

# Lectures on the Future Circular Collider – Part 1+2

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Bonn, 1-2 October 2025

# **Lecture 1 – FCC-ee design concepts**

1.0 colliders – revisited

1.1 luminosity – revisited

1.2 synchrotron radiation

1.3 beam-beam effects

1.4 luminosity revised

1.5 FCC concept(s)

# 1.0 colliders

## why high(er) energy?

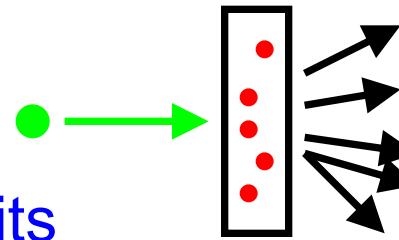
- quantum mechanics: de Broglie wavelength  $\lambda=h/p$   
→ examining matter at smaller distance requires higher momentum particles
- many of the particles of interest to particle physics are heavy  
→ high-energy collisions are needed to create these particles

# colliding beams

centre-of-mass energy:

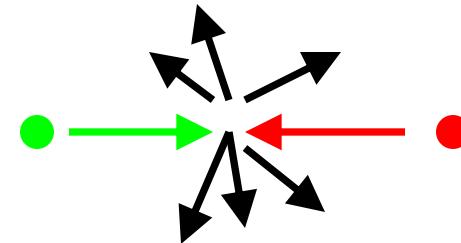
$$E_{\text{c.m.}} = \sqrt{2 E_{\text{beam}} M_{\text{target}} c^2}$$

beam hits  
a “fixed target”



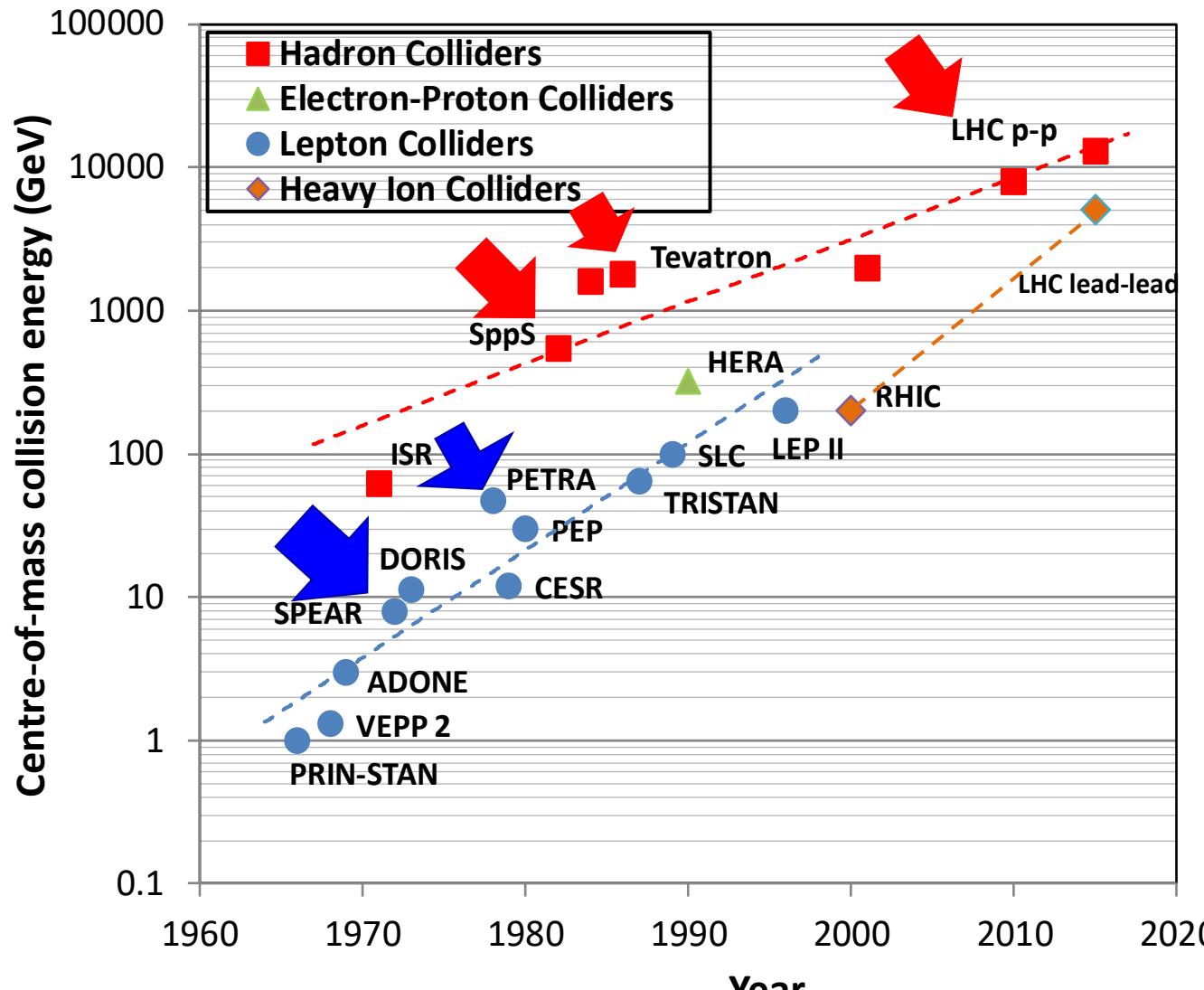
$$E_{\text{c.m.}} = 2 E_{\text{beam}}$$

two beams collide



colliding two beams against each other can provide  
much higher centre-of-mass energies than fixed target!

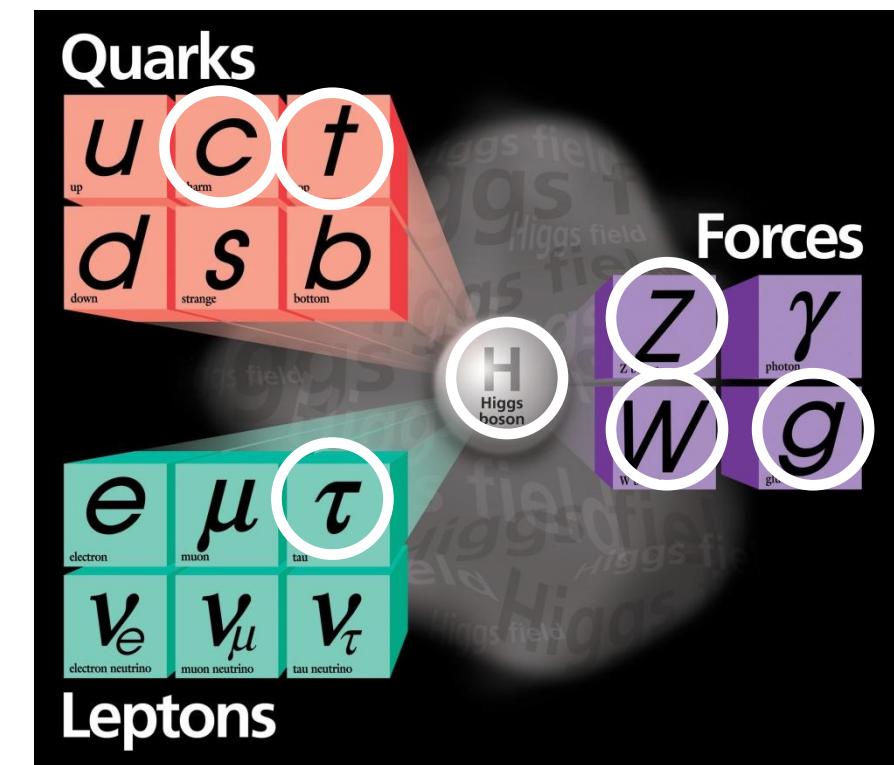
# colliders and discoveries



powerful instruments for discovery  
and precision measurement

## Standard Model Particles and forces

A. Ballarino



**“An  $e^+e^-$  storage ring in the range of a few hundred GeV  
in the centre of mass can be built with present  
technology...” “...the most useful project on the horizon.”**



B. Richter, 1976

# LEP/LEP2: highest energy so far

**circumference 27 km**

**in operation from 1989 to 2000**

**maximum c.m. energy 209 GeV**

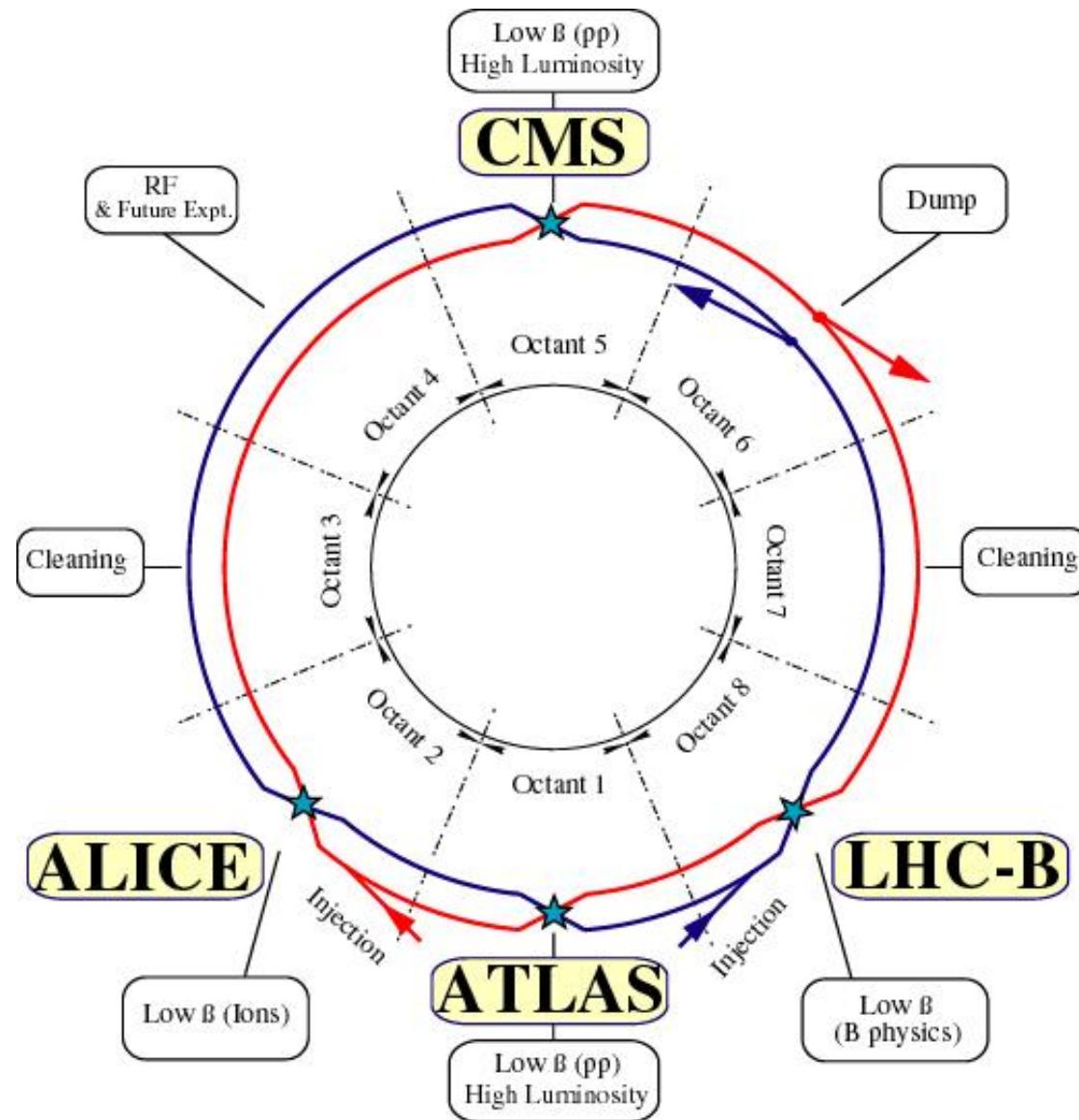
**maximum synchrotron radiation power 23 MW**



# FCC-ee physics requirements

- **beam energy range from 35 GeV to  $\approx 200$  GeV**
- **highest possible luminosities** at all working points
- physics programs / energies:
  - $Z$  (45.5 GeV)  $Z$  pole, 'TeraZ' and high precision  $M_Z$  &  $\Gamma_Z$
  - $W$  (80 GeV)  $W$  pair production threshold, high precision  $M_W$
  - $H$  (120 GeV)  $ZH$  production (maximum rate of  $H$ 's)
  - $t$  (175 GeV):  $t\bar{t}$  threshold,  $H$  studies
  - more ( $\alpha_{QED}$  etc.)
- possibly  $H$  (63 GeV) direct  $s$ -channel production w. **monochromatization**
- **some polarization up to  $\geq 80$  GeV** for beam energy calibration

# present flagship: Large Hadron Collider (LHC)



installed in the  
LEP tunnel ! –  
circumference  
27 km

world's highest  
energy p-p collider  
at CERN/Geneva

*running  
extremely well*

4 July, 13 years ago

LHC has already produced  
> 30 million Higgs bosons

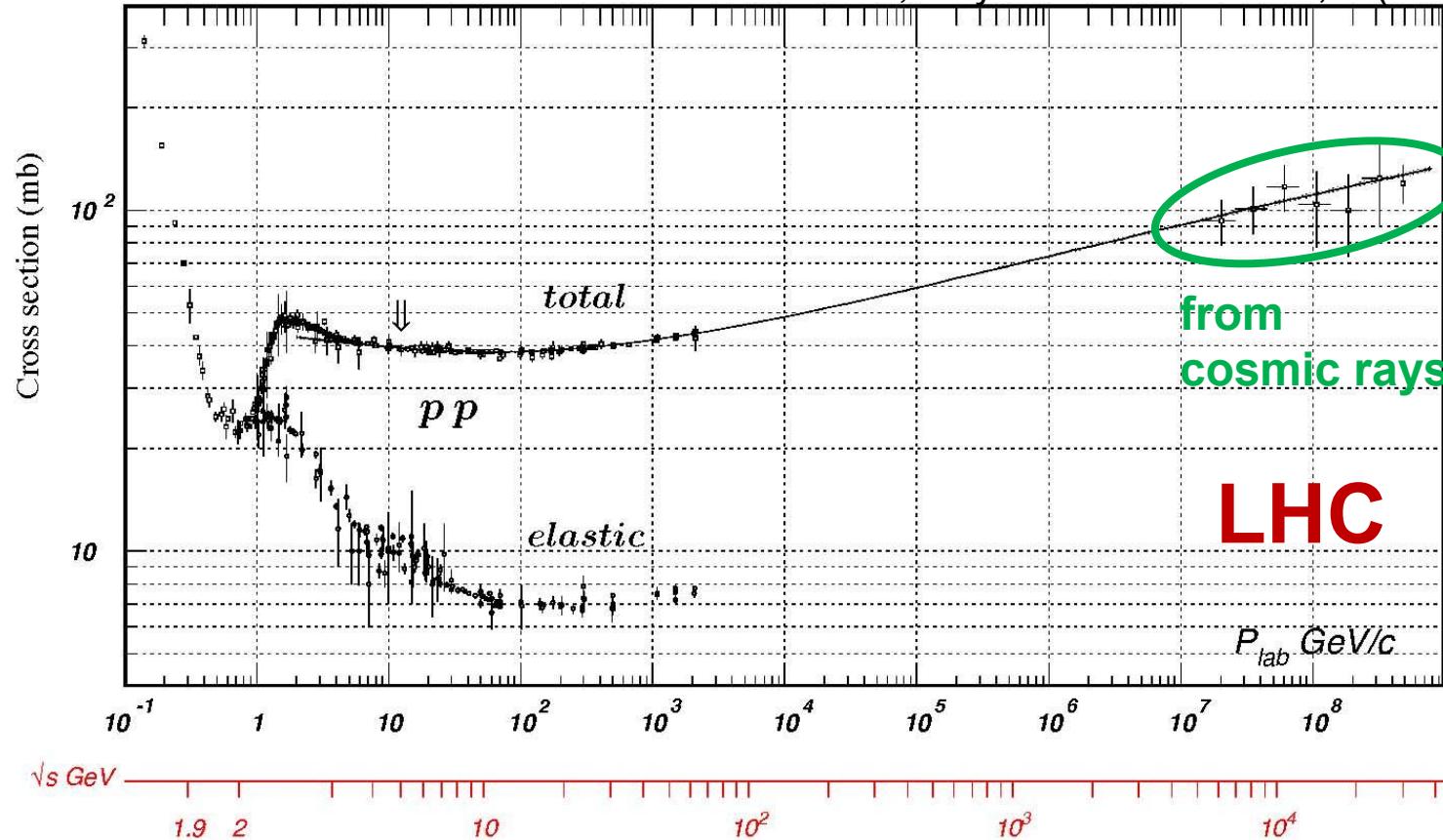


# 1.1 luminosity

$$R = \sigma L$$

reaction rate      cross section      luminosity

C. Amsler *et al.*, Physics Letters B667, 1 (2008)



$$\sigma_{\text{tot}} \sim 100 \text{ mbarn} \sim 10^{-25} \text{ cm}^2$$

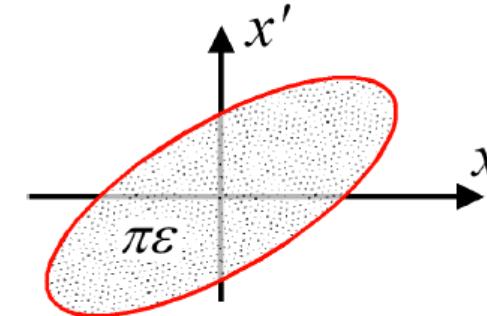
determines  
proton  
consumption  
(beam  
lifetime)

$\sigma_{\text{Higgs, p-p}} \sim 30 \text{ pb} \rightarrow \text{with } 300 \text{ fb}^{-1} \text{ LHC produces } \sim 10 \text{ million Higgs}$

# luminosity for collision of flat beams

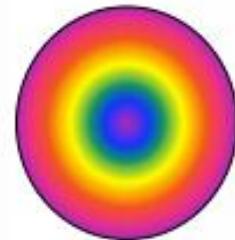
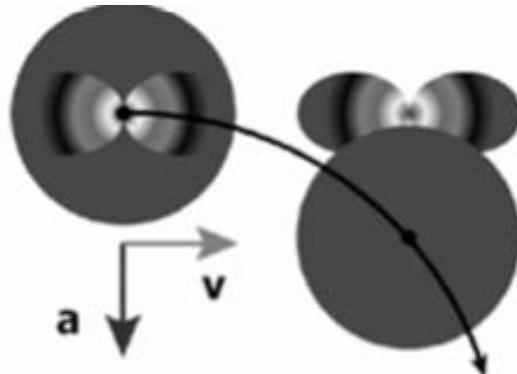
$$L = \frac{n_b N_b^2 f_{\text{rev}}}{4\pi \sigma_x^* \sigma_y^*} F = \frac{n_b N_b^2 f_{\text{rev}}}{4\pi \varepsilon_x \sqrt{\beta_x^* \beta_y^* \kappa}} F$$

$N_b$	number of particles per bunch
$n_b$	number of bunches per beam
$f_{\text{rev}}$	revolution frequency
$\sigma_{x,y}^*$	hor./vert. beam size at interaction point
$F$	reduction factor due to crossing angle and hourglass effect
$\varepsilon_x$	hor. emittance (from optics)
$\kappa$	emittance coupling $\kappa = \varepsilon_y / \varepsilon_x$
$\beta^*$	hor./vert, beta function at IP



$$\sigma^* = \sqrt{\beta^* \varepsilon}$$

# 1.2 Synchrotron Radiation (SR) transverse and longitudinal acceleration



Radiation field quickly  
separates itself from the  
Coulomb field

$$P_{\perp} = \frac{q^2}{6\pi\epsilon_0 m_0^2 c^3} \gamma^2 \left( \frac{d\mathbf{p}_{\perp}}{dt} \right)^2$$

Radiation field cannot  
separate itself from the  
Coulomb field

$$P_{\parallel} = \frac{q^2}{6\pi\epsilon_0 m_0^2 c^3} \left( \frac{d\mathbf{p}_{\parallel}}{dt} \right)^2$$

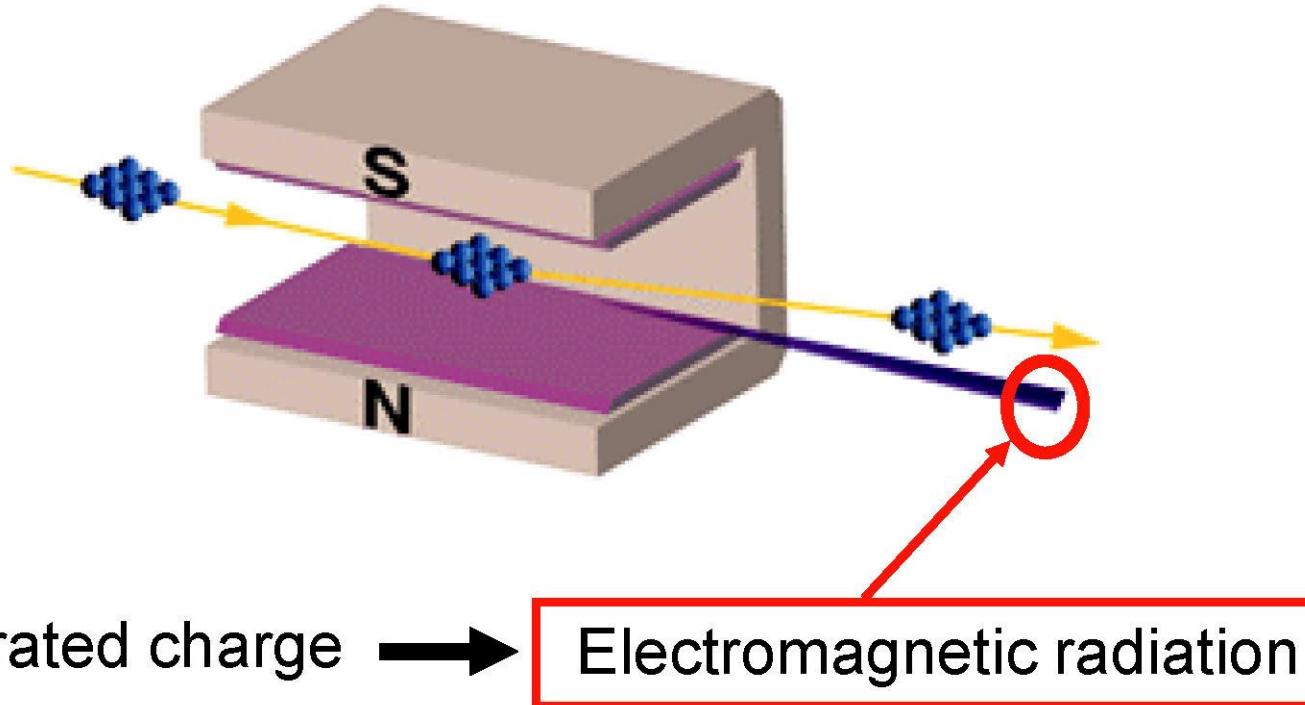
**negligible!**

$$P_{\perp} = \frac{c}{6\pi\epsilon_0} q^2 \frac{(\beta\gamma)^4}{\rho^2} \quad \rho = \text{curvature radius}$$

W. Barletta, USPAS

excellent news for high-gradient acceleration!

# curved orbit of $e^-$ in magnetic field

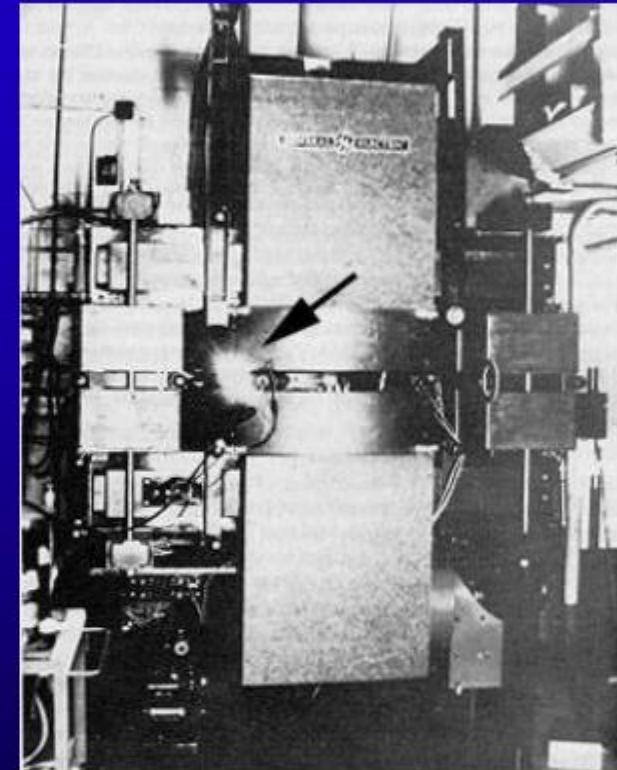


**Crab Nebula**  
**6000 light years away**



**First light observed  
1054 AD**

**GE Synchrotron  
New York State**



**First light observed  
1947**

# classical theory of electromagnetic radiation

Liénard-Wiechert potentials

$$\varphi(t) = \frac{1}{4\pi\epsilon_0} \frac{q}{[r(1 - \vec{n} \cdot \vec{\beta})]_{ret}}$$

retarded time defined by implicit equation

$$t_{ret} = t - \frac{1}{v} |\vec{r} - \vec{r}_s(t_{ret})|$$

$$\vec{A}(t) = \frac{q}{4\pi\epsilon_0 c^2} \left[ \frac{\vec{v}}{r(1 - \vec{n} \cdot \vec{\beta})} \right]_{ret}$$

Lorentz gauge

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$



$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla} \varphi - \frac{\partial \vec{A}}{\partial t}$$

# energy loss for a particle in a ring

Rate of energy loss (GeV/s) for single particle:

$$P_\gamma = \frac{e^2 c^3}{2\pi} C_\gamma E^2 B^2 = \frac{c C_\gamma E^4}{2\pi \rho^2}$$

with  $C_\gamma = \frac{4\pi}{3} \frac{r_e}{(mc^2)^3} \approx 8.85 \times 10^{-5} \text{ m GeV}^{-3}$

$\rho = \frac{p}{eB}$  bending radius ;

$E$  particle energy ;  $m$  particle mass ;

$r_e$  classical particle radius ( $2.8 \times 10^{-15}$  m for electrons)



Energy lost in a ring over one turn (GeV):

$$U_0 = \frac{C_\gamma E_0^4}{2\pi} \oint G^2(s) ds$$

with  $G \equiv 1/\rho$

With a constant guide field  $G_0 \equiv 1/\rho_0$  along the curved path of the trajectory and zero elsewhere :

Energy radiated per turn

$$U_0 = \frac{C_\gamma E_0^4}{\rho_0}$$

Average power

$$\langle P_\gamma \rangle = \frac{U_0}{T_0} = \frac{c C_\gamma E_0^4}{L \rho_0}$$

$T_0 = L/c$  with  $L$  circumference

An electron that is not on the ideal orbit radiates at a different rate. But if magnetic field varies linearly with position, radiated power averaged over a betatron oscillation cycle is the same as that for an electron on the design orbit. The same is not true for an electron with an energy different from  $E_0$  (next slide).

For ultra-relativistic electrons the radiation is emitted primarily along the direction of motion. Most of the radiation is emitted within the angle  $1/\gamma$ . The accompanying momentum change is nearly exactly opposite to the direction of motion. Then only radiation effect is to decrease e<sup>-</sup> energy w/o changing its direction of motion.

# is this an issue for high energy accelerators?

energy loss per particle per turn  $U_0 = \frac{C_\gamma E_0^4}{\rho_0}$   $\rightarrow$  SR power  $P_{SR} = \frac{I_{beam}}{e} U_0$

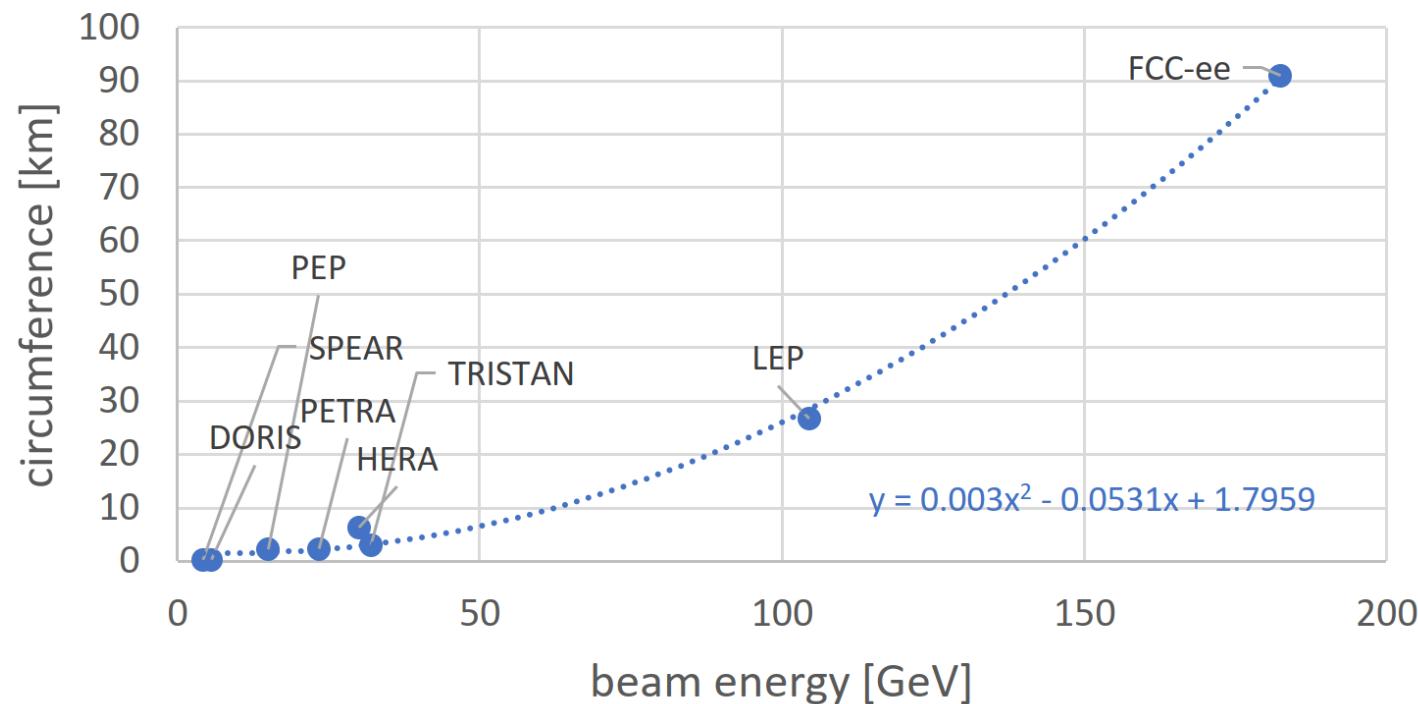
$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(mc^2)^3} \approx 8.85 \times 10^{-5} \text{ m GeV}^{-3}$$

**e<sup>±</sup>:**  $P_{SR} = 23$  MW for LEP (former e<sup>+</sup>e<sup>-</sup> collider in the 27 km LHC tunnel),  
100 MW for FCC-ee (new ~90 km ring, imposed as design constraint),

**protons:**  $P_{SR} = 0.01$  MW for LHC,  
up to 5 MW for FCC-hh (new ~90 km ring, ~10x collision energy of the LHC)  
– this may require >100 MW cryoplant power (FCC-hh CDR, 2018)

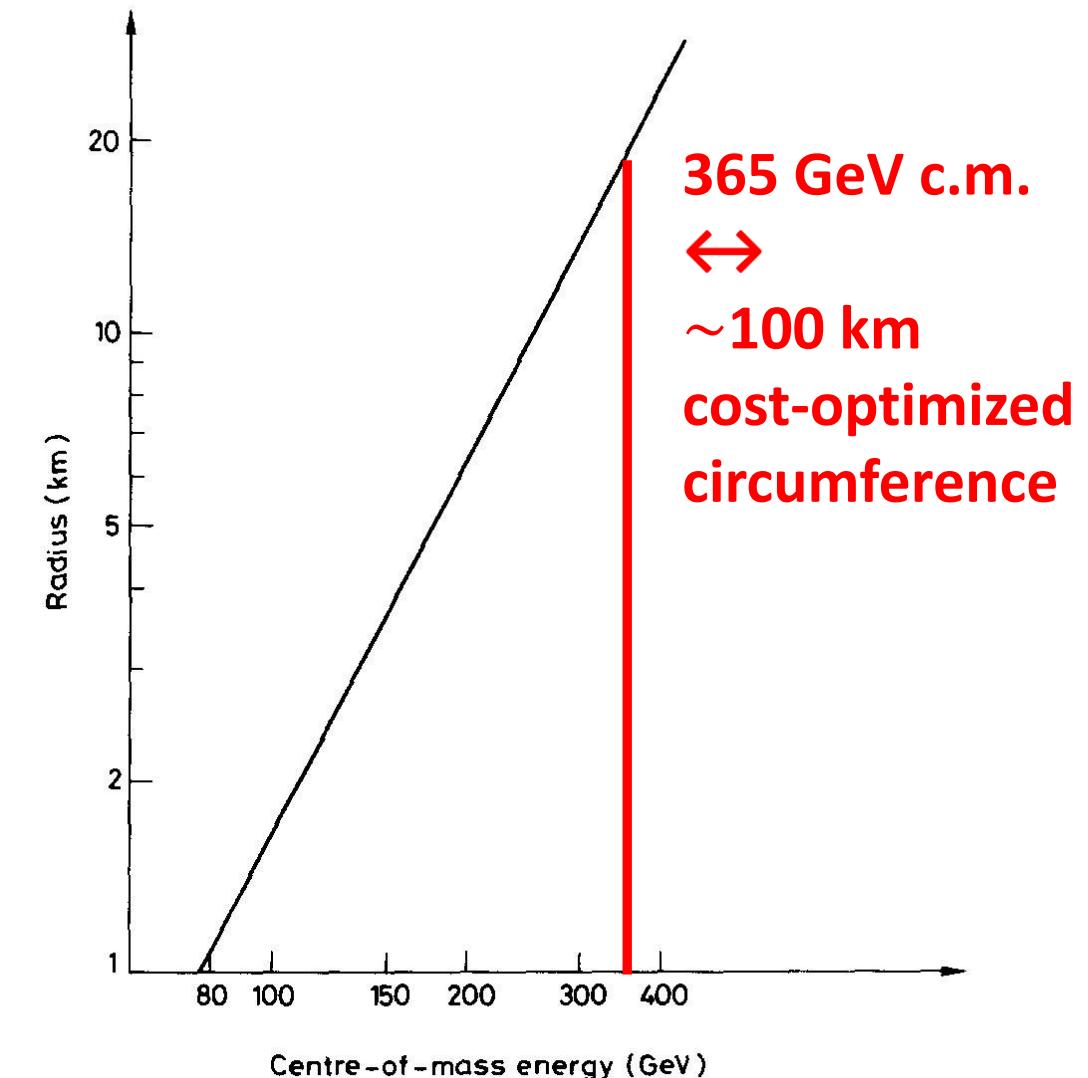
# SR → size of circular $e^+e^-$ colliders

lepton ring circumference versus beam energy



Data points from S. Myers, "FCC - Building on the Shoulders of Giants", Eur. Phys. J. Plus (2021) **136**: 1076

**Serendipitously, 90-100 km is exactly the size required for a 100 TeV hadron collider and optimum tunnel size in the Lake Geneva basin !**



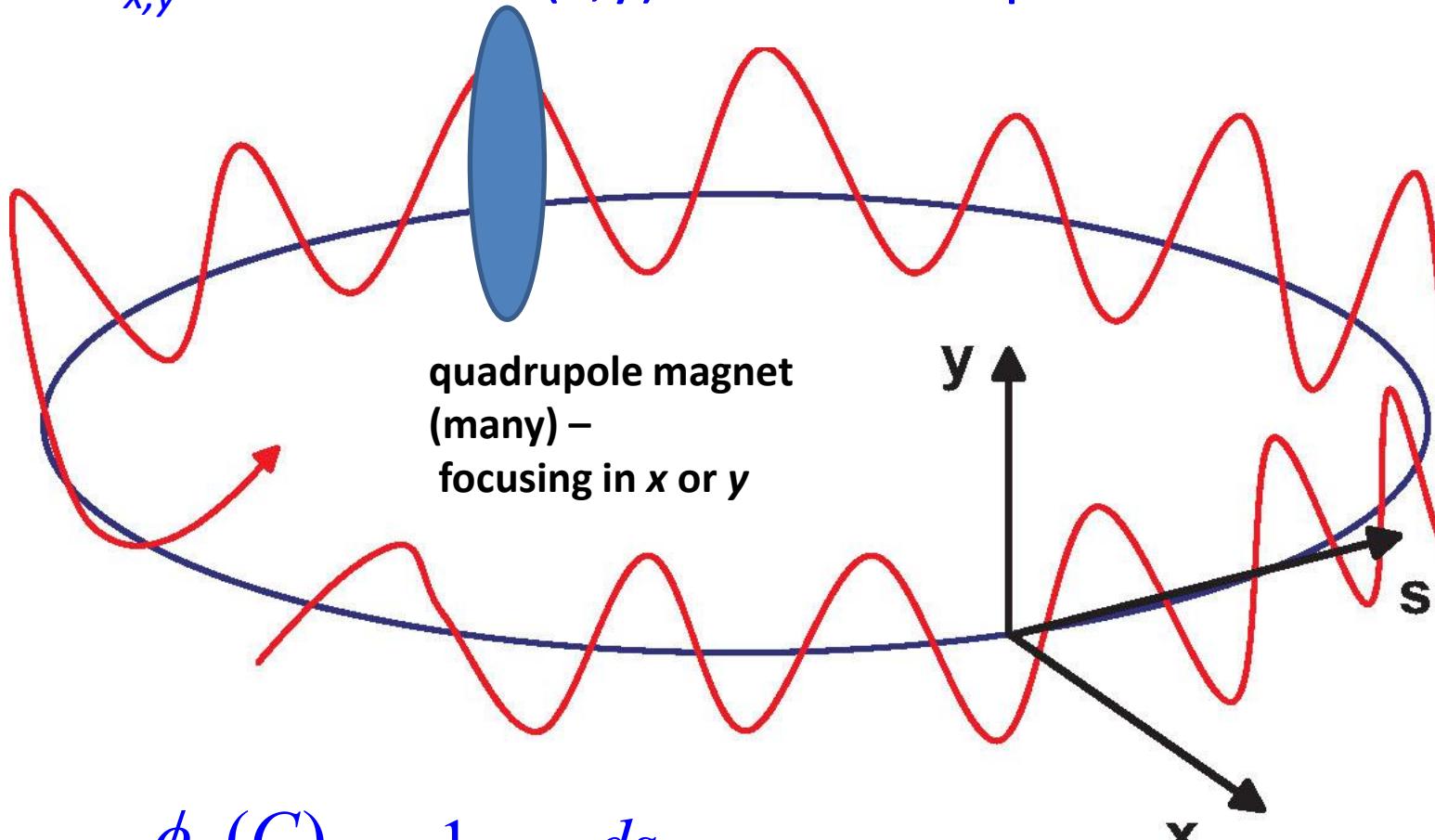
B. Richter, "Very High Energy Electron-Positron Colliding Beams for the Study of Weak Interactions", NIM 136 (1976) 47-60

**circular colliders**

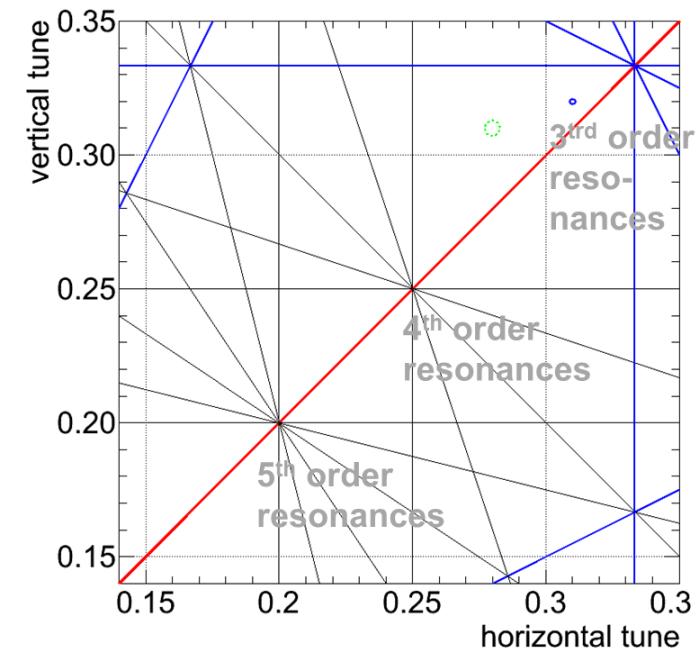
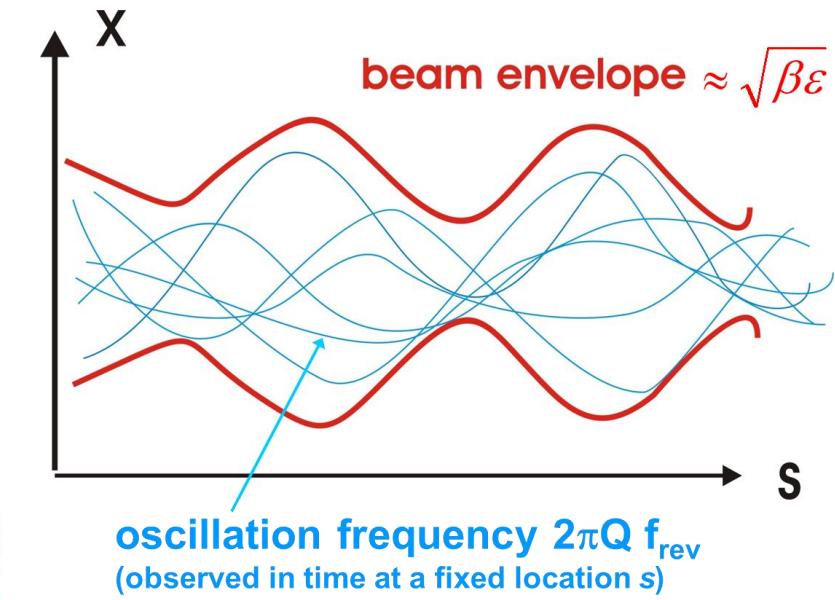
# betatron motion

schematic of betatron oscillation around storage ring

tune  $Q_{x,y}$  = number of (x,y) oscillations per turn



$$Q = \frac{\phi_\beta(C)}{2\pi} = \frac{1}{2\pi} \oint_C \frac{ds}{\beta(s)}$$



## quadrupole strength

$$\frac{d^2 x}{ds^2} = -k(s)x$$

$\Delta x' = -K x$  “kick approximation”

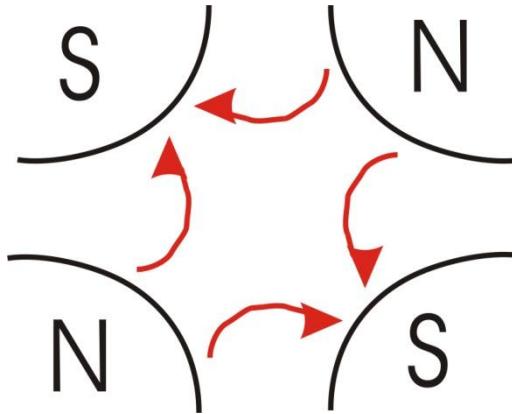
$$k = \frac{B_T}{(B\rho)a}$$

$B_T$ : pole-tip field  
 $a$ : pole-tip radius

$$(B\rho) = p/e = 3.356 \text{ T m} \quad p [\text{GeV}/c]$$

$$K = k l_Q$$

quadrupole length



relation between  
beta function,  
quadrupole strength,  
and betatron tune  
 $\Delta K \rightarrow \Delta Q$  - perhaps  
the oldest technique,  
documented in 1975

ISR-TH-AH-BZ-amb

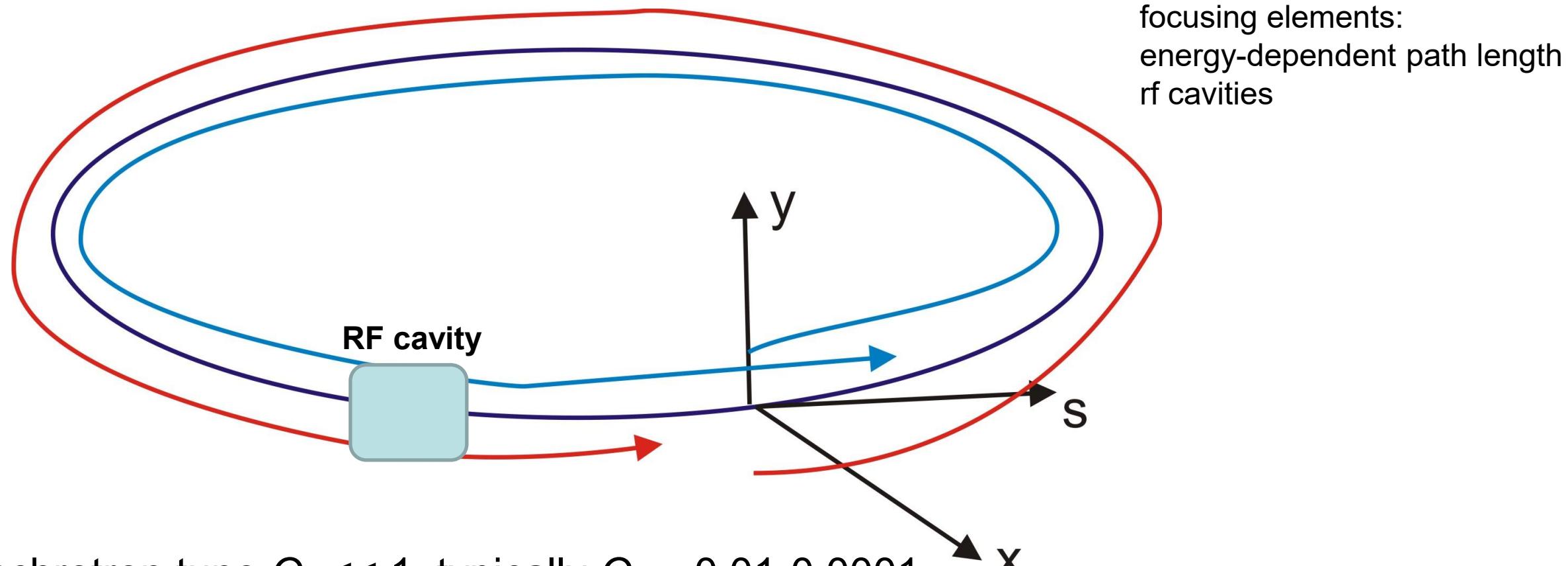
change quadrupole  
strength by  $\Delta K$  ;  
and detect tune shift

$$\Delta Q \approx \frac{\beta_Q \Delta K}{4\pi}$$

# synchrotron motion

schematic of “longitudinal oscillation” around storage ring

tune  $Q_s$  = number of synchrotron oscillations per turn



synchrotron tune  $Q_s \ll 1$ , typically  $Q_s \sim 0.01-0.0001$

betatron tune  $Q_{x,y} > 1$ , typically  $Q_{x,y} \sim 2 - 70$

# radiation damping of synchrotron oscillations

$$\frac{d^2\epsilon}{dt^2} + 2\alpha_\epsilon \frac{d\epsilon}{dt} + \Omega^2 \epsilon = 0$$

$\epsilon$  small energy deviation

## synchrotron motion with damping

$$\alpha_\epsilon = \frac{D}{2T_0}$$

## general damping decrement

$$\Omega^2 = \frac{\alpha_C e V_{\text{rf},0}}{T_0 E_0} \quad \text{angular synchrotron frequency}$$

# angular synchrotron frequency

## synchrotron radiation:

$$U_{\text{rad}} = U_0 + \epsilon D \quad \text{with} \quad D = \left( \frac{dU_{\text{rad}}}{dE} \right)_0$$

$U_{\text{rad}}$  from integrating  $P_\gamma$  around one complete off-energy orbit:

$$U_{\text{rad}} = \frac{1}{c} \oint \left( 1 + \frac{D_x}{\rho} \frac{\epsilon}{E_0} \right) P_\gamma ds$$

$$\frac{dU_{rad}}{dE} = \frac{1}{c} \oint \left\{ 2 \frac{P_\gamma}{E} + 2 \frac{P_\gamma}{B} \frac{D_x}{E_0} \frac{dB}{dx} + \frac{P_\gamma}{E} \frac{D_x}{\rho} \right\} ds$$

When energy of an electron deviates from  $E_0$ , the energy radiated in one revolution changes because

- (1) SR on nominal orbit
- (2) traveling through different magnetic field
- (3) different path length

$$\alpha_\epsilon = \frac{U_0}{2T_0E_0}(2+\mathcal{D})$$

$$\text{with } \mathcal{D} = \frac{\oint D_x G (G^2 + 2K_1) ds}{\oint G^2 ds}$$

$$J_\epsilon = 2 + \mathcal{D}$$

partition number

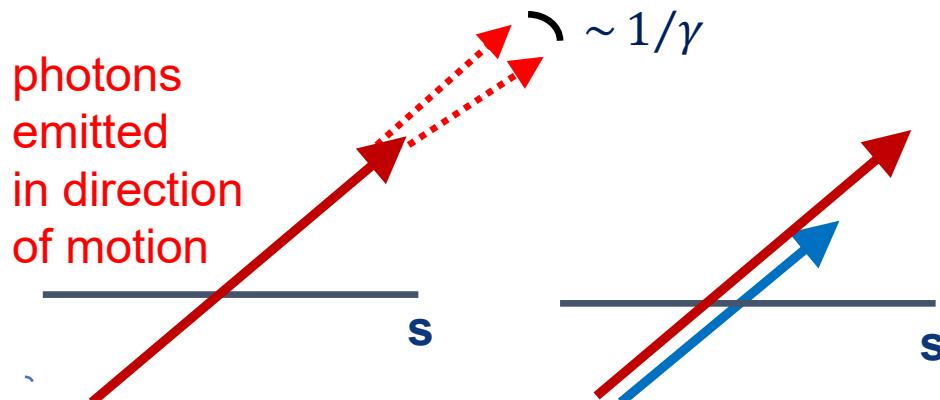
typically  $\mathcal{D} \ll 1$  and

$$\alpha_\epsilon \approx \frac{U_0}{T_0 E_0} = \frac{\langle P_\gamma \rangle}{E_0}$$

**damping time for energy oscillations**  
**= the time it takes an electron to radiate**  
**away its total energy !**

# radiation damping of betatron oscillations (sketch)

synchrotron  
radiation  
in magnetic field



both transverse and longitudinal momentum of emitting electron reduced

longitudinal acceleration in radiofrequency cavities



only longitudinal momentum is restored by RF cavities

→ net damping

$$\alpha_y = \frac{U_0}{2T_0 E_0}$$

$J_y = 1$

$$\alpha_x = \frac{U_0}{2T_0 E_0} (1 - \mathcal{D})$$

$J_x = 1 - \mathcal{D}$

effect of horizontal dispersion

Note: Robinson sum rule

$$\alpha_x + \alpha_y + \alpha_\epsilon = 4 \frac{U_0}{2T_0 E_0}$$

$$\alpha_x + \alpha_y + \alpha_\epsilon = 4$$

Robinson, 1956-58

theorem for arbitrary dissipative force:

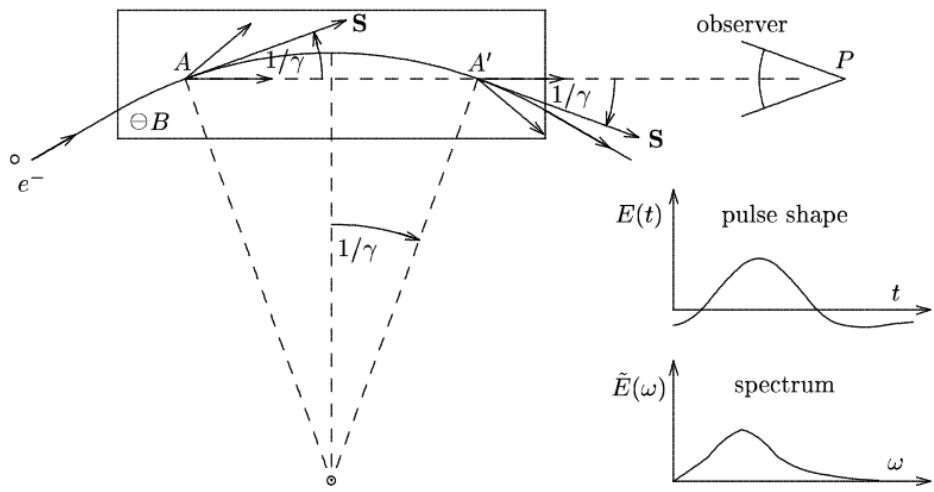
$$\alpha_x + \alpha_y + \alpha_\epsilon = \frac{1}{2} \frac{W}{pc} \left[ 2 + \frac{\partial \ln W}{\partial \ln p} \right]$$

$W$ : rate of energy loss  
 $p$ : particle momentum

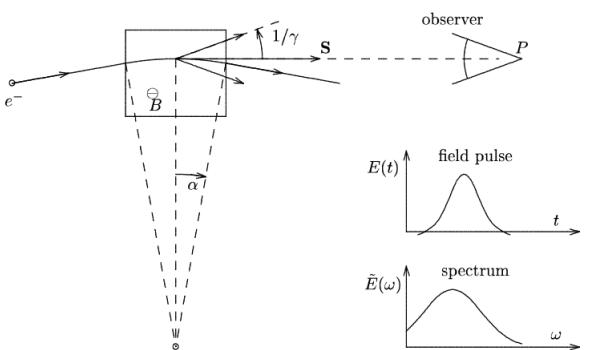
Pestrikov

# SR spectrum

typical frequency of the spectrum emitted in long magnets



typical frequency of the spectrum emitted in a short magnet



Hofmann, CAS 1996

arc dipole magnets in electron storage rings

length of the light pulse seen by an observer

$$\Delta t = t_e - t_\gamma = \frac{2\rho}{\beta\gamma c} - \frac{2\rho \sin(1/\gamma)}{c} \approx \frac{4}{3} \frac{\rho}{c\gamma^3}$$

typical frequency

$$\omega_{typ} \sim \frac{2\pi}{\Delta t} \sim \frac{3\pi c\gamma^3}{2\rho}$$

length of the light pulse seen by an observer

$$\Delta t_{sm} = \frac{L}{\beta c} - \frac{L}{c} \approx \frac{L}{2c\gamma^2}$$

typical frequency

$$\omega_{sm} \sim \frac{2\pi}{\Delta t_{sm}} \sim \frac{4\pi c\gamma^2}{L}$$

this could be important for future hadron colliders

Zimmermann, IPAC2022

# SR spectrum & photon emission

$$P_\gamma = \int_0^\infty \wp(\omega) d\omega$$

↑  
power spectrum

$$\wp(\omega) = \frac{P_\gamma}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

$$\boxed{\omega_c = \frac{3c\gamma^3}{2\rho}}$$

critical frequency  
(half of the power each is emitted  
at lower or higher frequencies)

$$S(\xi) = \frac{9\sqrt{3}}{8\pi} \xi \int_\xi^\infty K_{5/3}(\bar{\xi}) d\bar{\xi}$$

$$\int_0^\infty S(\xi) d\xi = 1$$

first obtained by Schwinger

radiation is emitted in the form of quanta  
(photons) of energy

$$u = \hbar\omega$$

$$u n(u) du = \wp(u/\hbar) d u / \hbar$$

$$n(u) = \frac{P_\gamma}{u_c^2} F\left(\frac{u}{u_c}\right)$$

quantum  
distribution  
function

$$u_c = \hbar\omega_c = \frac{3\hbar c\gamma^3}{2\rho}$$

photon emission rate

$$\mathcal{N} = \frac{15\sqrt{3}}{8} \frac{P_\gamma}{u_c}$$

critical photon energy

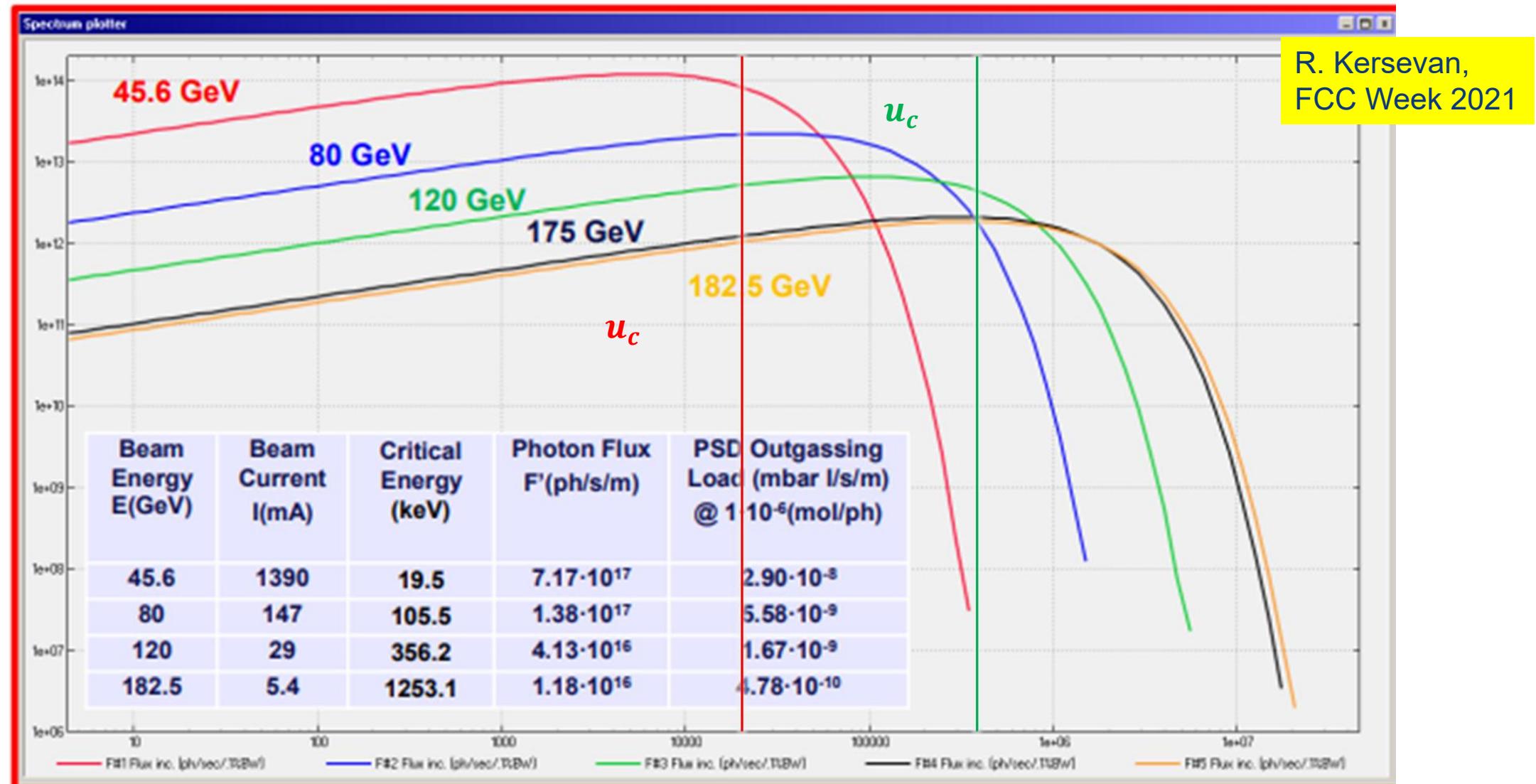
$$\langle u \rangle = \frac{8}{15\sqrt{3}} u_c$$

$$\langle u^2 \rangle = \frac{11}{27} u_c^2$$

mean number of quanta per radian deflection:  
 $5/(2\sqrt{3})\alpha\gamma$  with  $\alpha \approx 1/137$  fine-structure constant

Sands, 1970

# example SR spectra for FCC-ee



Units: Vertical: photons/s/(0.1% bandwidth)/m; Range  $[10^6 - 2 \cdot 10^{14}]$   
Horizontal eV; Range  $[4 - 5 \cdot 10^6]$

# SR in collision: beamstrahlung

synchrotron radiation in the strong field of the opposing beam (“beamstrahlung”)

collision of Gaussian bunches:

Chen & Yokoya, 1992 and before

average Upsilon

$$\Upsilon_{av} \approx \frac{5}{6} \frac{Nr_e^2 \gamma}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

where  $\Upsilon = \frac{2 \hbar \omega_c}{3 E_0} = \gamma \frac{|B| + |E/c|}{B_c}$

linear colliders:  
 $0.2 \leq \Upsilon \leq 100$

circular colliders:  
 $\Upsilon \sim 10^{-4}$

$$n_\gamma \approx \frac{2.16 \alpha r_e N}{\sigma_x + \sigma_y} \frac{1}{(1 + \Upsilon_{av}^{2/3})^{1/2}}$$

photons emitted per electron

historical design constraint for  
linear colliders:

$$n_\gamma < 1$$

$$\delta_E \approx 2.09 \frac{r_e^3 N^2 \gamma}{\sigma_z} \left( \frac{2}{\sigma_x + \sigma_y} \right)^2 \frac{1}{(1 + (1.5 \Upsilon_{av})^{2/3})^2}$$

average relative energy loss

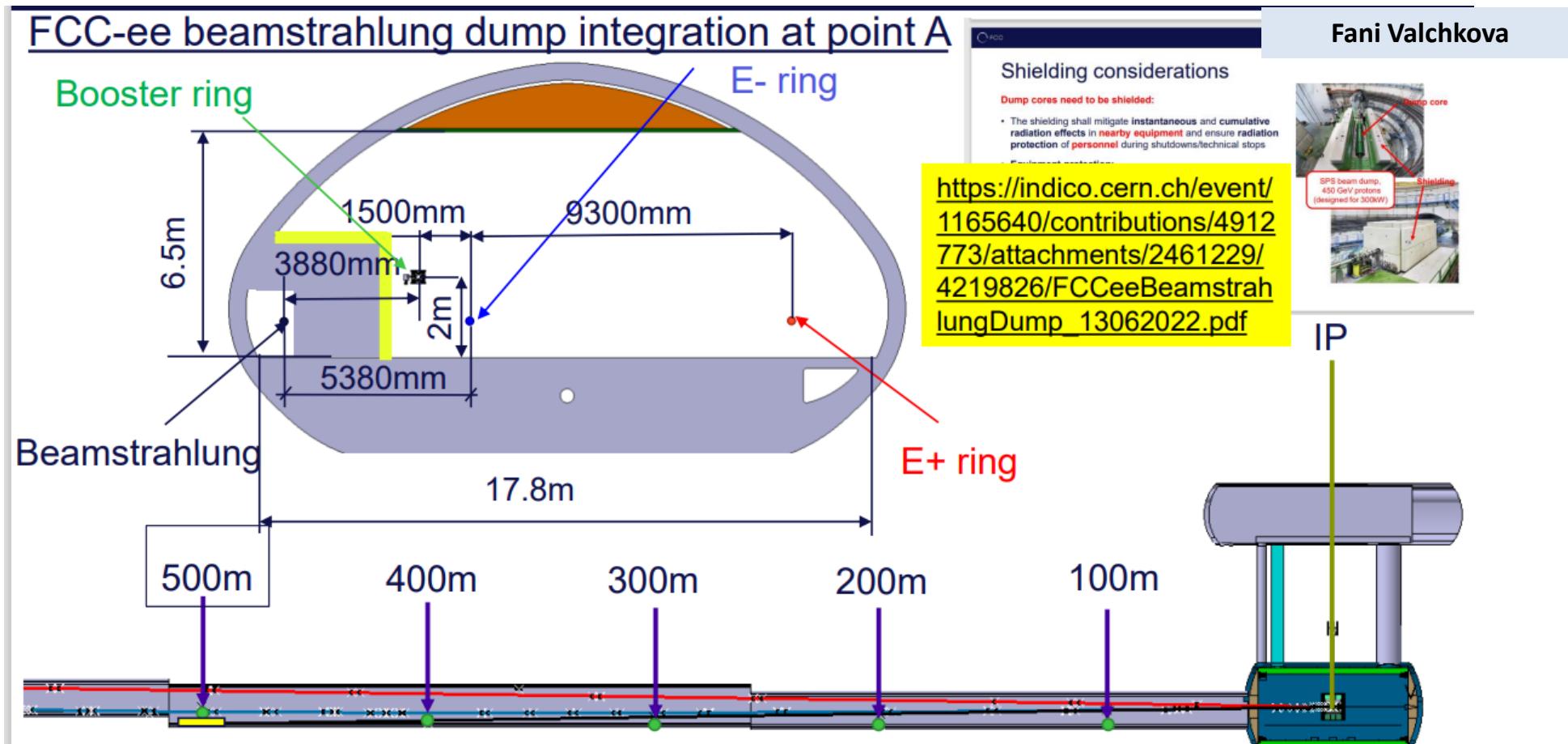
how to reduce beamstrahlung: (1) flat beams  $\sigma_x \gg \sigma_y$  (!), and/or (2) very short bunches ?

circular  
colliders

beamstrahlung increases equilibrium beam energy  
spread and/or limits the beam lifetime

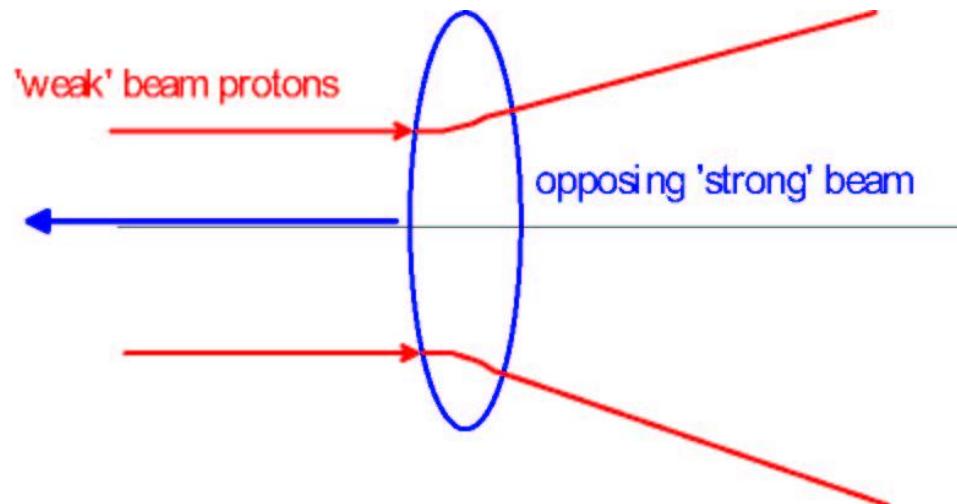
# handling of beamstrahlung at FCC-ee

up to 0.5 MW beamstrahlung photon power per beam per IP,  
requires dedicated shielded photon beam dumps



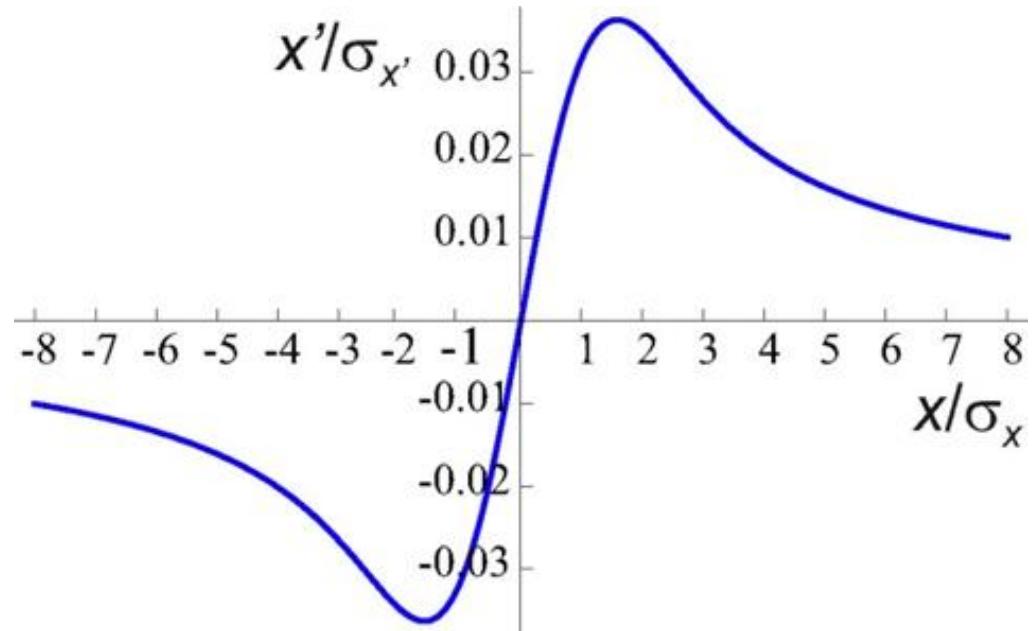
# 1.3 Beam-beam effects

## collider figure of merit: beam-beam tune shift



head-on beam-beam  
collision in the LHC

**(nonlinear) beam-beam force**



at small amplitude similar to effect of defocusing quadrupole

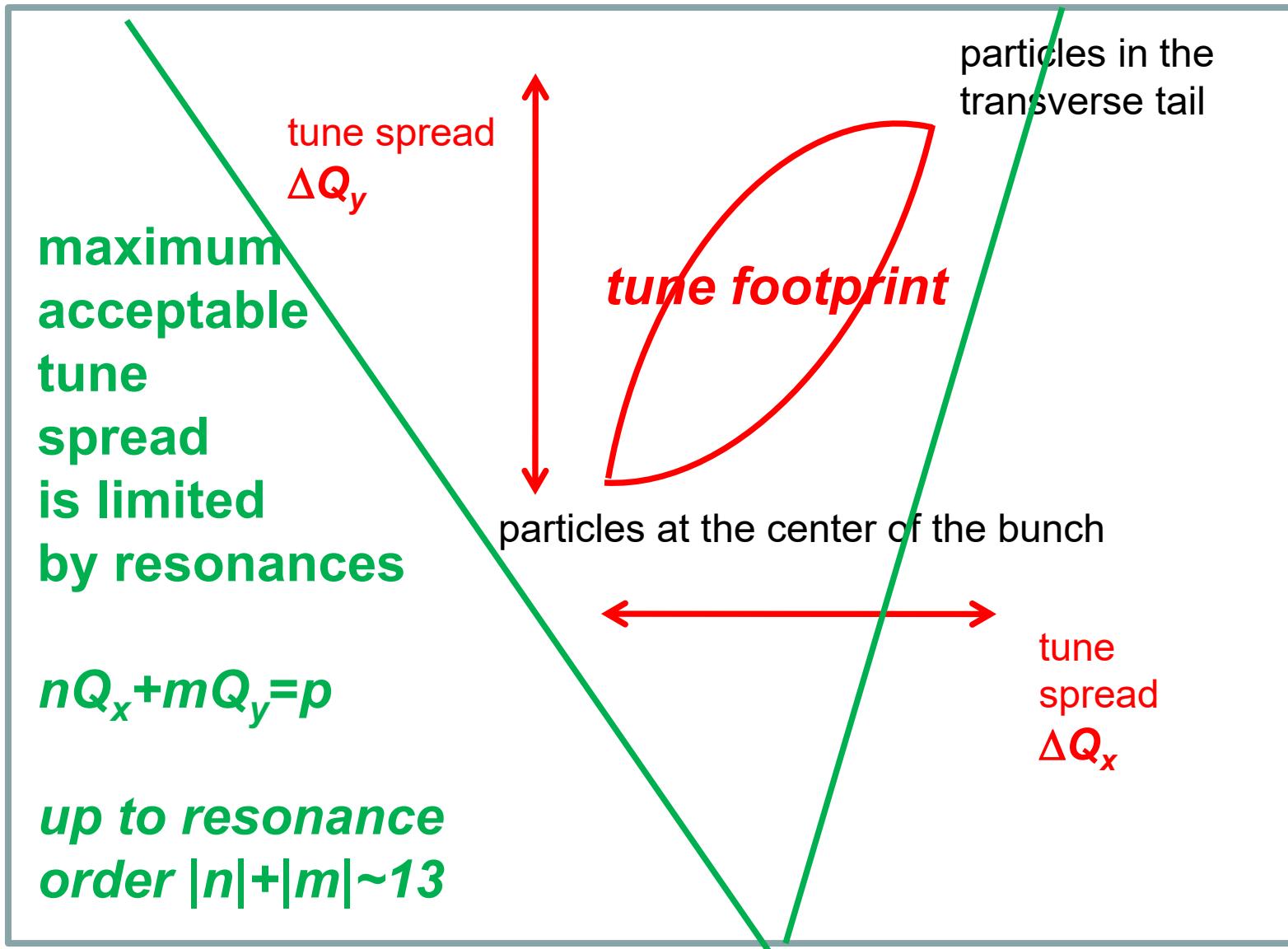
for pure head-on collision

$$\Delta Q_{x,y;\max} = \xi_{x,y} = \frac{2N_b r_0 \beta^*}{4\pi\gamma(2\sigma^*)^2} = \frac{N_b}{\varepsilon_N} \frac{r_0}{4\pi}$$

for single  
collision  
(nominal  
LHC  $\sim 0.0033$ )

# beam-beam tune spread

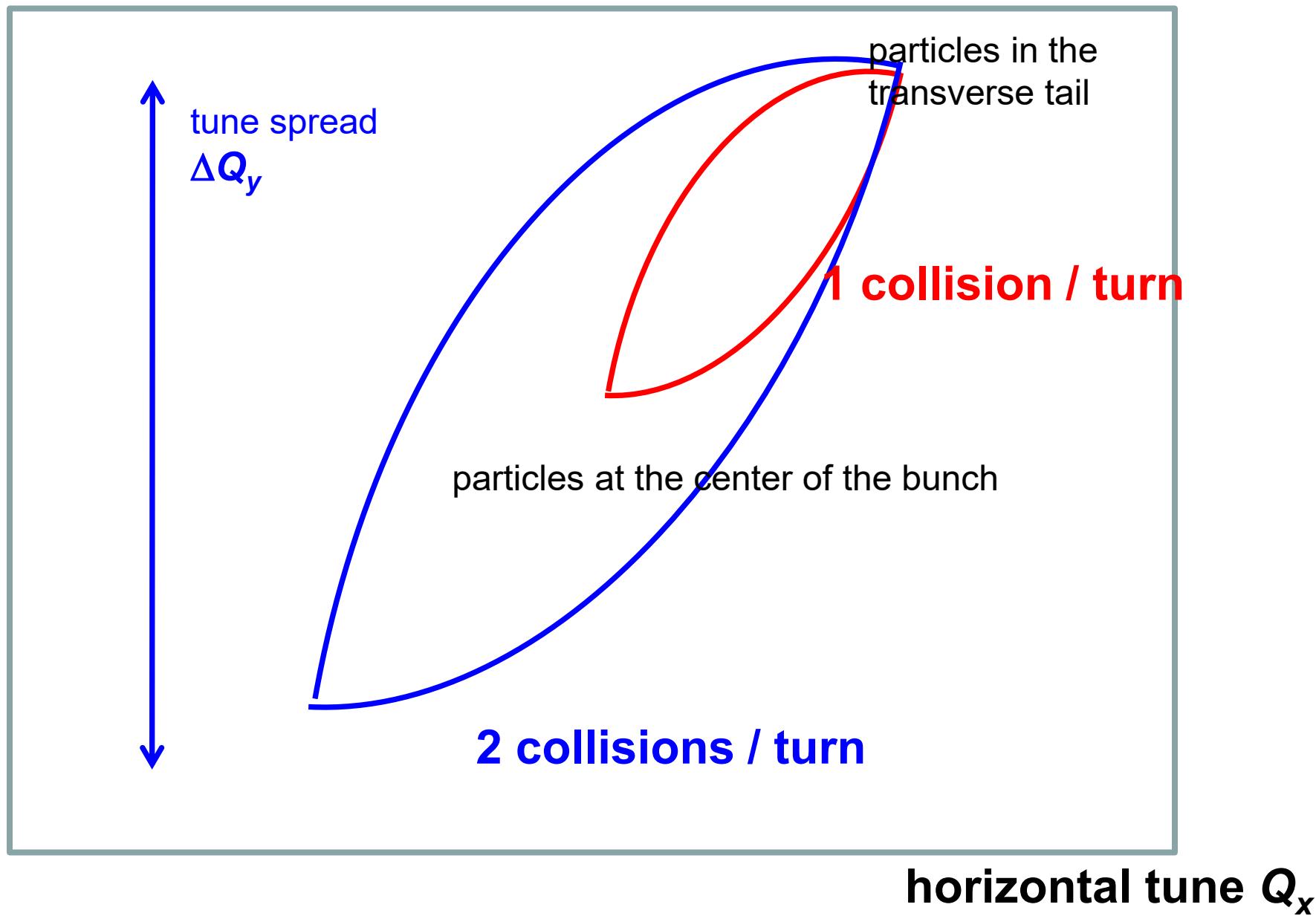
vertical  
tune  $Q_y$



horizontal tune  $Q_x$

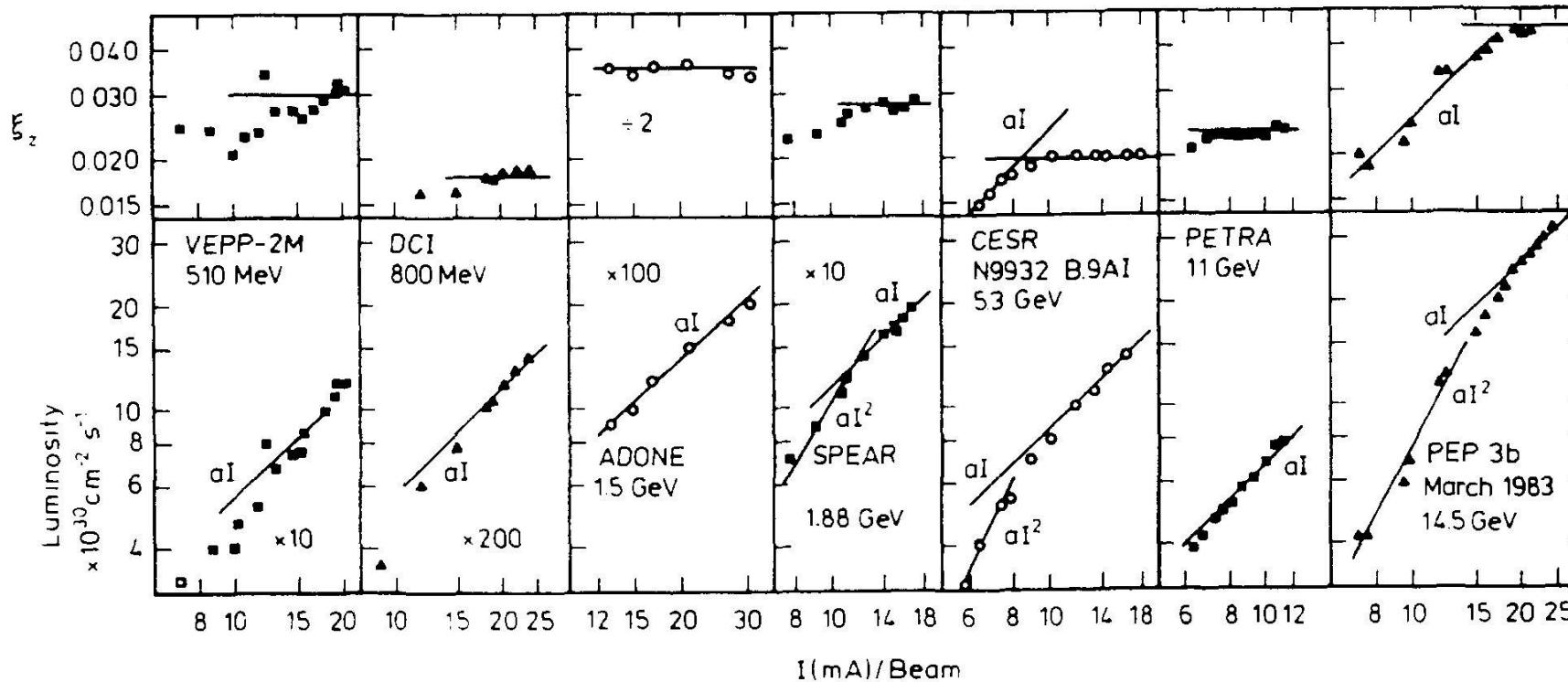
# multiple interaction points

vertical  
tune  $Q_y$



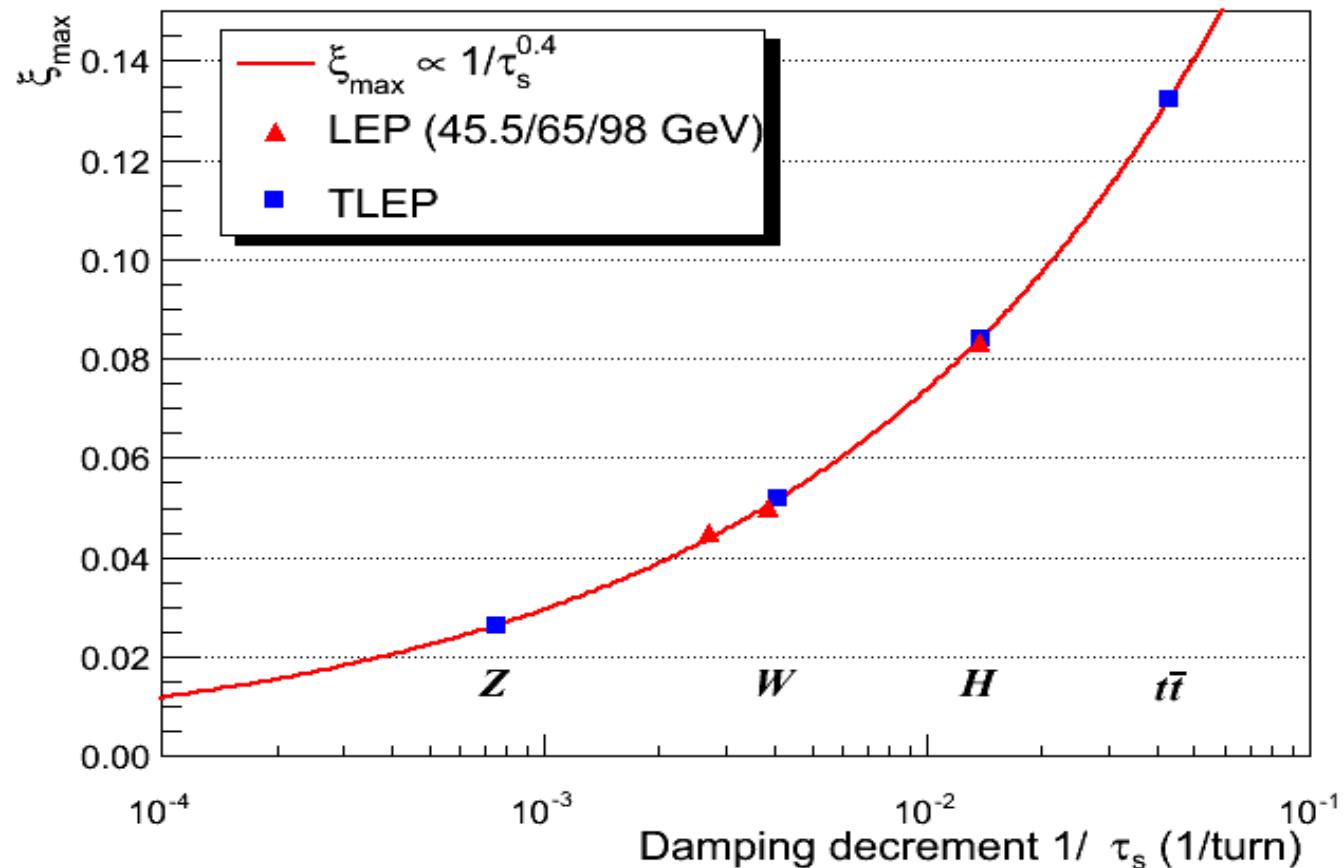
# beam-beam limit in $e^+e^-$ colliders

J. Seeman



luminosity and vertical tune-shift parameter versus beam current for various electron-positron colliders; the tune shift saturates at some current value, above which the luminosity grows linearly

# beam-beam limit w strong SR damping



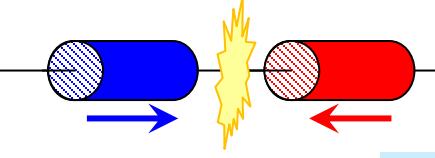
R. Assmann

$$\lambda_d = 1/(f_{rev} \cdot \tau \cdot n_{ip}) \quad \xi_y^\infty \propto (\lambda_d)^{0.4}$$

damping decrement per IP

# 1.4 Luminosity revisited

scaling: larger  $E$  &  $\rho$

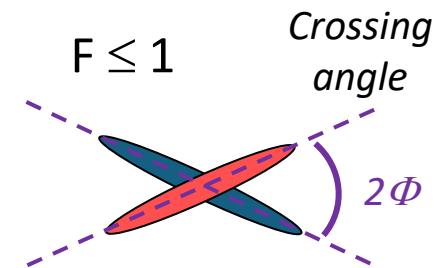
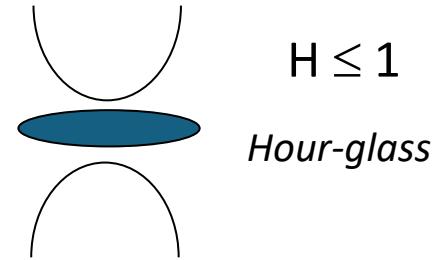


$efkN = \text{beam current} \propto \frac{1}{E^4}$

$$L = \frac{fkN^2}{4\pi\sigma_x\sigma_y} FH$$

$\xi_y \propto \frac{\beta_y^* N}{E\sigma_x\sigma_y} \leq \xi_y^{\max}(E)$  Beam-beam parameter

$$L \propto \frac{\rho P_{SR}}{E^3} \frac{\xi_y}{\beta_y^*}$$



$\sigma$  = beam size  
 $k$  = no. bunches  
 $f$  = rev. frequency  
 $N$  = bunch population  
 $P_{SR}$  = synch. rad. power  
 $\beta^*$  = betatron fct at IP  
 (beam envelope)

# luminosity scaling: damping

- beam-beam parameter  $\xi$  measures strength of field sensed by the particles in a collision
- beam-beam parameter limits can be scaled from LEP data (4 IPs); crab waist allows for higher  $\xi$

$$\xi_y \propto \frac{\beta_y^* N}{E \sigma_x \sigma_y} \leq \xi_y^{\max}(E)$$

$$\xi_y^{\max}(E) \propto \frac{1}{\tau_s^{0.4}} \propto E^{1.2}$$

FCC-ee  
vs LEP

$$L \propto \frac{\rho P_{SR}}{E^{1.8}} \frac{1}{\beta_y^*}$$

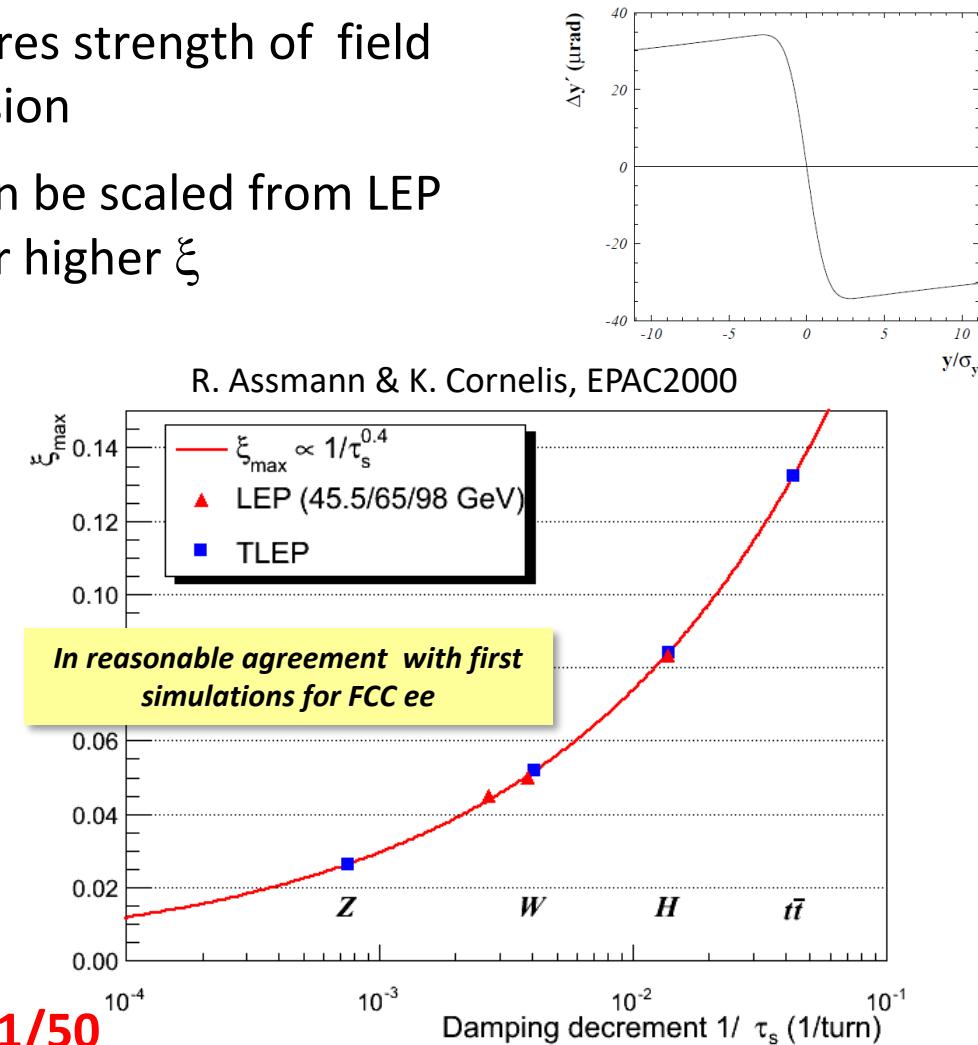
x4.5

x3.7

<2

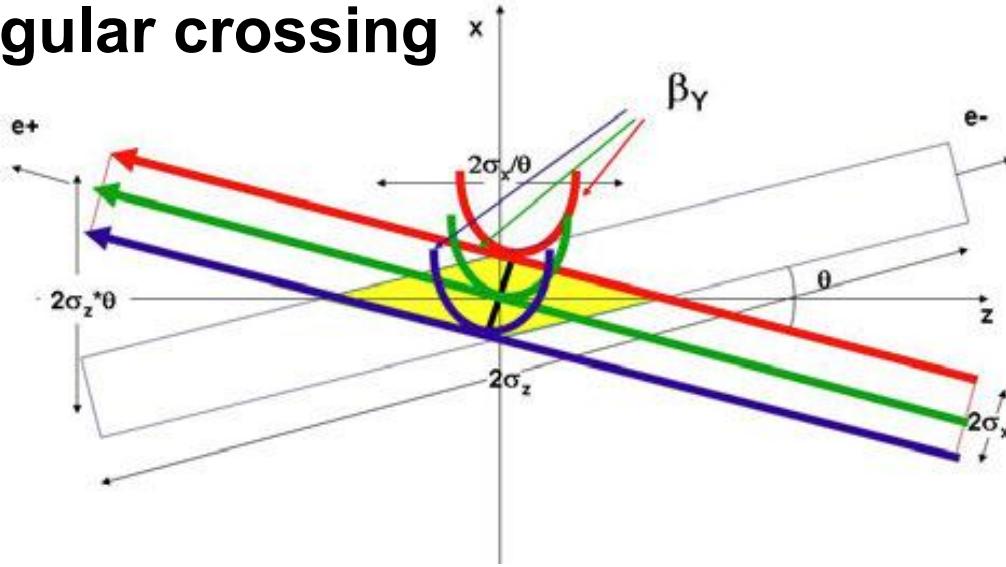
x1/25-1/50

→ extremely high luminosity

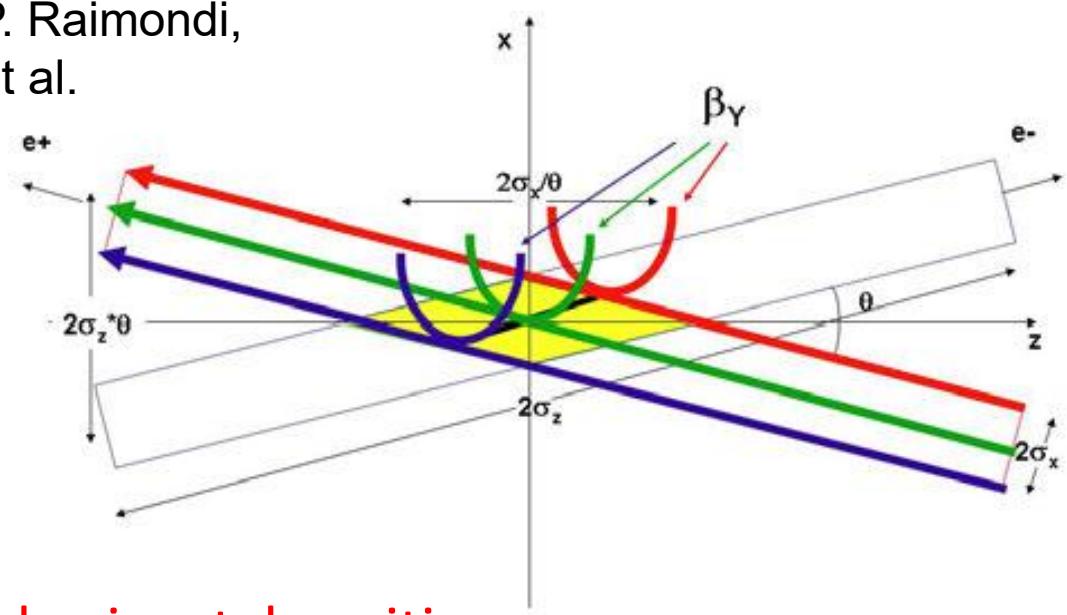


# crab-waist crossing for flat beams

## regular crossing



P. Raimondi,  
et al.



**crab waist** - vertical waist position in  $s$  varies with horizontal position  $x$

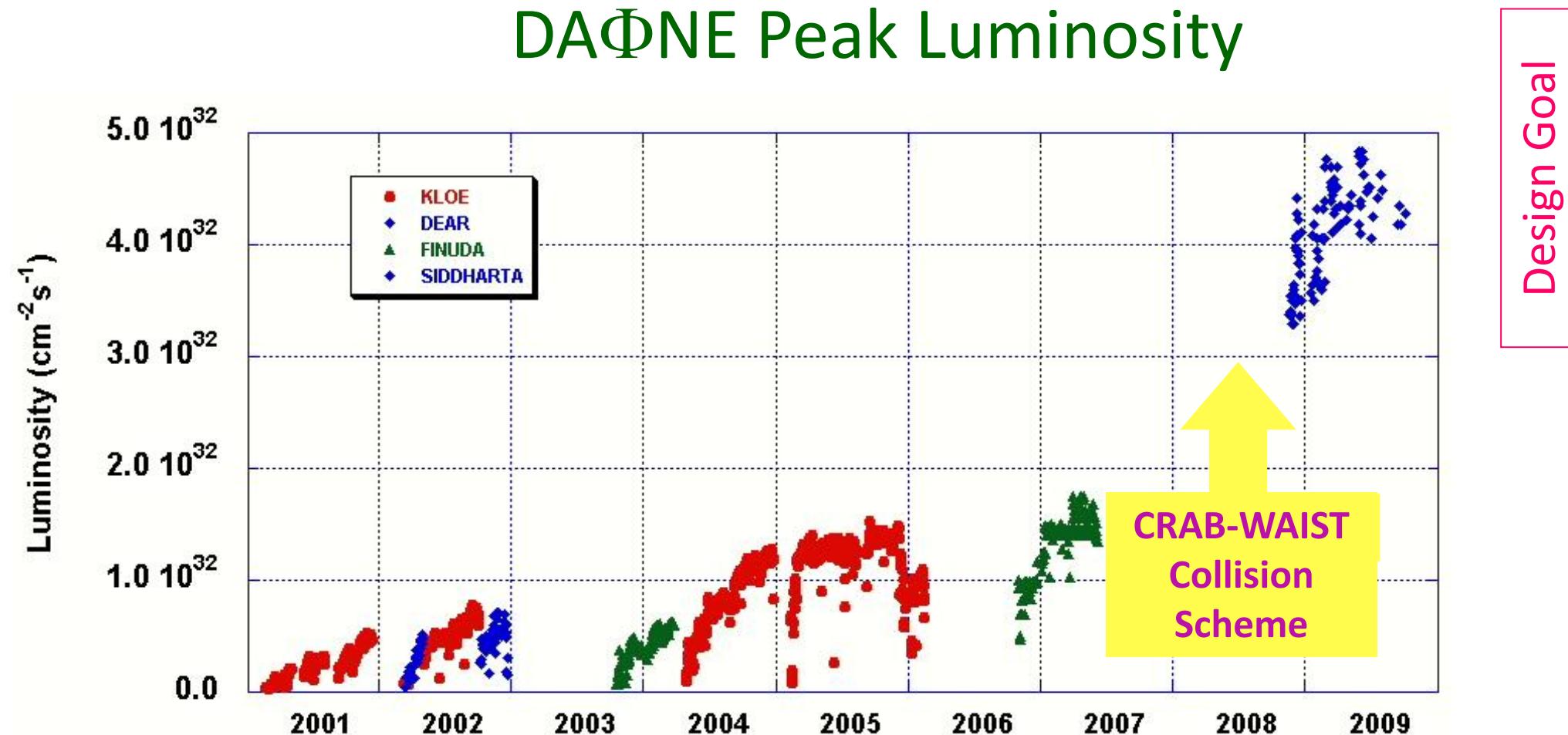
- allows for small  $\beta_y^*$  and for small  $\varepsilon_{x,y}$
- and avoids betatron resonances ( $\rightarrow$ higher beam-beam tune shift)

$$\Phi \equiv \frac{\sigma_z \theta}{2\sigma_x^*} \text{ "Piwinski angle"}$$

$$L = \frac{N_b f_0}{4\pi\sigma_x\sigma_y} \left[ \frac{N^2}{\sqrt{1+\Phi^2}} \right]; \quad \xi_y = \frac{r_e \beta_y}{2\pi\gamma\sigma_x\sigma_y} \left[ \frac{N}{\sqrt{1+\Phi^2}} \right]; \quad \xi_x = \frac{r_e \beta_x}{2\pi\gamma\sigma_x^2} \left[ \frac{N}{1+\Phi^2} \right]$$

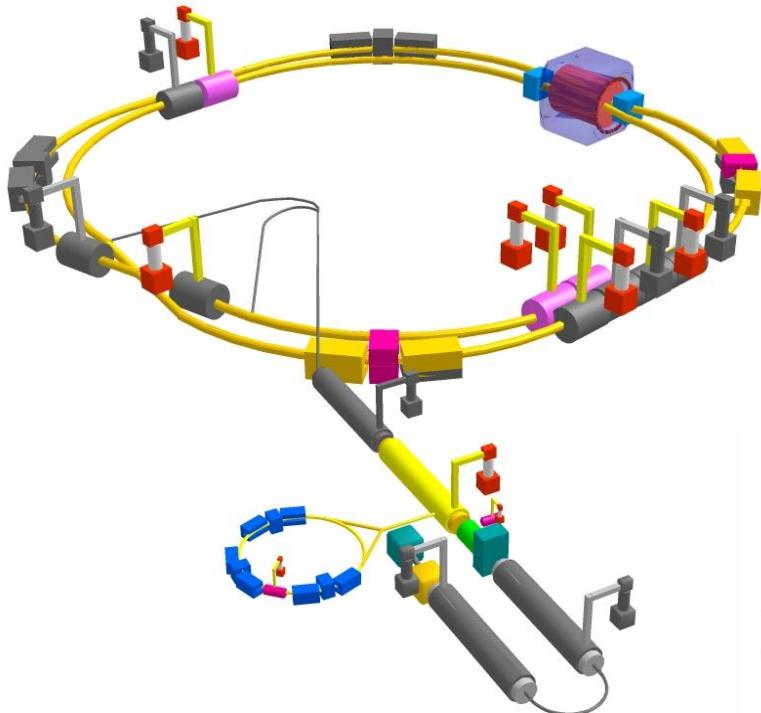
$$\rightarrow L = \frac{I_{beam}}{e} \frac{\gamma}{2r_e} \frac{\xi_y}{\beta_y^*}$$

# DAΦNE: “crab waist” collisions

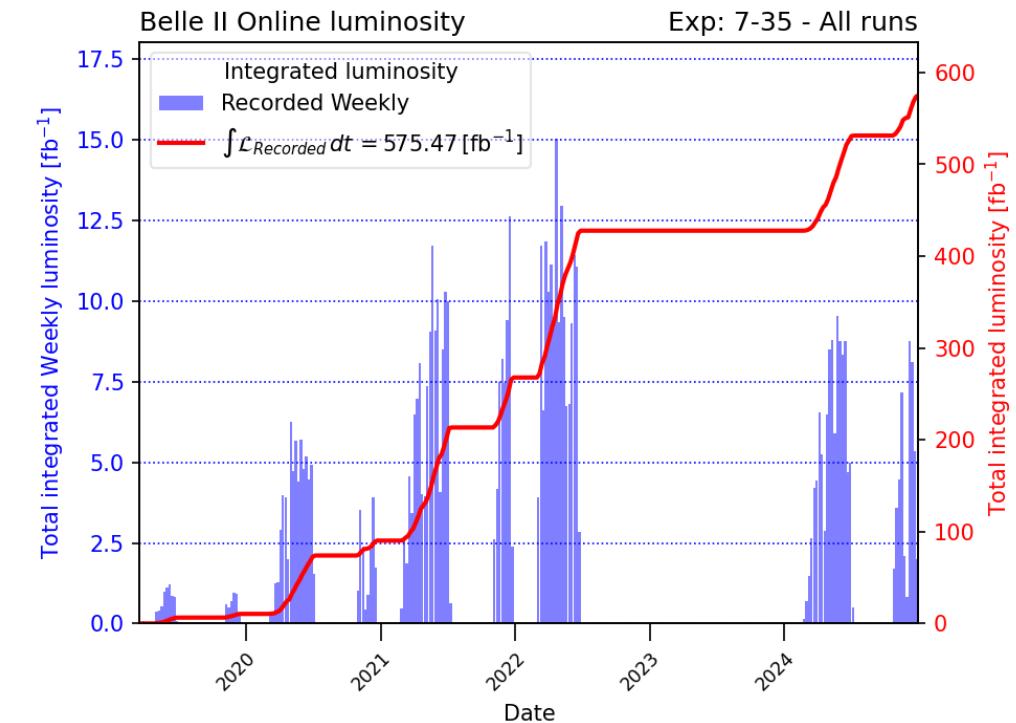
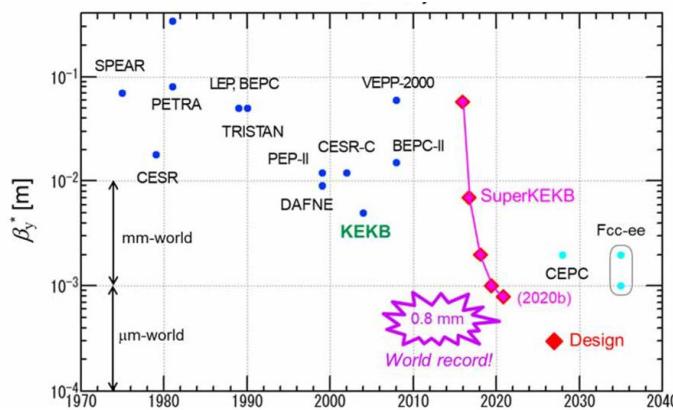


# SuperKEKB 4 GeV $e^+$ vs 7 GeV $e^-$ in Japan

circumference 3 km



world's highest  
luminosity &  
lowest  $\beta^*$   $e^+e^-$   
collider at  
KEK/Tsukuba



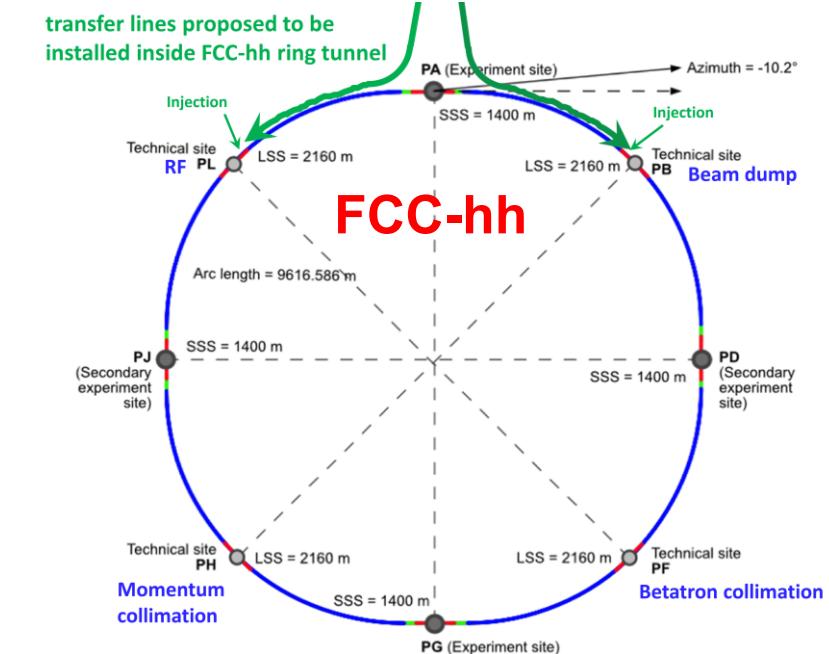
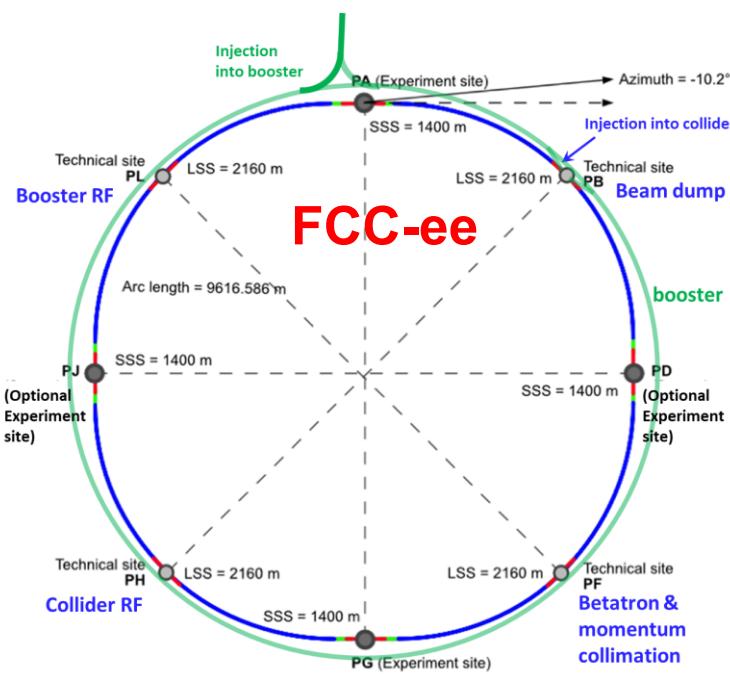
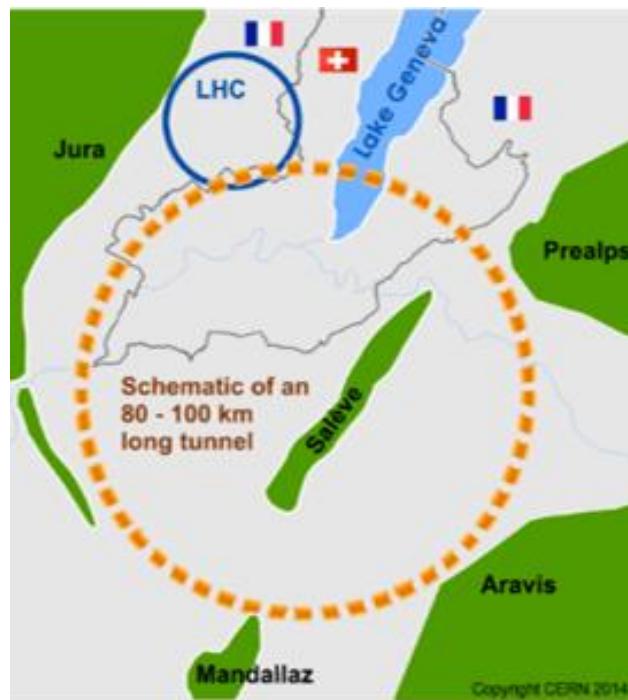
total integrated luminosity so far  
 $\sim 575 \text{ fb}^{-1}$  over  $\sim 6$  years

world record luminosity of  $4.71 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ ,  $\beta_y^* = 1.0 \text{ mm}$  routinely, also  $\beta_y^* = 0.8 \text{ mm}$  shown  
– with “virtual” crab-waist collision scheme originally developed for FCC-ee (K. Oide)

# 1.5 FCC concepts

## FCC integrated program

- inspired by the successful LEP/LHC programs at CERN
- **stage 1: FCC-ee (Z, W, H,  $t\bar{t}$ ) as Higgs factory, electroweak & top factory at highest luminosities**
- **stage 2: FCC-hh (~100 TeV) as natural continuation at energy frontier, pp & AA collisions; e-h option**
- highly synergetic and complementary programme maximising the physics opportunities
- common civil engineering and technical infrastructures, building on and reusing CERN's existing infrastructure
- FCC integrated project allows the start of a new, major facility at CERN within a few years of the end of HL-LHC



2020 - 2045

2045 - 2065

2070 -

# FCC-ee key design concepts

## double ring collider

- many bunches, high current, like LHC and B factories, different from LEP

## crab-waist collision scheme

- successfully demonstrated at DAΦNE (Italy) and SuperKEKB (Japan)

## top-up injection

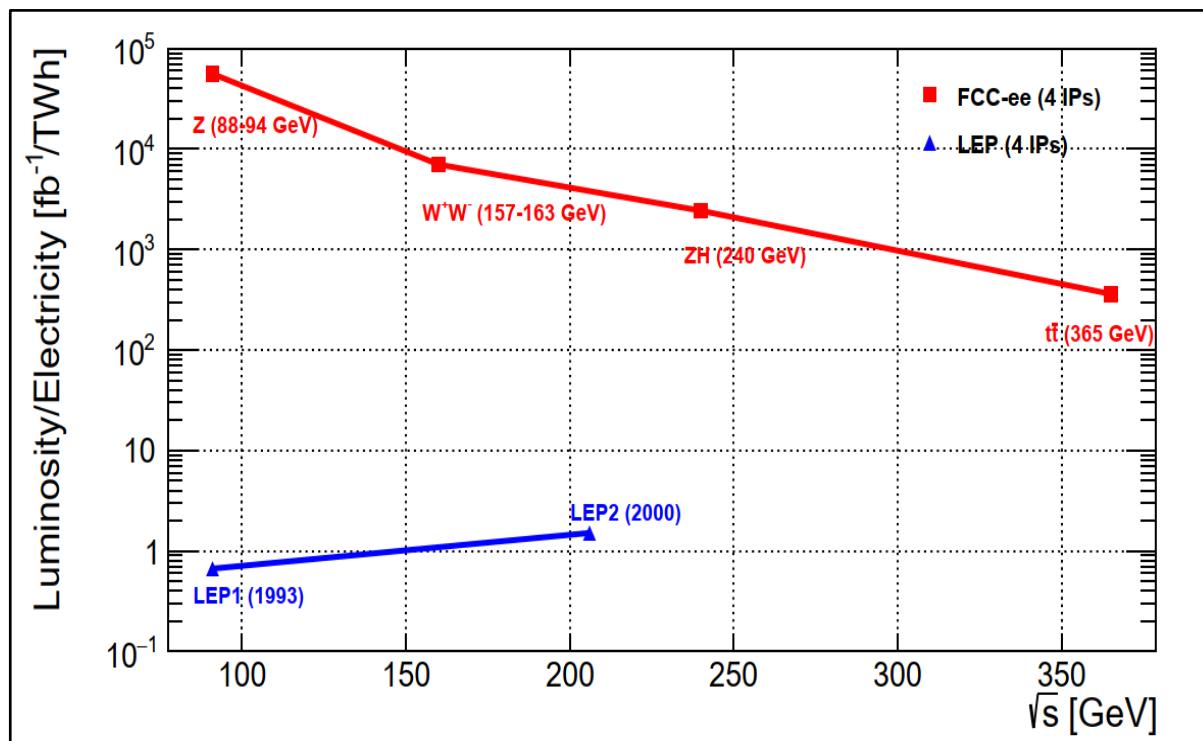
- standard at modern light sources, like SLS
- used at recent  $e^+e^-$  colliders, PEP-II (USA), KEKB (Japan), BEPCII (China)

## SC radiofrequency system

- Nb/Cu 400 MHz SC cavities pioneered at former CERN LEP
- bulk Nb 800 MHz SC cavities similar to ESS (Sweden), EuXFEL (Germany)
- revolutionary highly efficient RF power sources
- new operation scheme for flexible energy switching & reduced complexity

Combining concepts from past and present lepton colliders yields giant step in efficiency:

→  $10^4 - 10^5 \times$  luminosity/energy of LEP  
→ sustainable physics



# FCC-ee design parameters

parameter	Z	WW	H (ZH)	tt
Collision energy $\sqrt{s}$ [GeV]	88, 91, 94	157, 163	240	340-350
synchrotron radiation/beam [MW]	50	50	50	50
beam current [mA]	1294	135	26.8	6.0
number bunches / beam	11200	1852	300	70
total RF voltage 400 / 800 MHz [GV]	0.08 / 0	1.0 / 0	2.1 / 0	2.1 / 7.4
luminosity / IP [ $10^{34} \text{ cm}^{-2}\text{s}^{-1}$ ]	144	20	7.5	1.8
luminosity / year [ $\text{ab}^{-1}$ ]	68	9.6	3.6	0.83
run time (including lumi ramp-up) [years]	4	2	3	1
total integrated luminosity [ $\text{ab}^{-1}$ ]	205	19.2	10.8	0.4
total number of events	$6 \cdot 10^{12} Z$	$2.4 \cdot 10^8 \text{ WW}$ (incl. WW at higher $\sqrt{s}$ )	$2.2 \cdot 10^6 \text{ ZH}$ $65 \text{ k WW} \rightarrow \text{H}$	$2 \cdot 10^6 \text{ tt} + 370 \text{ k ZH}$ $+ 92 \text{ k WW} \rightarrow \text{H}$



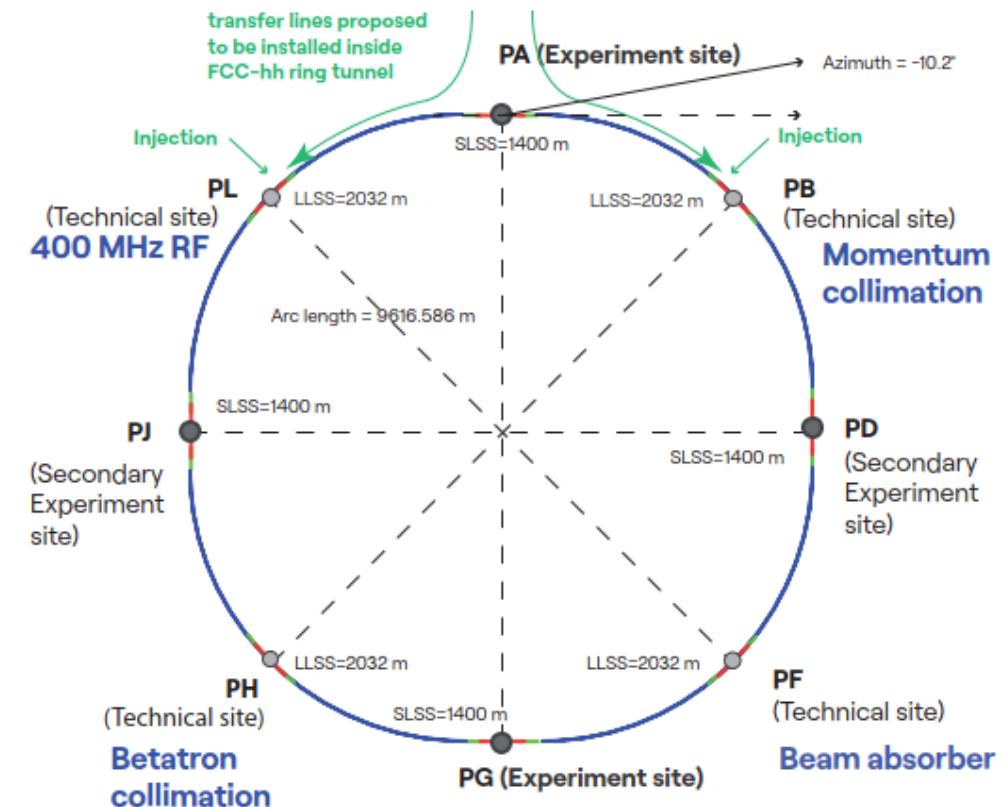
# Stage 2: hadron collider FCC-hh

- Parameter optimization to lower electricity consumption (~max. consumption of FCC-ee)
- Magnetic field considered realistic with today's technologies (Nb<sub>3</sub>Sn, ~14T; alternative: HTS)

Main parameters 2025

parameter	FCC-hh	HL-LHC
<b>collision energy cms [TeV]</b>	<b>85</b>	14
<b>dipole field [T]</b>	<b>14</b>	8.33
<b>circumference [km]</b>	<b>90.7</b>	26.7
<b>beam current [A]</b>	<b>0.5</b>	1.1
<b>synchr. rad. per ring [kW]</b>	<b>1200</b>	7.3
<b>peak luminos. [10<sup>34</sup> cm<sup>-2</sup>s<sup>-1</sup>]</b>	30	<b>5 (lev.)</b>
<b>events/bunch crossing</b>	1000	132
<b>stored energy/beam [GJ]</b>	6.5	0.7
<b>integr. luminosity / IP [fb<sup>-1</sup>]</b>	20000	3000

FCC-hh functional layout



## **Lecture 2 – FCC-ee optics**

2.1 arc emittance

2.2 final focus

2.3 full ring optics

2.4 errors, DA, MA, BBA, etc.

2.5 beam-beam performance

# 2.1 arc emittance

# equilibrium excitation & damping

equilibrium energy spread and bunch length:

$$\sigma_\epsilon \approx \sqrt{E_0 u_c}$$

$$\sigma_z \approx \frac{c \alpha_c}{\Omega_s} \frac{\sigma_\epsilon}{E_0}$$

horizontal equilibrium emittance for bending angle  $\theta$  per cell:

$$\varepsilon_x = \frac{C_q}{1 - \mathcal{D}} F_T \gamma^2 \theta^3$$

$$C_q = \frac{55 \hbar c}{32 \sqrt{3} m_e c^2} \approx 3.84 \times 10^{-13} \text{ m}$$

$$F_T = \frac{\rho^2}{l^3} \langle \mathcal{H} \rangle_{\text{dipole}}$$

$$\langle \mathcal{H} \rangle_{\text{dipole}} = \frac{1}{2\pi\rho} \oint (\gamma_x D_x^2 + 2\alpha_x D_x D'_x + \beta_x D'^2_x) ds$$

$F_T \approx 2.5 L/l$  for 90 deg FODO cell  
 $\approx 0.1$  for “useful & realistic” TME cell

vertical equilibrium emittance normally determined by spurious vertical dispersion and betatron coupling – intrinsic limit is set by  $1/\gamma$  opening angle of the synchrotron radiation:

$$\varepsilon_y \approx C_q \beta_{\text{typical}} / \rho$$

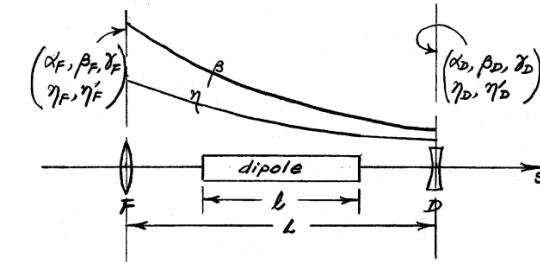
Sands, 1970

$$\varepsilon_y = \frac{13}{55} C_q \frac{\oint \frac{\beta}{\rho^3} ds}{\oint \frac{1}{\rho^2} ds}$$

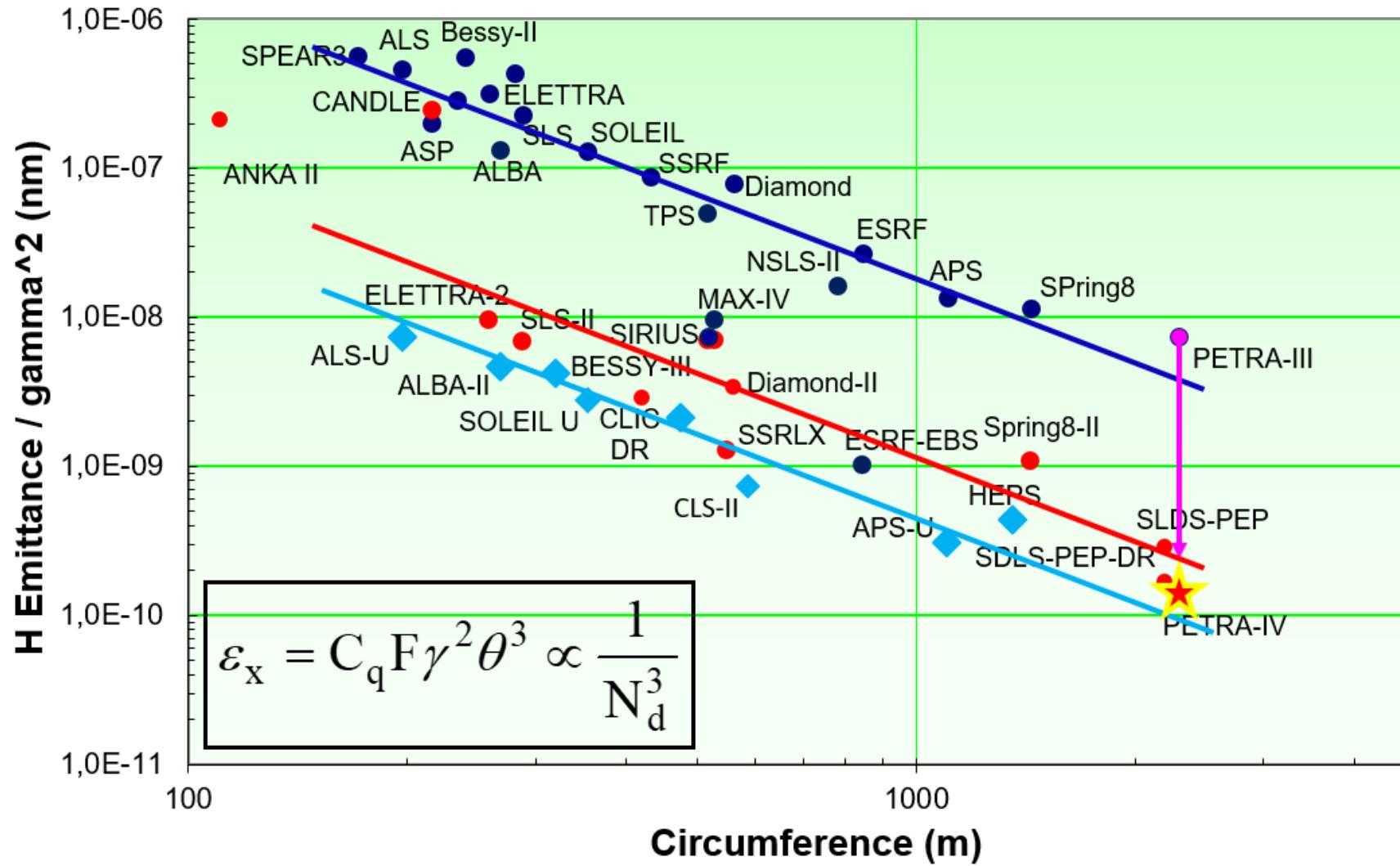
taking into account correlation between photon emission angle and energy

T. Raubenheimer, 1991  
K. Hirata, 1993

Teng, TM-1269, 1984

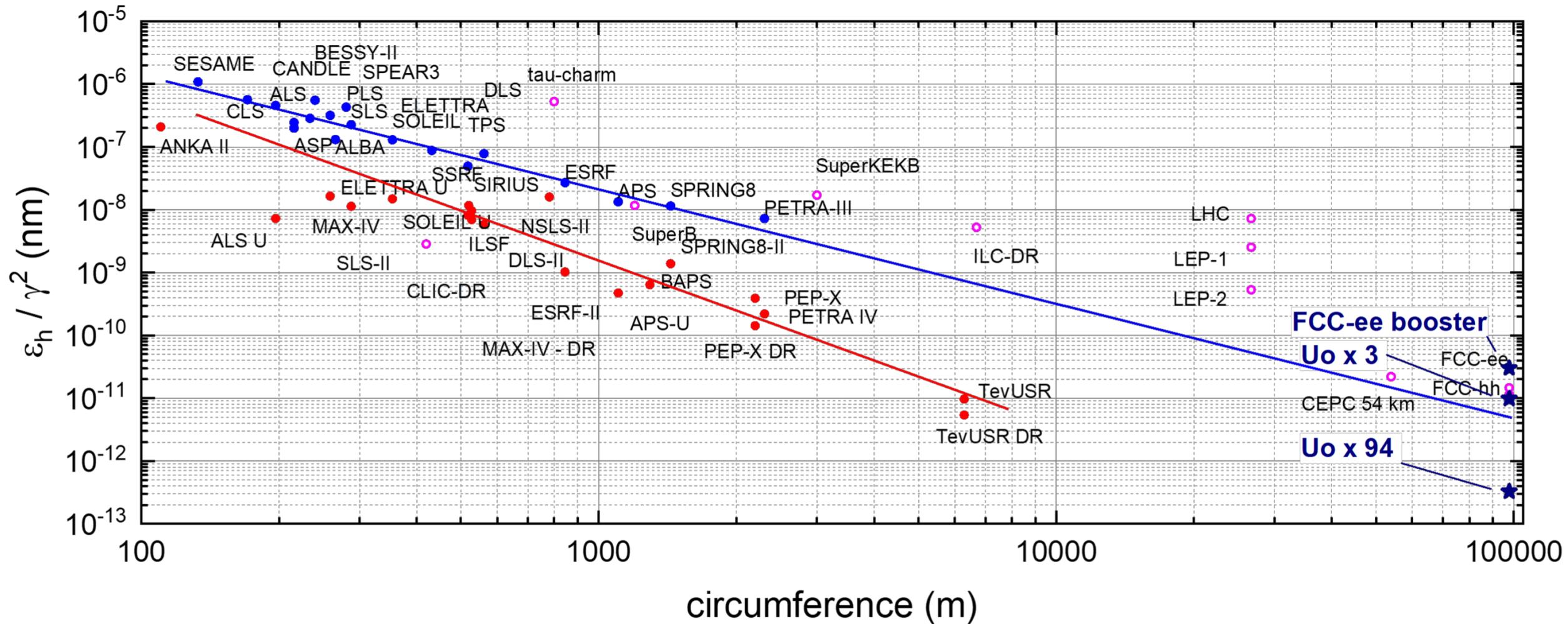


# low-emittance rings – the state of the art



R. Bartolini --  
iFAST Annual Meeting,  
Trieste, April 2023

# low-emittance rings – bring on the FCC-ee !



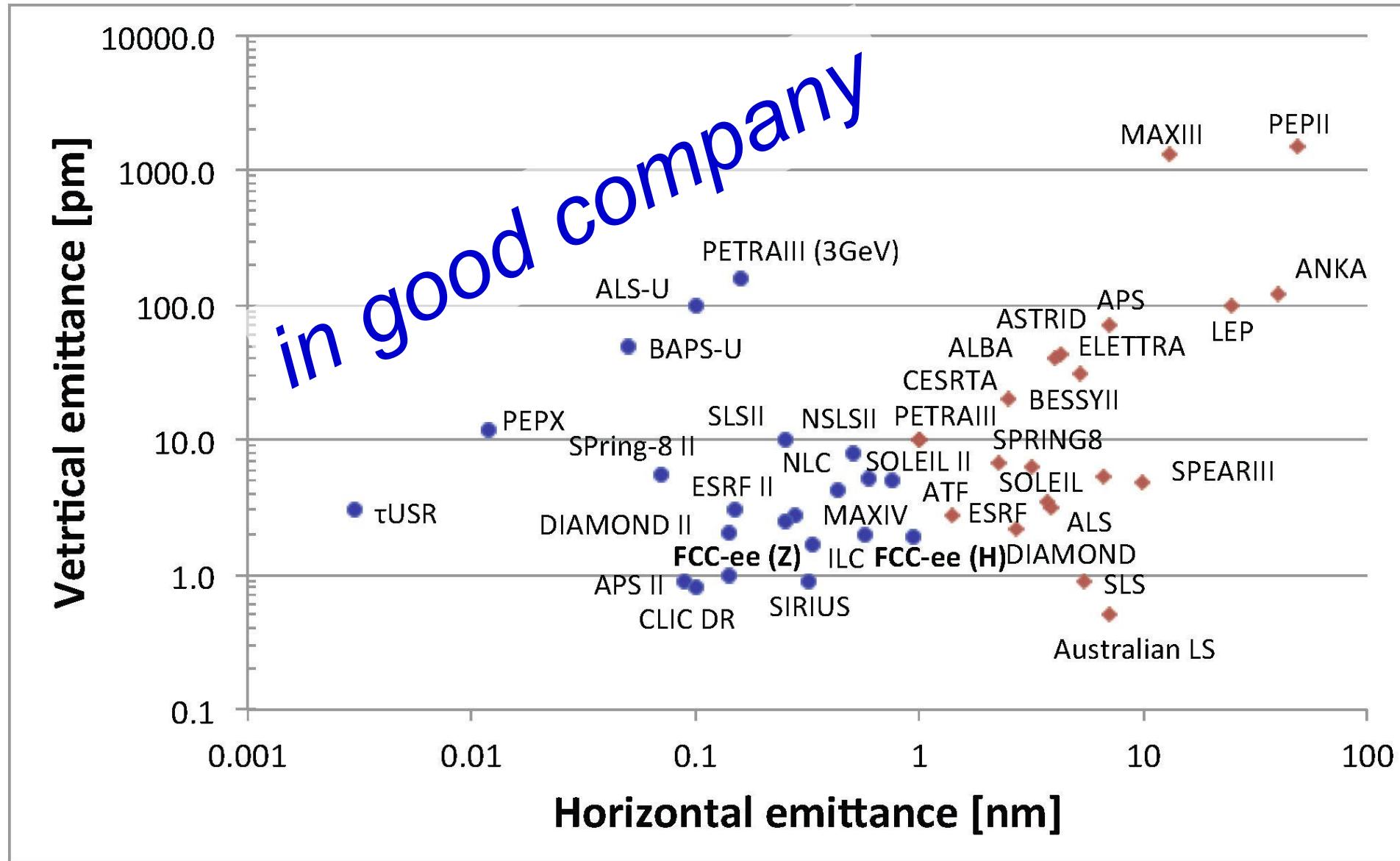
# ring equilibrium emittance

$$\varepsilon_x \sim 10^{-12} \left( \frac{l_b}{\rho} \right)^3 \gamma^2 \text{ [m]}$$

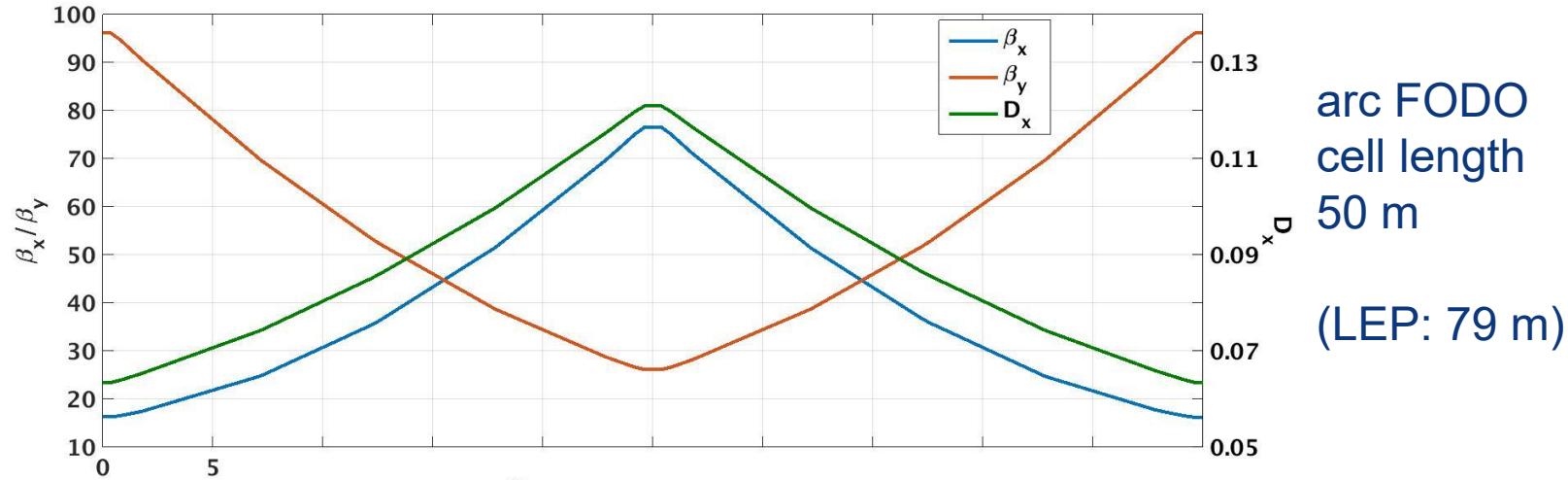
	bending radius $\rho$ [km]	beam energy [GeV]	$\gamma$	$l_b$ (~1/2 cell length)	$\varepsilon_x$ [nm]
LEP2	3.1	104	$2.0 \times 10^5$	39.5	22
FCC-ee-Z	10	45.6	$8.9 \times 10^4$	~46	0.7
FCC-ee-H	10	120	$2.4 \times 10^5$	~20	0.66
FCC-ee-t	10	182.5	$3.6 \times 10^5$	~20	1.5

we can either use 90 degree FODO optics and half the cell length going to Higgs or higher energy, or we can change the phase advance per cell

# FCC-ee also needs a small vertical emittance $\sim 1$ pm



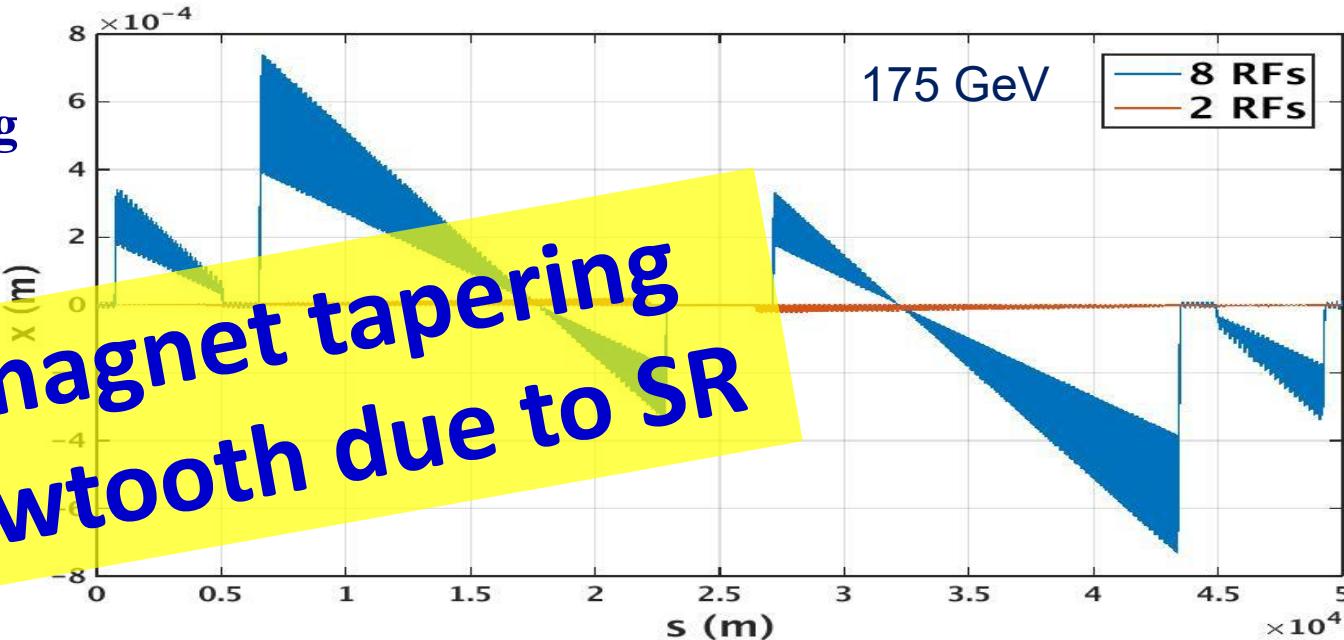
# arc optics & sawtooth tapering



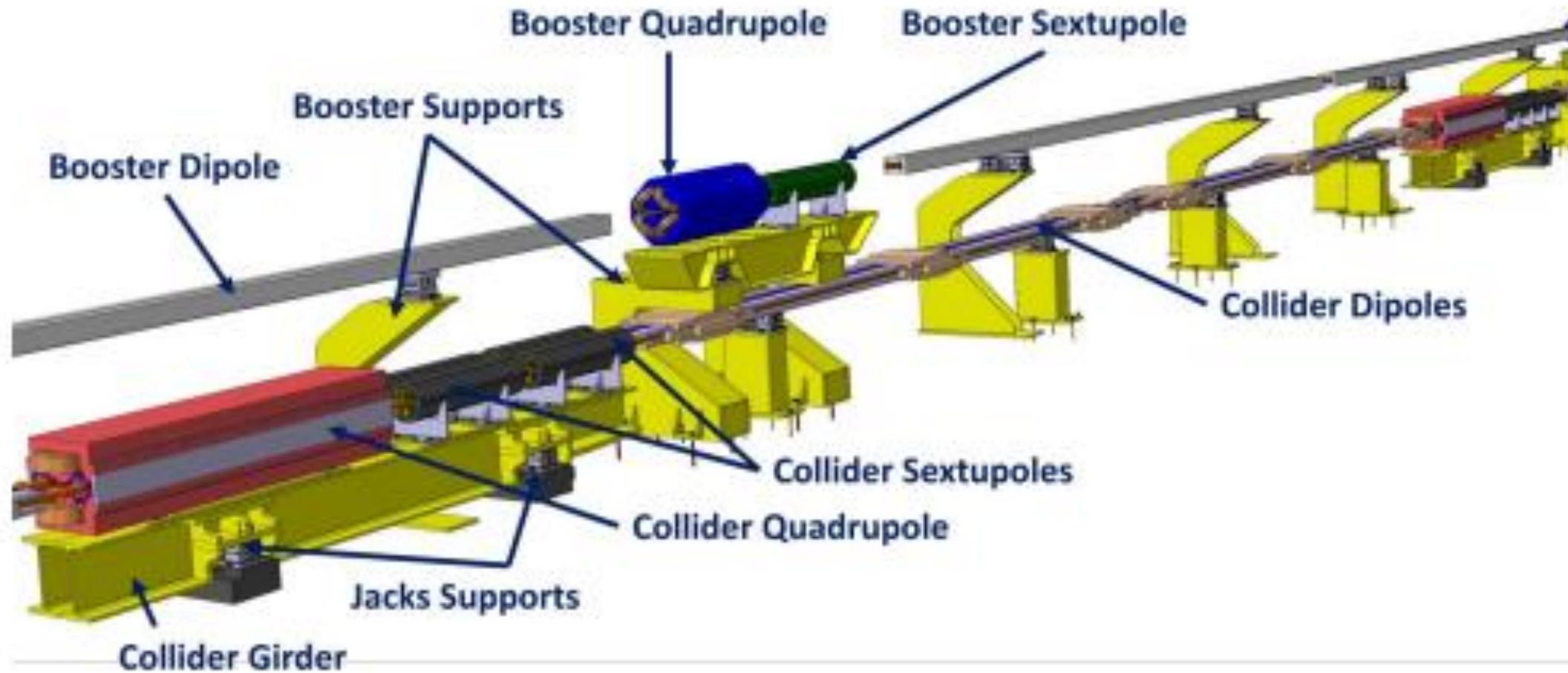
Comparison:  
8 RF w/o tapering  
2 RF with  
tapered dipoles

double ring and magnet tapering  
remove energy sawtooth due to SR

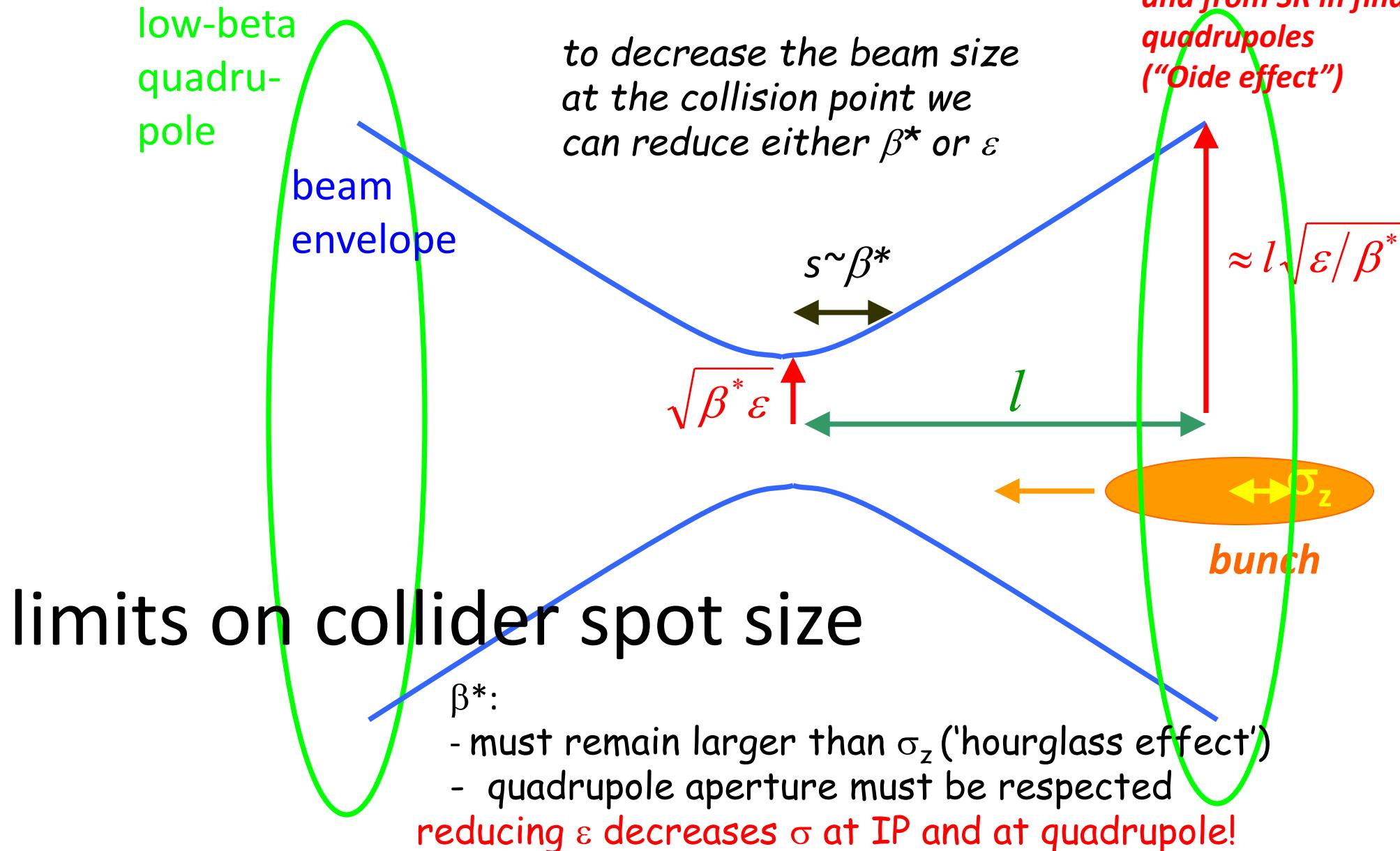
Sandra Aumont  
Pascale Berger,  
Andreas Doblhammer  
Bernhard Hensch



# arc half cell



## 2.2 final focus



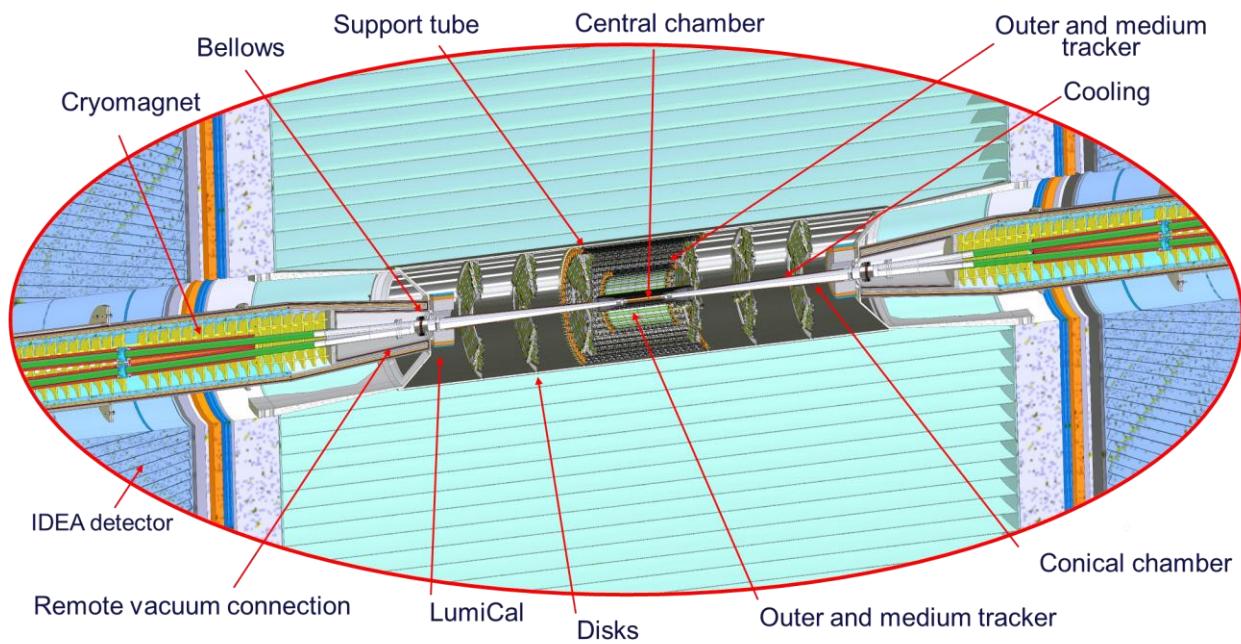
# Machine-Detector Interface

## Key topics:

SC IR magnet system  
& Cryostat design

3D integration

IR mock-up / INFN



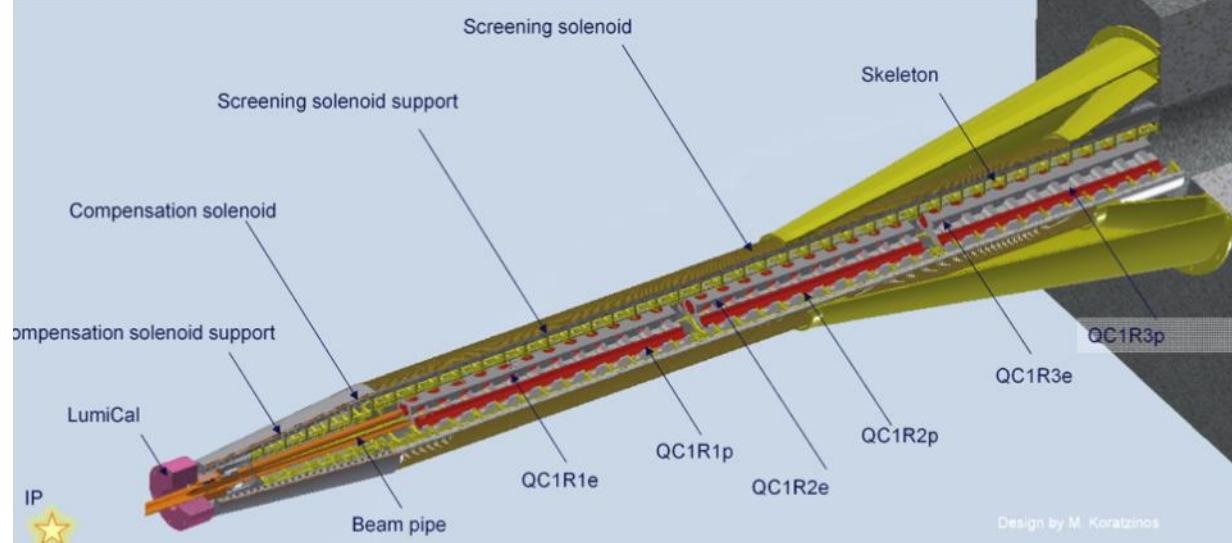
M. Boscolo, F. Palla, INFN

Machine		FCCee	CEPC	ILC	SuperKEKB
Crossing-angle	mrad	30	33	14	83
$L^*$	m	2.2	1.9	3.5	0.935
Vertical $\beta_y^*$ at IP	mm	0.7-1.6	0.9-2.7	0.4	0.3
Detector soln field	T	2/3	3	3.5/5	1.5
Detector stay clear	mrad	100	118/141	90	350/436
Two beam $\Delta X$ at $L^*$	mm	66	62.7	49	77.6
He temperature	K	1.9	4.2	4.5	4.5



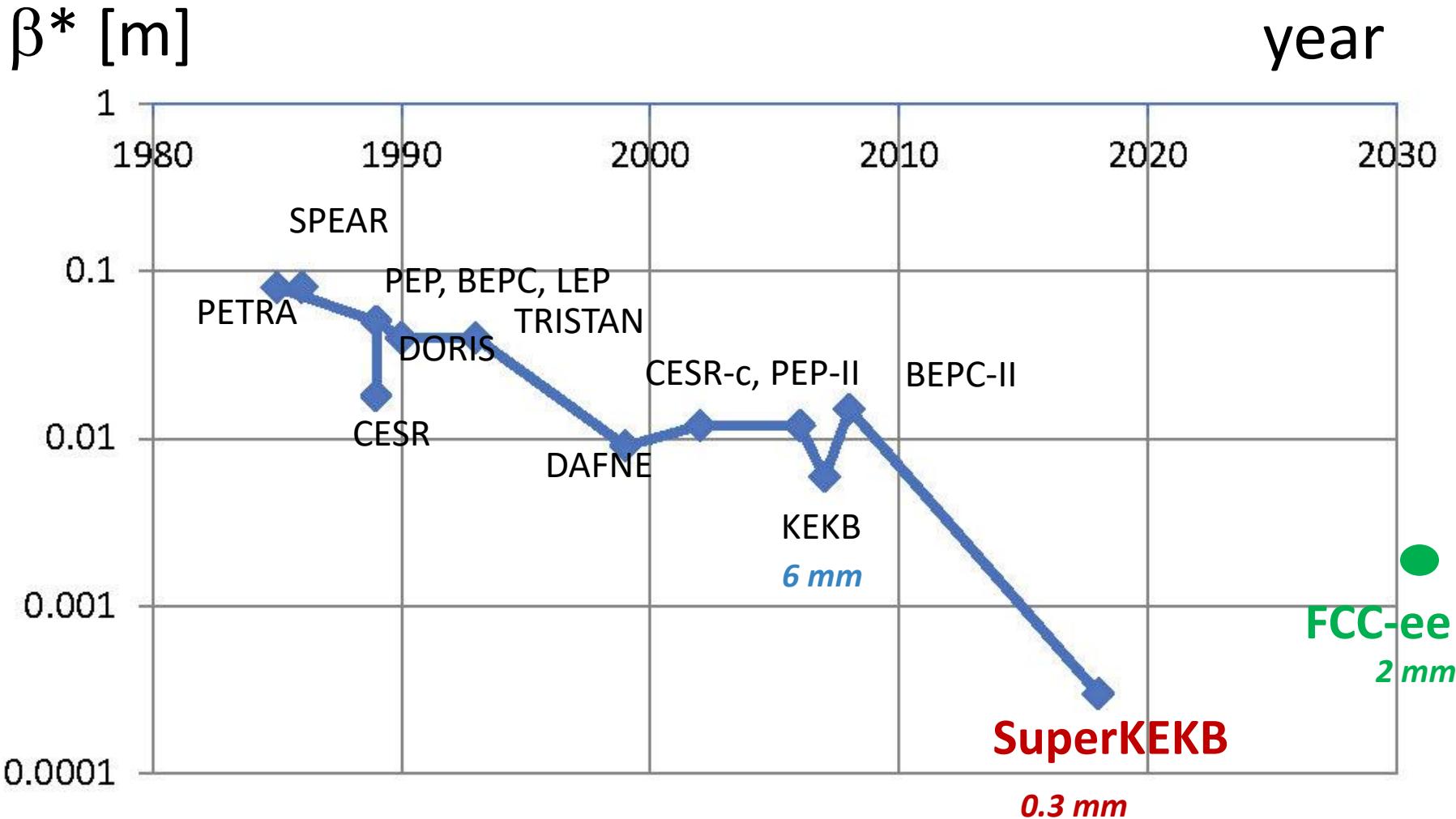
J. Seeman, A. Novokhatski, SLAC  
B. Parker, Vikas Teotia, BNL  
P. Tavares, CERN

M. Koratzinos, PSI



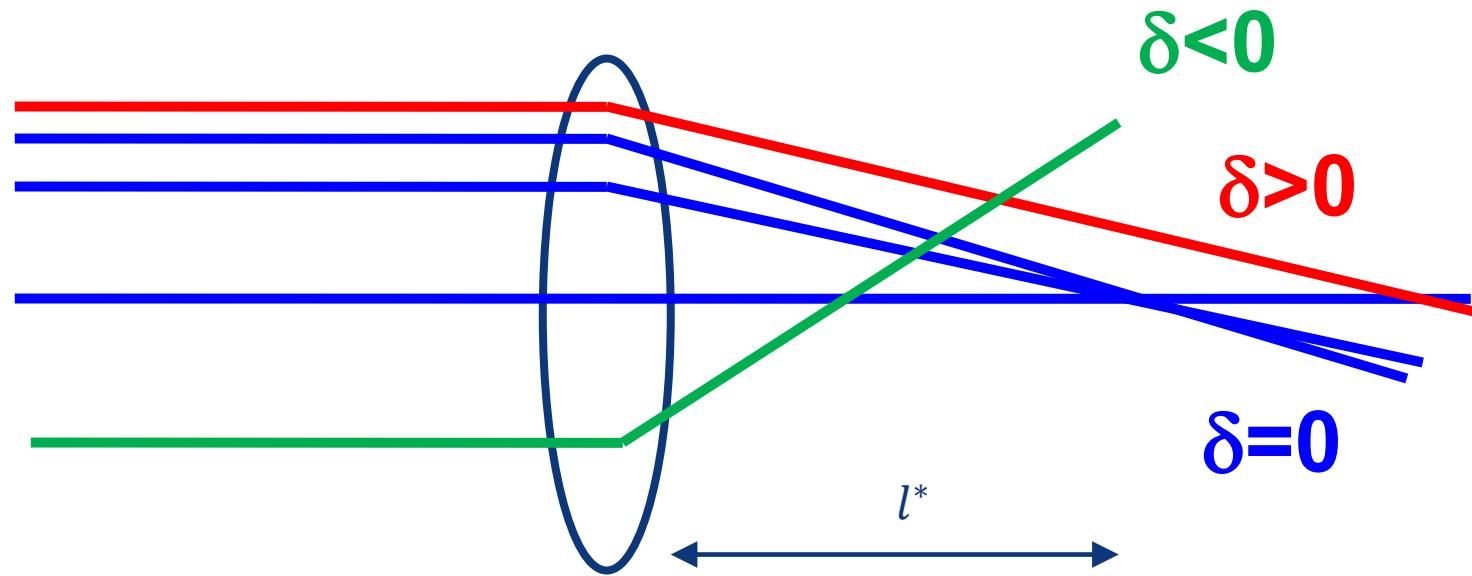
Design by M. Koratzinos

# $\beta_y^*$ evolution over 40 years



entering a new regime for ring colliders –  
SuperKEKB will pave the way towards  $\beta^* \leq 2$  mm

# final focus chromaticity



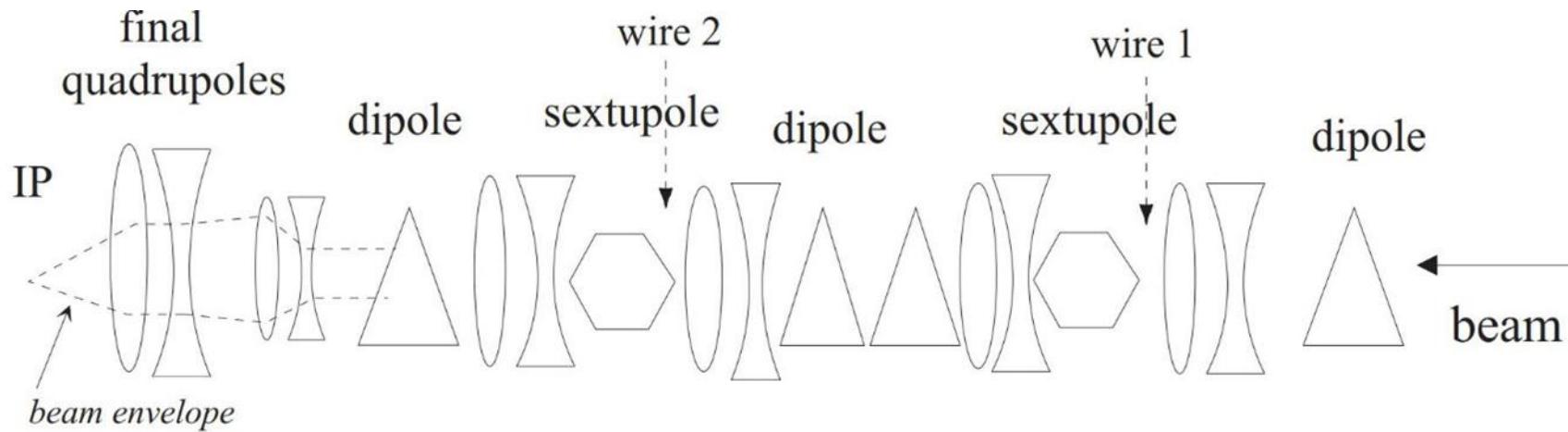
spot size increase due to  
(uncorrected) chromaticity,

$$\frac{\Delta\sigma_y^*}{\sigma_{y0}^*} = \xi\delta_{rms}$$

$$\sigma_{y0}^* \equiv \sqrt{\beta_y^* \varepsilon_y}, \quad \xi \approx \frac{l^*}{\beta^*}$$

# schematic of a final focus system

sextupole magnets at location with nonzero dispersion  
to correct the chromaticity of the final quadrupoles

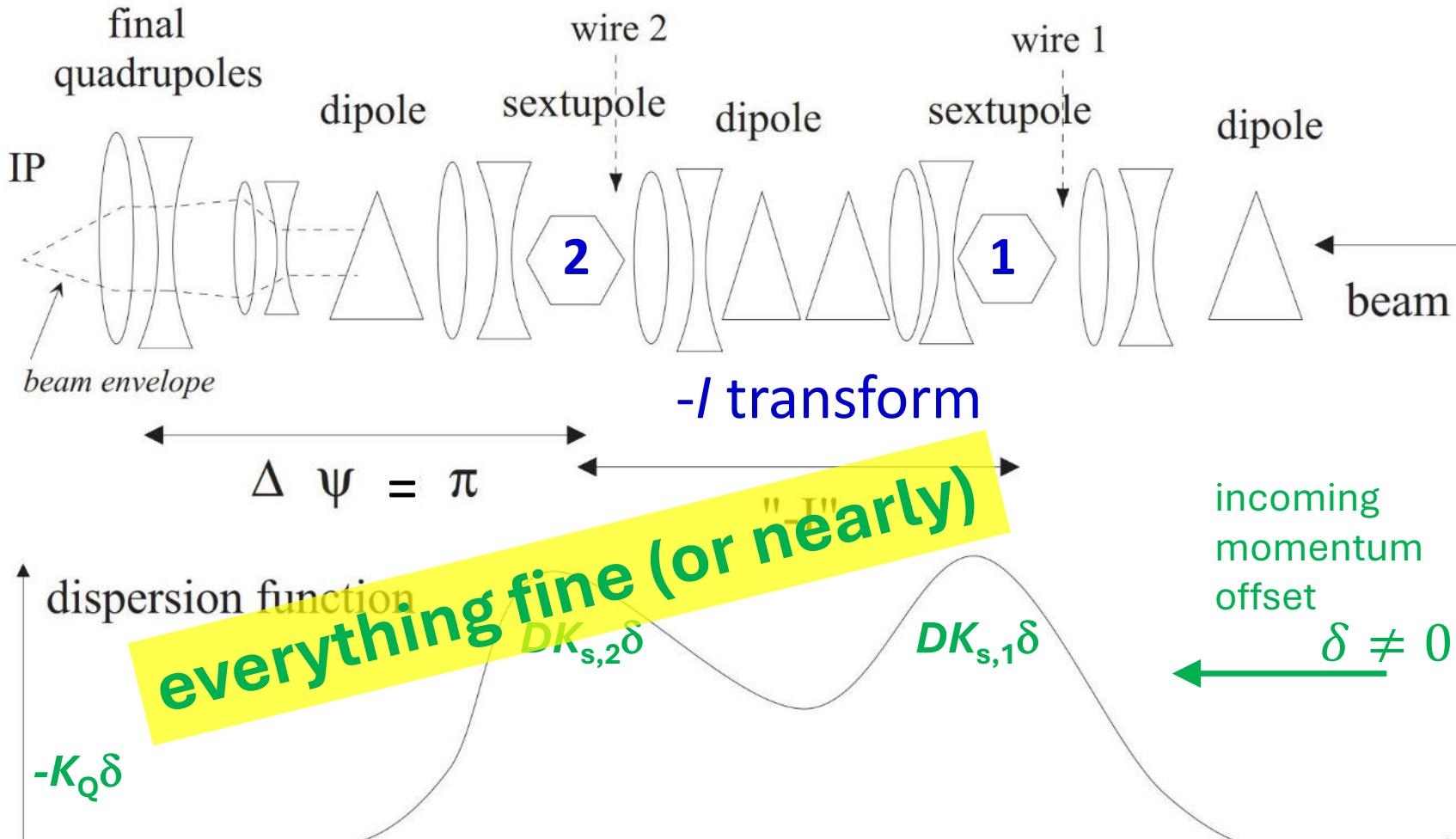


IP

final doublet

dipoles,  
quadrupoles,  
sextupoles, ...

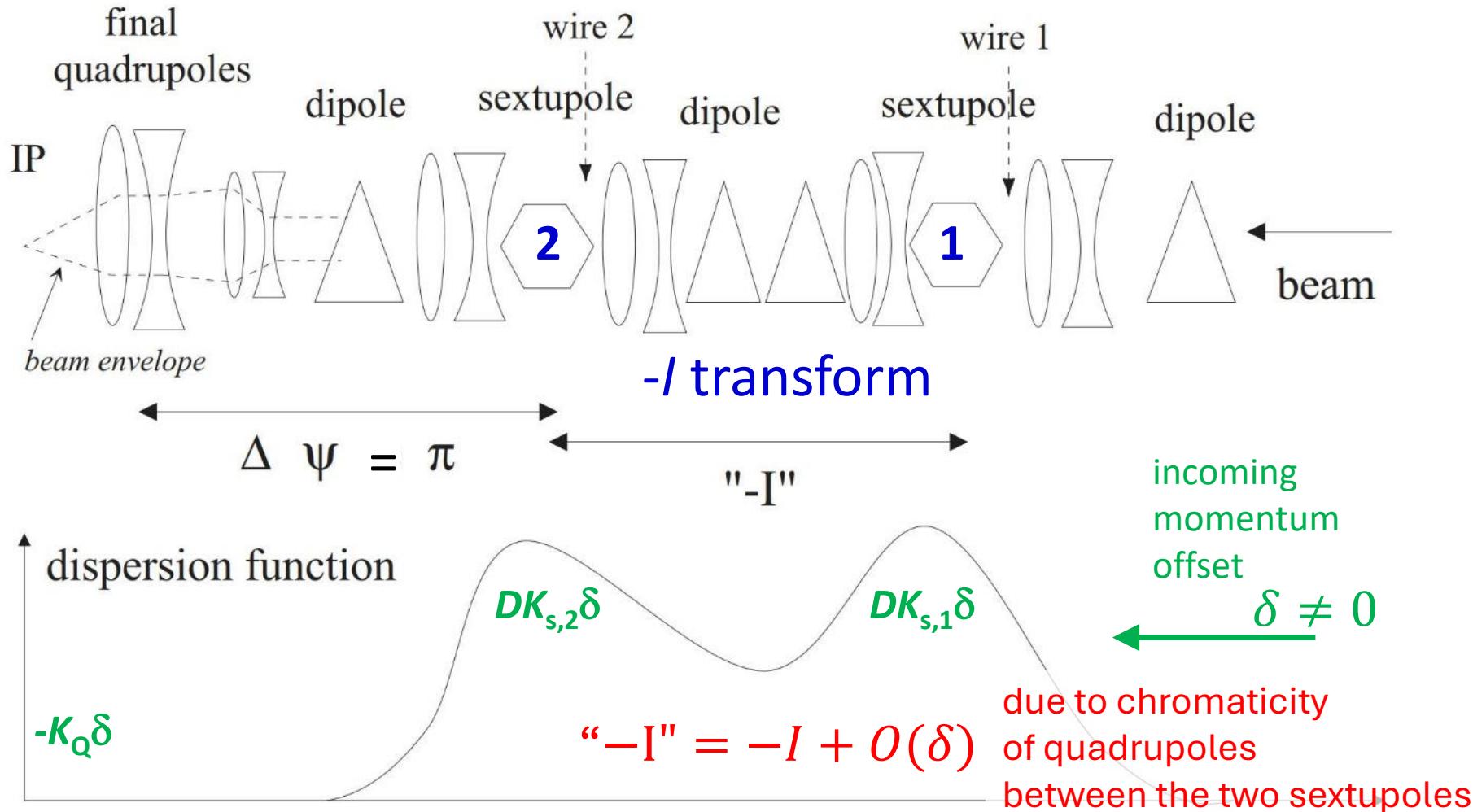
# chromatic correction



$$x'_2 = -x'_\beta - \frac{1}{2} K_{s,1} (x_\beta^2 + 2x_\beta D_i \delta + D_i^2 \delta^2) + \frac{1}{2} K_{s,2} (x_\beta^2 - 2x_\beta D_i \delta + D_i^2 \delta^2)$$

$$K_{s,1} = K_{s,2}: \quad x'_2 = -x'_\beta - 2K_s x_\beta D_i \delta.$$

# one problem: chromatic breakdown of -I



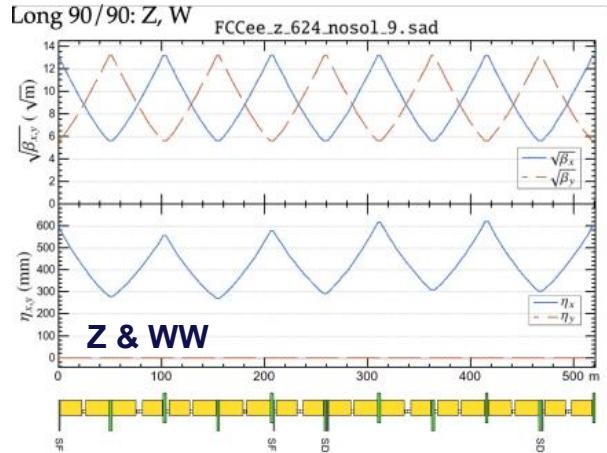
$$x'_2 = -x'_\beta - \frac{1}{2} K_{s,1} (x_\beta^2 + 2x_\beta D_i \delta + D_i^2 \delta^2) + \frac{1}{2} K_{s,2} (x_\beta^2 - 2x_\beta D_i \delta + D_i^2 \delta^2)$$

$$K_{s,1} = K_{s,2}: \quad x'_2 = -x'_\beta - 2K_s x_\beta D_i \delta + c_1 x' \delta + c_2 x^3 \delta^2 + \dots$$

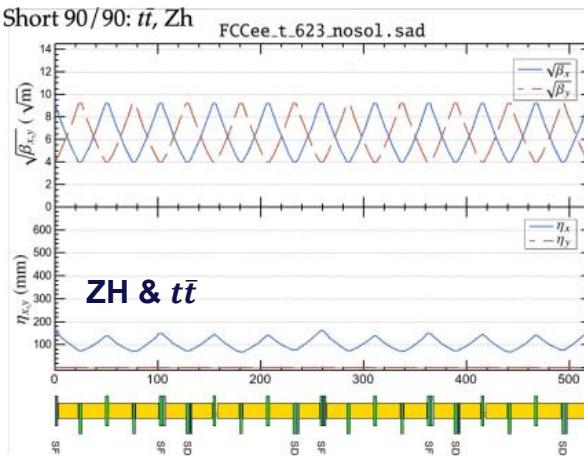
# 2.3 full ring optics

## FCC-ee optics baseline GHC

arc

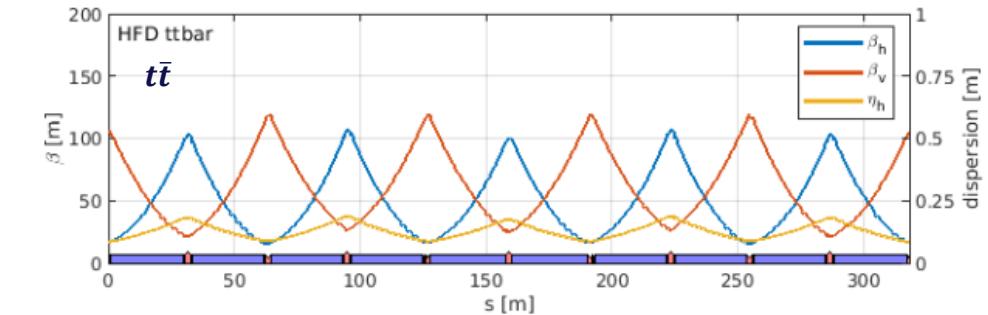


K. Oide et al.

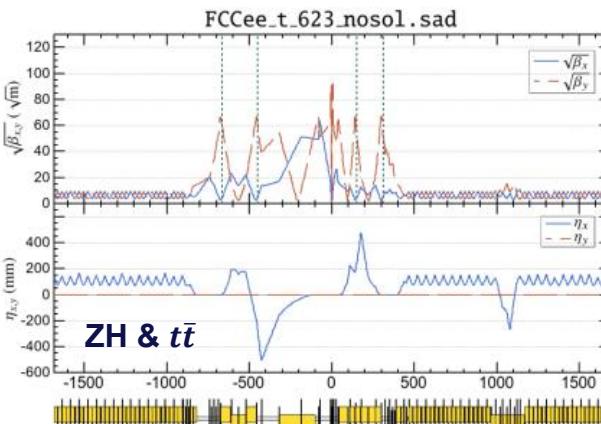
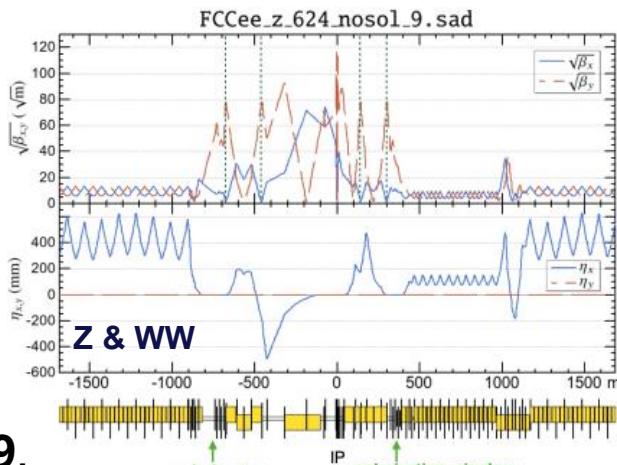


## alternative LCC

P. Raimondi, S. Liuzzo, et al.

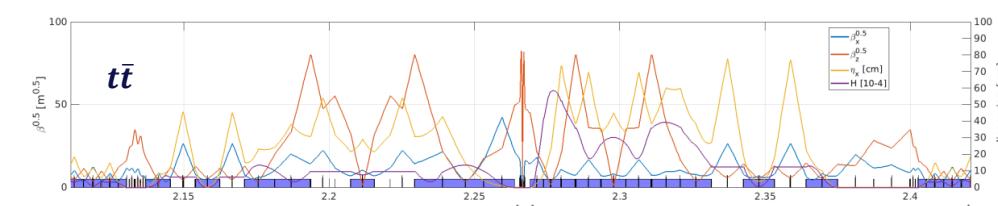


experimental straight



PRAB 19,  
111005 (2016)

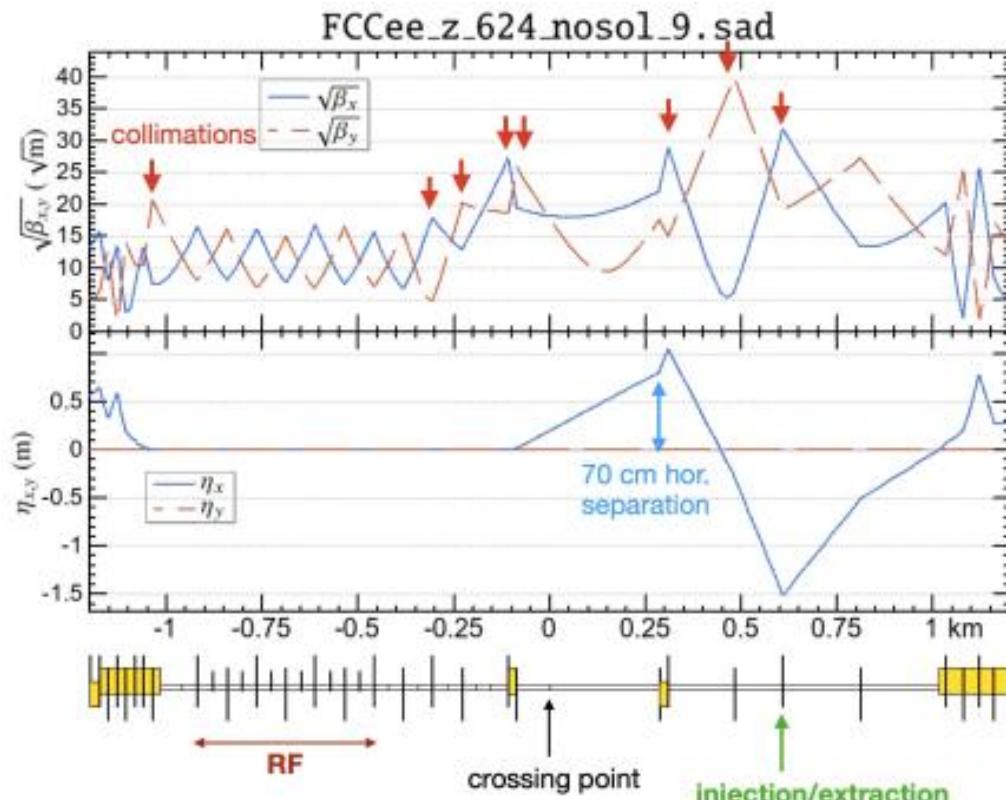
→ March 2026: Decision on Optics



PRAB 28,  
021002 (2025)

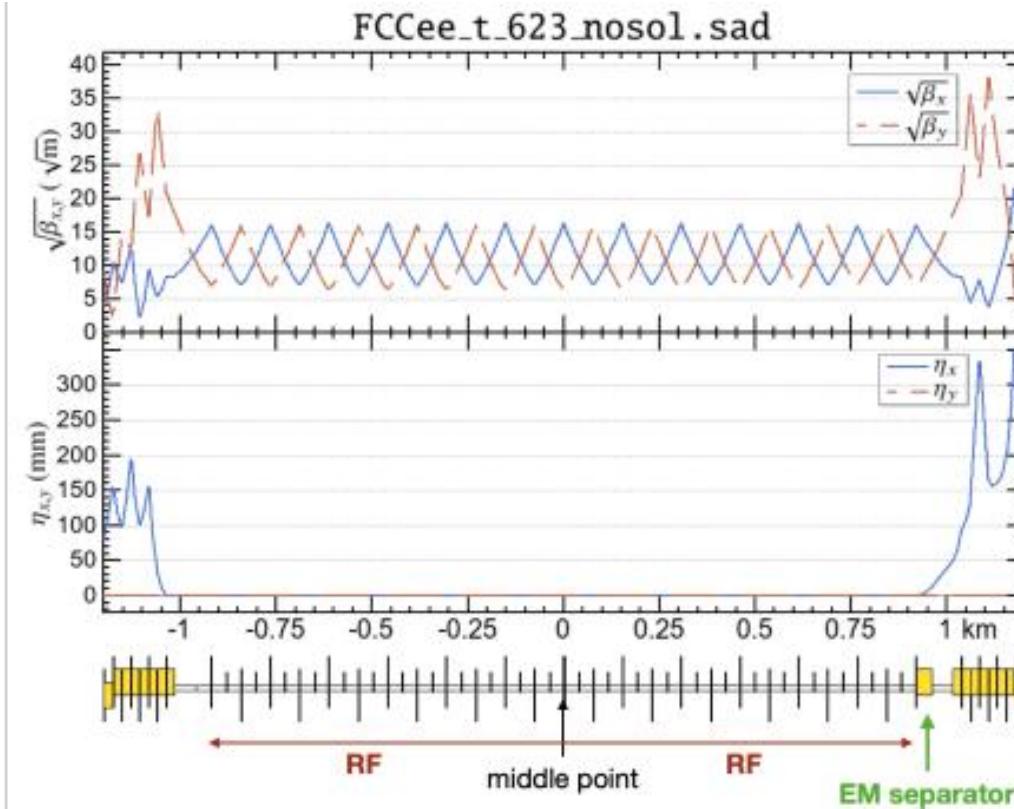
# GHC baseline optics: technical straights

**Universal optics** for all technical insertions in Z/W operation: RF, injection/extraction, & collimation



Maintaining perfect superperiodicity

Optics for the common RF section in ZH/ttbar operation

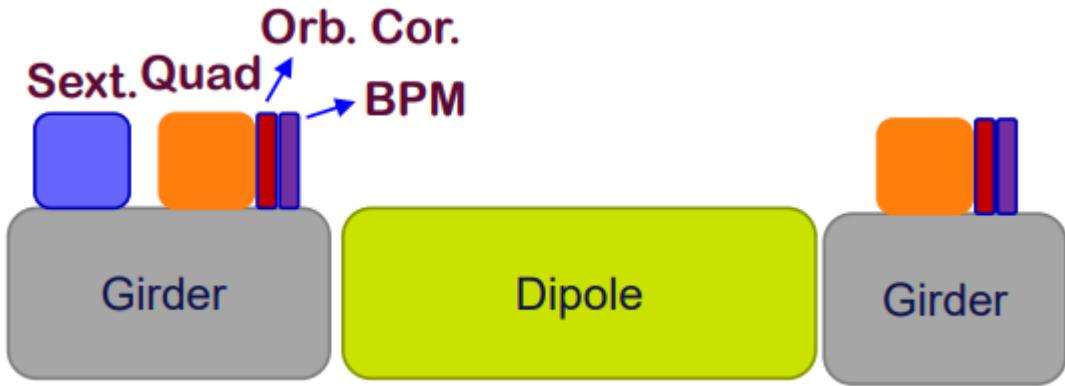


# FCC design with Xsuite

<https://xsuite.readthedocs.io/en/latest/>



## 2.4 errors etc.



Arc alignment and strength tolerances. These values correspond to one sigma Gaussian distribution truncated at 2.5 sigma. Quadrupoles and sextupoles are placed on top of common girders.

Element	$\sigma_{x/y} [\mu\text{m}]$	$\sigma_{\theta/\psi/\phi} [\mu\text{rad}]$	$\Delta k/k [10^{-4}]$
Arc quads & sext.	50	50	2
Dipoles	1000	1000	2
Girders	150	150	-
BPMs-to-quad	100	-	-

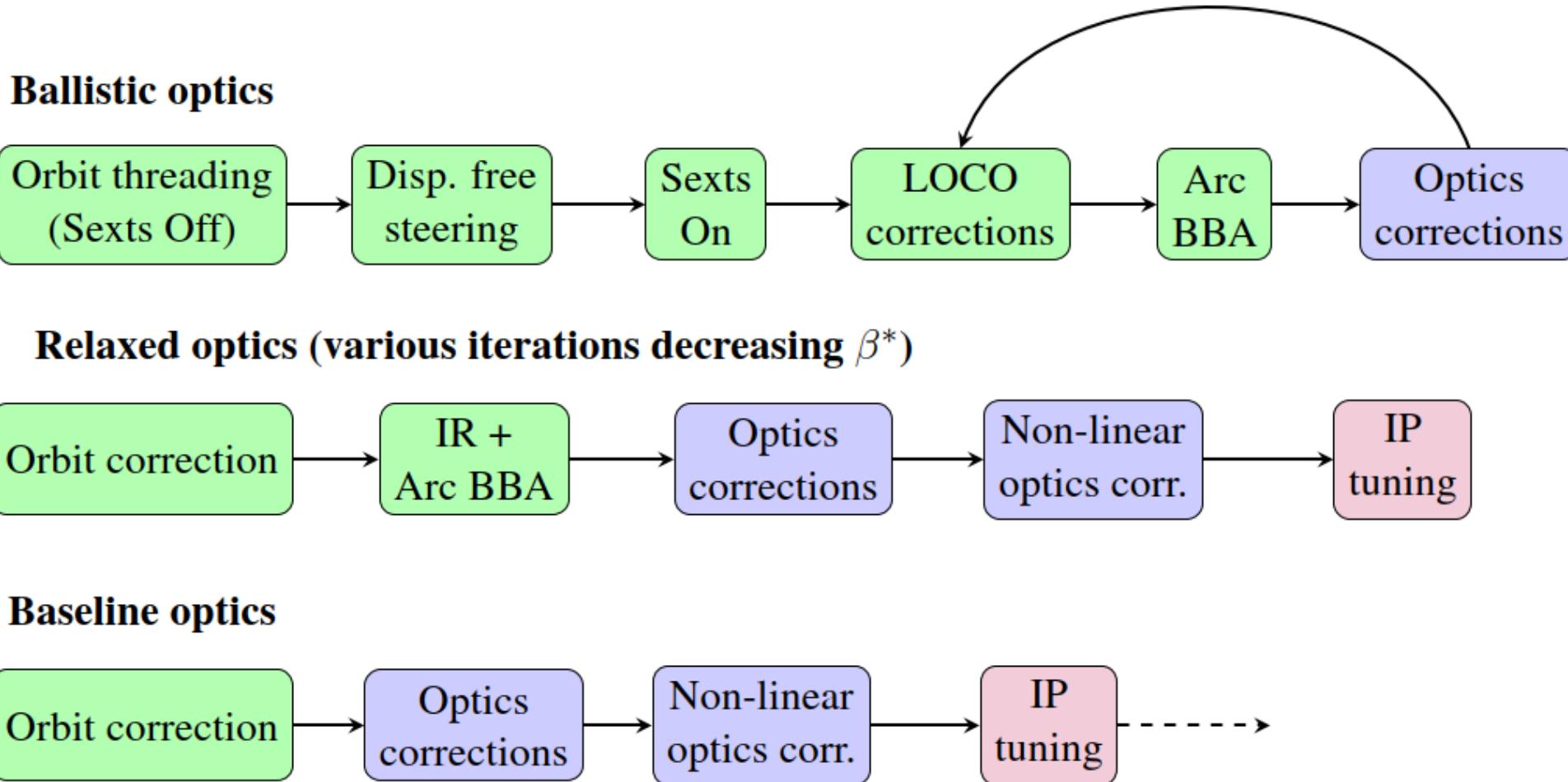
Arc BPM performance specifications.

Closed orbit resolution	$0.1 \mu\text{m}$
Turn-by-turn (TbT) position resolution	$1 \mu\text{m}$
Number of turns in TbT mode	50000

## Location of arc magnet correctors and BPMs.

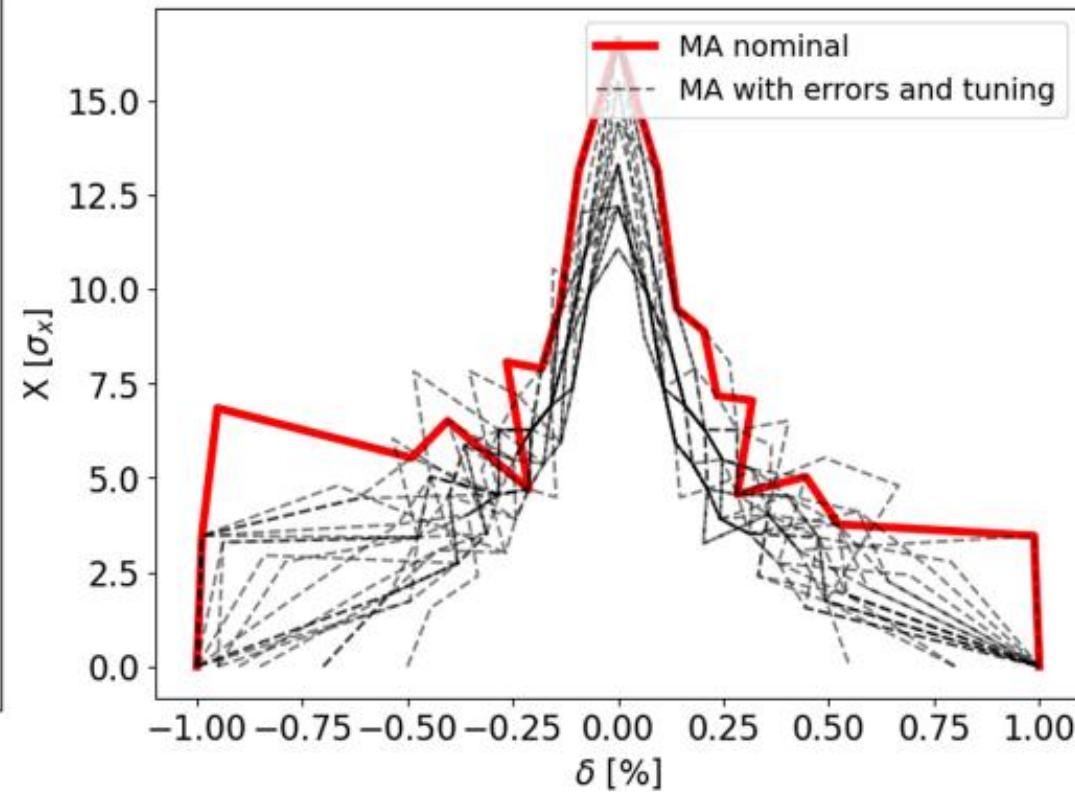
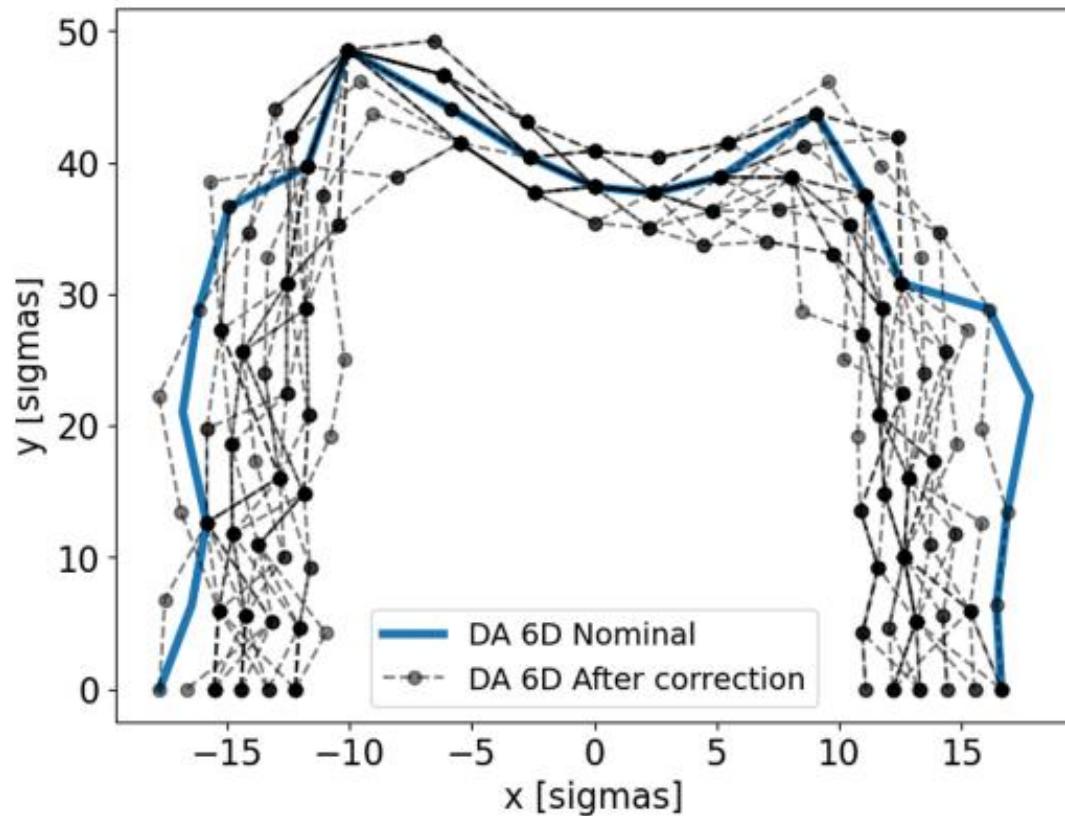
<b>Device</b>	<b>Location</b>
Horizontal orbit corrector	Embedded at the edge of main dipole next to main quadrupole
Vertical orbit corrector	Embedded at the sextupole or stand-alone
Quadrupolar corrector	Trim coil in all main quadrupoles
Skew quadrupole	Embedded at the sextupole
BPM (H & V)	Attached to the main quadrupole

# Optics commissioning sequence



Steps during the FCC-ee optics commissioning starting with the most relaxed optics, the ballistic optics.

# Simulated performance of baseline optics with errors

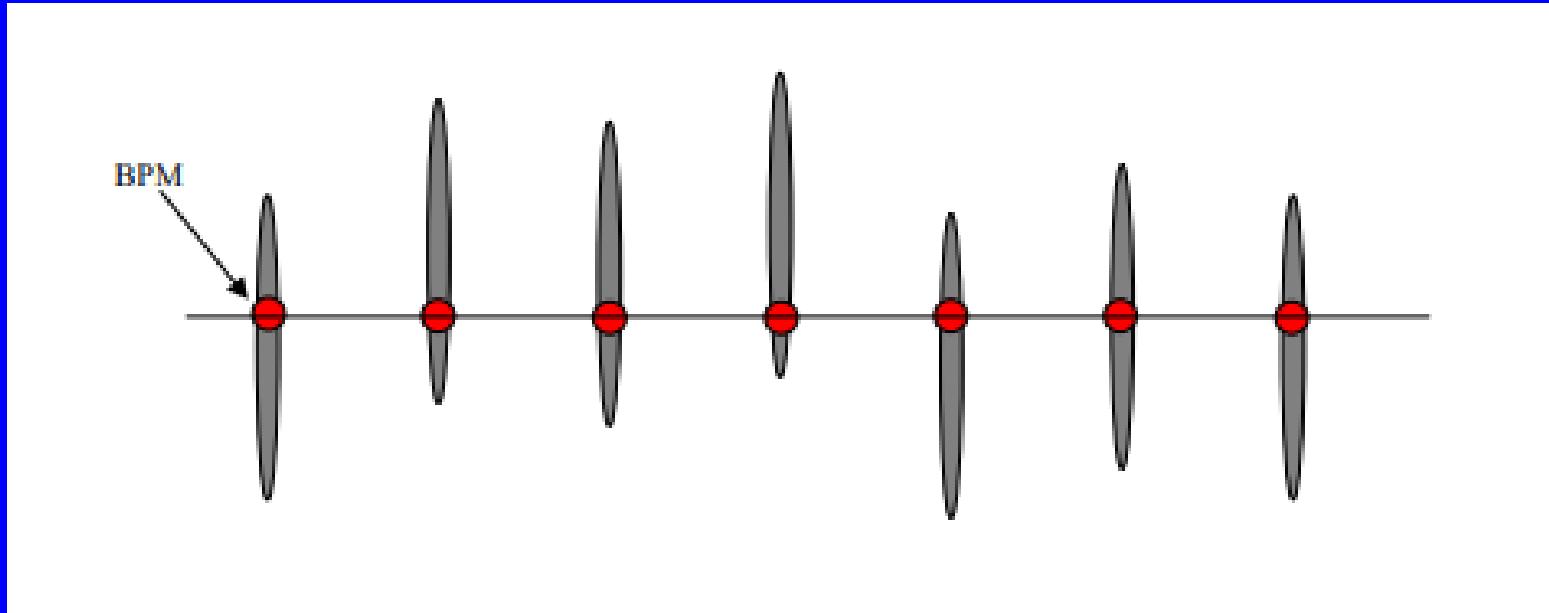


Dynamic Aperture (left) and Momentum Acceptance (right) after linear optics corrections having used the errors described before.

Median rms values of several optics parameters before (after sextupole ramping) and after linear optics correction (nominal lattice).  $\Delta\dot{\psi}$  stands for phase advance deviations between nearby BPMs.

Parameter	Before correction (rms)	After correction (rms)
horizontal orbit (μm)	120.2	120.5
vertical orbit (μm)	217.5	217.6
$\Delta\beta_x/\beta_x$ (%)	7.41	0.29
$\Delta\beta_y/\beta_y$ (%)	15.79	2.81
$\Delta D_x$ (mm)	57.79	0.28
$\Delta D_y$ (mm)	62.24	2.80
$\varepsilon_h$ (nm)	0.72	0.71
$\varepsilon_v$ (pm)	26.01	0.57
horiz. $\Delta\psi$ [2π]	$1.1 \times 10^{-2}$	$2.9 \times 10^{-4}$
vert. $\Delta\psi$ [2π]	$1.9 \times 10^{-2}$	$2.3 \times 10^{-3}$
$\text{Re } f_{1001}$	$4.9 \times 10^{-2}$	$1.7 \times 10^{-4}$
$\text{Im } f_{1001}$	$4.4 \times 10^{-2}$	$5.2 \times 10^{-5}$
$\text{Re } f_{1010}$	$3.7 \times 10^{-2}$	$1.3 \times 10^{-4}$
$\text{Im } f_{1010}$	$3.7 \times 10^{-2}$	$1.3 \times 10^{-4}$

# Scenario 1: Quad offsets, but BPMs aligned



Assuming:

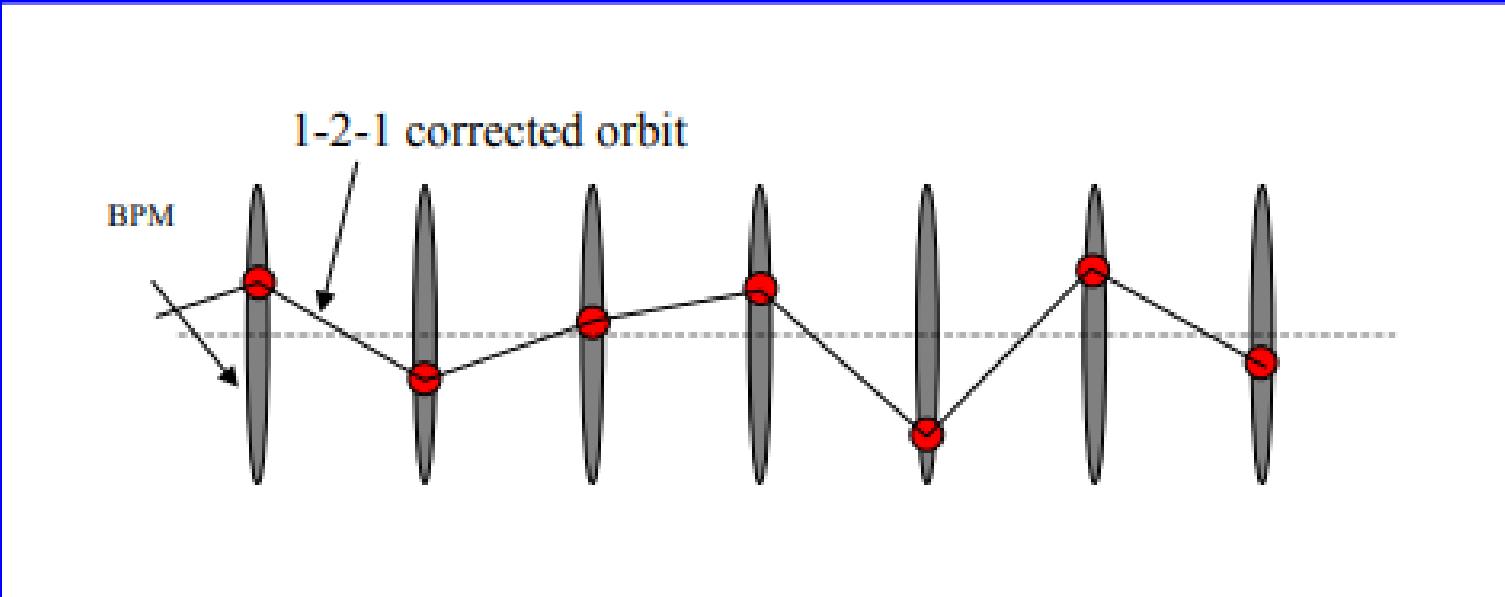
- a BPM adjacent to each quad
- a 'steerer' at each quad

simply apply one to one steering to orbit

steerer { quad mover  
dipole corrector

N. Walker,  
USPAS 2003,  
Santa Barbara

## Scenario 2: Quads aligned, BPMs offset



one-to-one correction BAD!

Resulting orbit not Dispersion Free  $\Rightarrow$  emittance growth

Need to find a steering algorithm which effectively puts  
BPMs on (some) reference line    **beam-based alignment**  
**followed by dispersion-free steering**

real world scenario: some mix of scenarios 1 and 2

# Beam-based alignment (BBA)

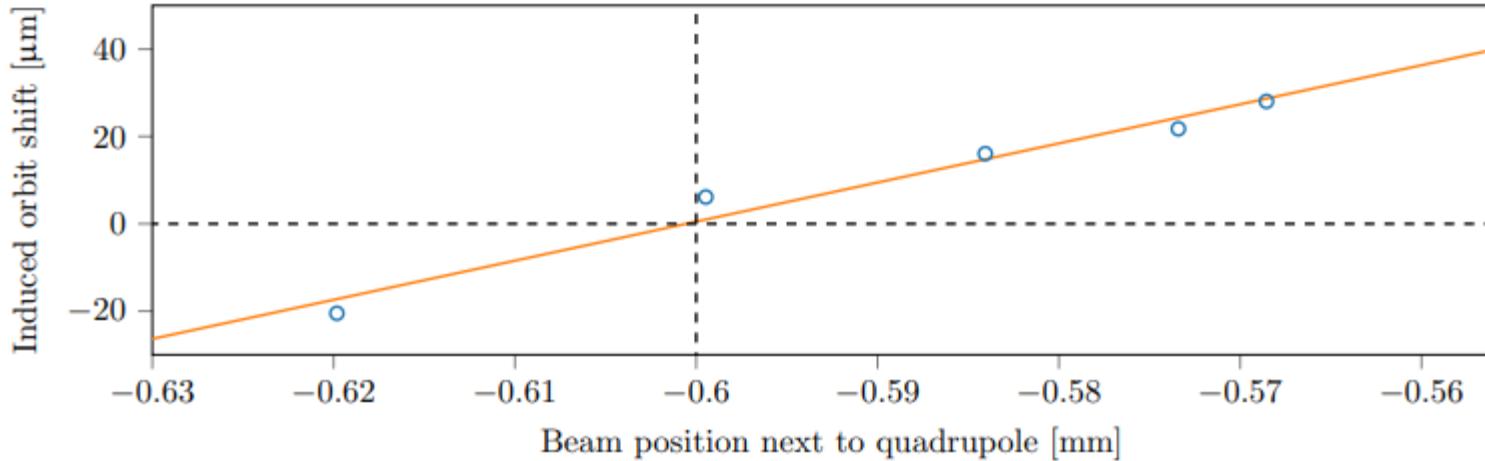
BBA can be performed after the sextupoles have been switched on

sheer size of the FCC → parallel approaches are explored, aimed at achieving approximately 10 to 20  $\mu\text{m}$  effective alignment after BBA

Using Parallel Quadrupole Modulation System (PQMS), a technique which has already successfully been tested at SLAC/SPEAR, is applied to the FCC-ee.

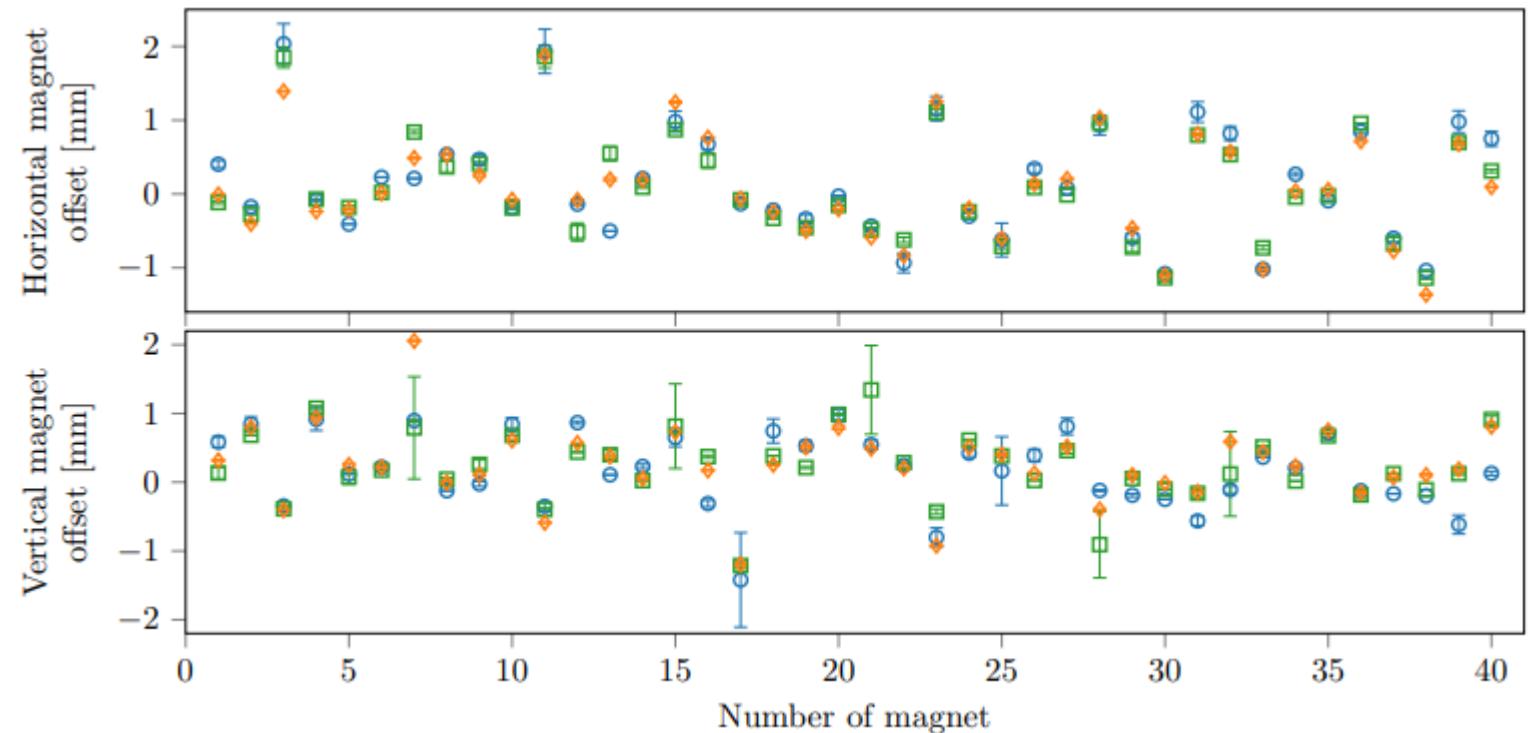
Modulating 10 quadrupoles in parallel with a  $\Delta K/K$  of 2%, distributed equally over one arc, and using a calibrated lattice with 1  $\mu\text{m}$  BPM resolution, an accuracy below 20  $\mu\text{m}$  for vertical and horizontal arc quadrupole BBA is expected.

# BBA tests for FCC-ee at KIT/KARA in Karlsruhe



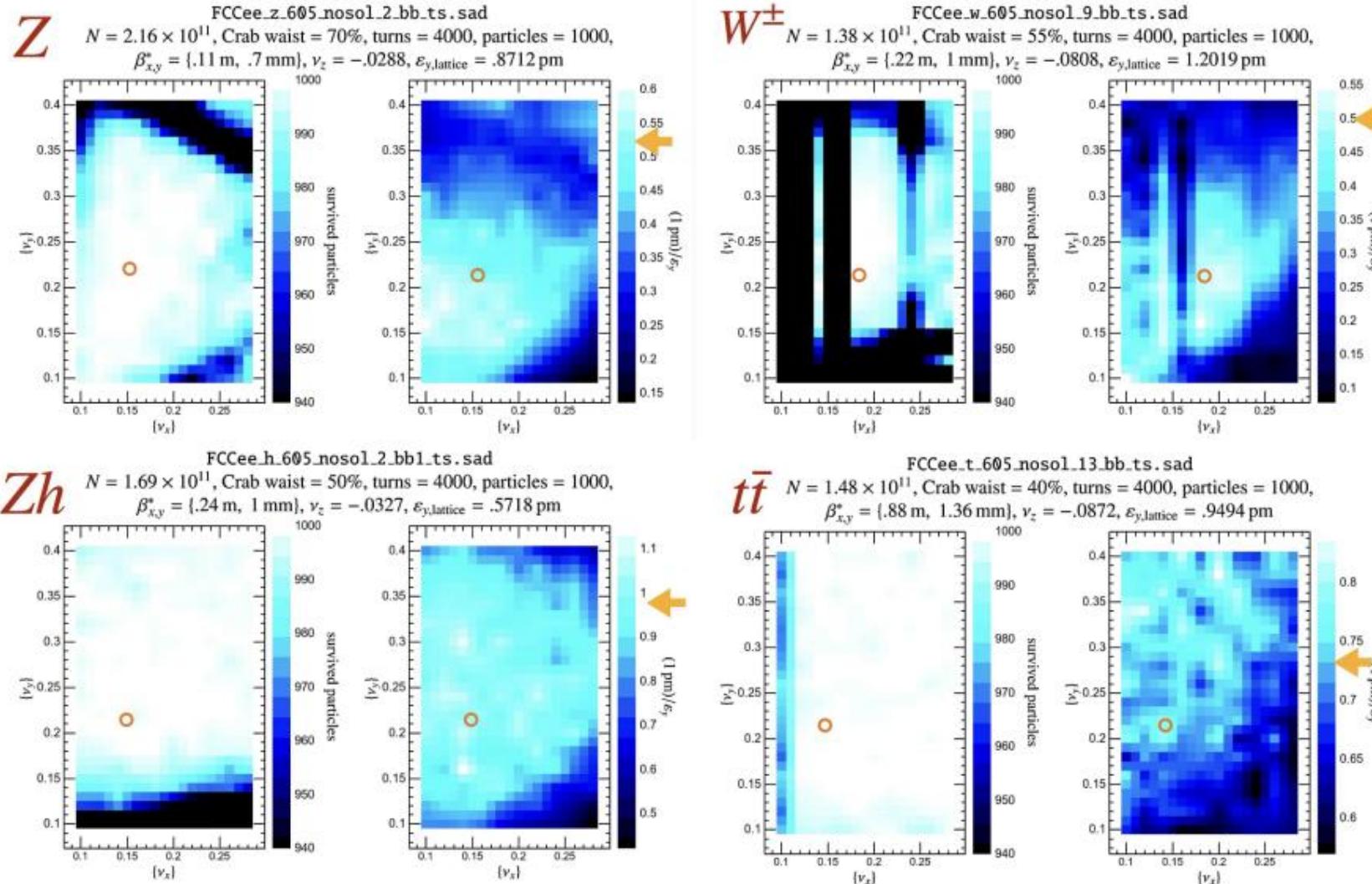
Dependence of measured induced orbit shift on beam position close to quadrupole.

Estimated magnet offsets with respect to closest BPM using linear (blue), rms (orange) and parallel BBA (green).



# 2.5 beam-beam performance

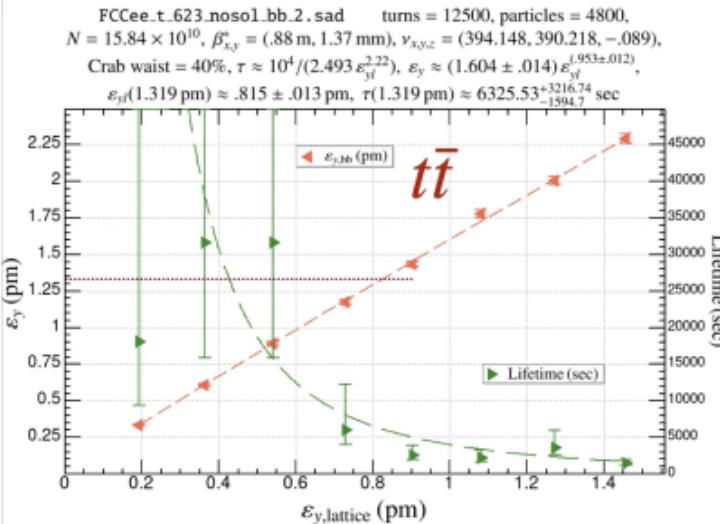
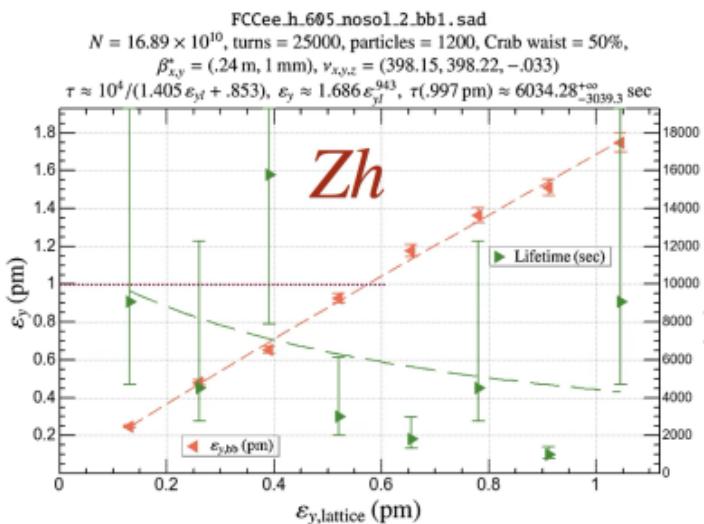
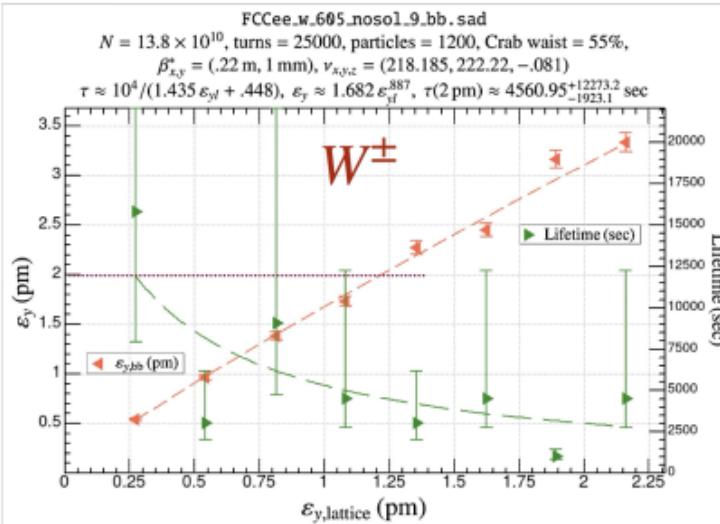
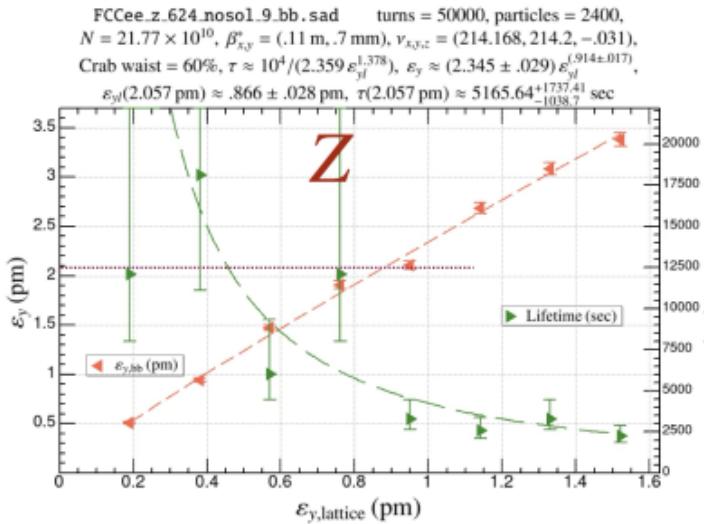
## GHC optics



tune scan of the beam-beam effect with the full lattice

K. Oide

# vertical bare lattice emittance required with beam-beam



results of beam-beam tracking with lattice and beamstrahlung for each energy

K. Oide

# Further Reading

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