# Exercise session 1

## 1. Laser propagation in plasma

Can a laser pulse with wavelength

$$\lambda_1 = 800 \text{ nm} \quad (a), \qquad \lambda_2 = 10\ \mu\text{m} \quad (b)$$

propagate in a plasma with electron density

$$n_{pe1} = 10^{17} \text{ cm}^{-3} \quad (c), \qquad n_{pe2} = 5 \times 10^{19} \text{ cm}^{-3} \quad (d)$$

## 2. Wakefield excitation length

(a) Estimate the distance $L$ over which a drive beam with an energy of 10 GeV per particle and a beam density, assuming a transformer ratio of 1 ($E_{decc,max} = E_{acc,max}$)

$$n_b = 5 \times 10^{16} \text{ cm}^{-3}$$

excites wakefields in a plasma of density

$$n_{pe} = 10^{16} \text{ cm}^{-3}$$

(b) Estimate the distance $L$ over which a laser pulse with peak intensity

$$I = 5 \times 10^{18} \text{ W/cm}^2$$

and wavelength

$$\lambda = 800 \text{ nm}$$

excites wakefields in a plasma of density

$$n_{pe} = 10^{18} \text{ cm}^{-3}$$

## 3. Group velocity and witness electron energy gain

(a) What is the group velocity of a laser pulse with

$$\lambda_1 = 800 \text{ nm} \quad (a), \qquad \lambda_2 = 10\ \mu\text{m} \quad (b)$$

in a plasma with electron density

$$n_{pe1} = 2 \times 10^{17} \text{ cm}^{-3} \quad (c), \qquad n_{pe2} = 3 \times 10^{19} \text{ cm}^{-3} \quad (d)$$

(b) Assuming a laser pulse with $a_0 = 0.5$, which combination of $\lambda$ and $n_{pe}$ from part (a) yields the highest witness electron energy gain?
*Hint: express the dephasing ($L_d$) and depletion lengths ($L_{pd}$) as a function of $n_{pe}$ and $\lambda_l$, as well as the accelerating fields.*

## 4. Rayleigh range and self-focusing power

(a) What is the Rayleigh range of a laser pulse with

$$\lambda_1 = 800 \text{ nm} \quad (a), \qquad \lambda_2 = 10 \ \mu\text{m} \quad (b)$$

and focal spot size

$$w_0 = 50 \ \mu\text{m}?$$

(b) What is the minimum laser power required for self-focusing in a plasma with electron density

$$n_{pe1} = 2 \times 10^{17} \text{ cm}^{-3} \quad (c), \qquad n_{pe2} = 5 \times 10^{18} \text{ cm}^{-3} \ ?$$

## 5. Dephasing length in PWFA

(a) Derive the dephasing length in the case of a PWFA, where the drive bunch propagates with a group velocity $v_{b,d} = c\sqrt{1 - 1/\gamma_d^2}$ and the witness $v_{b,w} = c\sqrt{1 - 1/\gamma_w^2}$, assuming that both the energy gain and loss over the dephasing length is negligible compared to the initial energy ($\gamma(t) = constant$).

## 6. Dispersion relation in vacuum

Derive the dispersion relation of an electromagnetic wave in vacuum:

$$\omega = ck.$$

# Exercise session 2

## 1. Efficiency in LWFA

You want to build an accelerator that can accelerate 1 nC of electrons to 1 TeV at a repetition rate of 1 kHz. You are planning to use less than 2.7 MW of power. Your wall-plug-to-driver efficiency is 50%. Your driver-to-wake efficiency is 90%. What does your wake-to-witness efficiency need to be ?

## 2. Staging

Between two LWFA stages a witness electron bunch drifts ballistically through a vacuum gap of length $L$. To enter the second plasma stage properly matched, the bunch must enter the second stage at a waist and with the same transverse size as it had at the exit of the first stage. A short focusing element (thin lens approximation) is placed at the center of the gap to satisfy this condition.

### Reminders

- In the transfer matrix formalism:

$$M_{\text{drift}}(d) = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}, \qquad M_{\text{lens}}(f) = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}.$$

- For a quadrupole magnet of length $L_q$ and magnetic gradient $G$, the equivalent of a thin-lens focal length is

$$\frac{1}{f} = \frac{GL_q}{B\rho},$$

  where $B\rho = p/q \sim 3.33\,\text{p[GeV/c]}$ is the magnetic rigidity.

(a) For a 10 cm-long quadrupole placed at the center of the gap, express the magnetic gradient G required to refocus a beam at the downstream entrance of the second stage as a function of electron energy and gap length.

(b) Suppose your electron bunch has an energy per particle of $E = 10\,\text{GeV}$ and the gap length is $L = 1.0$ m, what is the magnetic gradient required to refocus the beam. Compare this number with typical, normal conducting quadrupoles.

(c) Consider implementing instead a plasma lens of 1 mm diameter. What would be the current required for to refocus the beam.

(d) Discuss the scaling of the current with beam energy. What is challenge for a TeV-scale plasma wakefield acceleration staging setup ?

## 3. Single stage acceleration schemes

Another way to accelerate electrons to very high energies in a plasma wakefield accelerator is to use a driver that carries a large amount of energy. This is the approach taken by the AWAKE experiment at CERN. Instead of using a laser or an electron beam, AWAKE uses high-energy proton bunches - specifically, 400 GeV protons produced by the Super Proton Synchrotron. These bunches contains about $3 \times 10^{11}$ protons and can serve as the drive beam to create the plasma wakefields needed to accelerate electrons to high energies in a **single plasma stage**.

(a) The proton bunch has a length of $\sigma_z \sim 5.1\,\text{cm}$ and a transverse size $\sigma_r \sim 200\,\mu\text{m}$. Calculate the wave-breaking field amplitude of the plasma, knowing that, ideally, $\sigma_z \sim \lambda_{pe}$. Comment on the result.

(b) Calculate now the wave-breaking field amplitude if the condition to fulfill is $1/k_{pe} > \sigma_r$ instead. Comment on the result.