factor dependent formula for weights
- Similar treatment possible for non-leptonic decays
- However, not practical for analysts beyond a few dominant modes