

Lectures on the Future Circular Collider – Part 1+2

An aerial photograph of a rugged, mountainous landscape with a large, circular collider ring overlaid. The ring is depicted with a dashed white line and a solid teal line, with several teal dots marking specific points along its circumference. A winding road or river is visible within the ring's area. The background shows steep, rocky slopes and a few small, snow-filled depressions.

Frank Zimmermann
Bonn, 1-2 October 2025

Lecture 1 – FCC-ee design concepts

1.0 colliders – revisited

1.1 luminosity – revisited

1.2 synchrotron radiation

1.3 beam-beam effects

1.4 luminosity revised

1.5 FCC concept(s)

1.0 colliders

why high(er) energy?

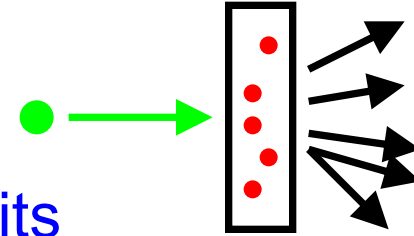
- quantum mechanics: de Broglie wavelength $\lambda = h/p$
→ examining matter at smaller distance requires higher momentum particles
- many of the particles of interest to particle physics are heavy
→ high-energy collisions are needed to create these particles

colliding beams

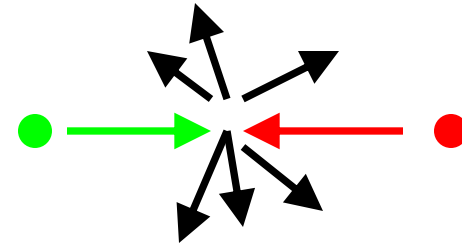
centre-of-mass energy:

$$E_{\text{c.m.}} = \sqrt{2 E_{\text{beam}} M_{\text{target}} c^2}$$

beam hits
a “fixed target”

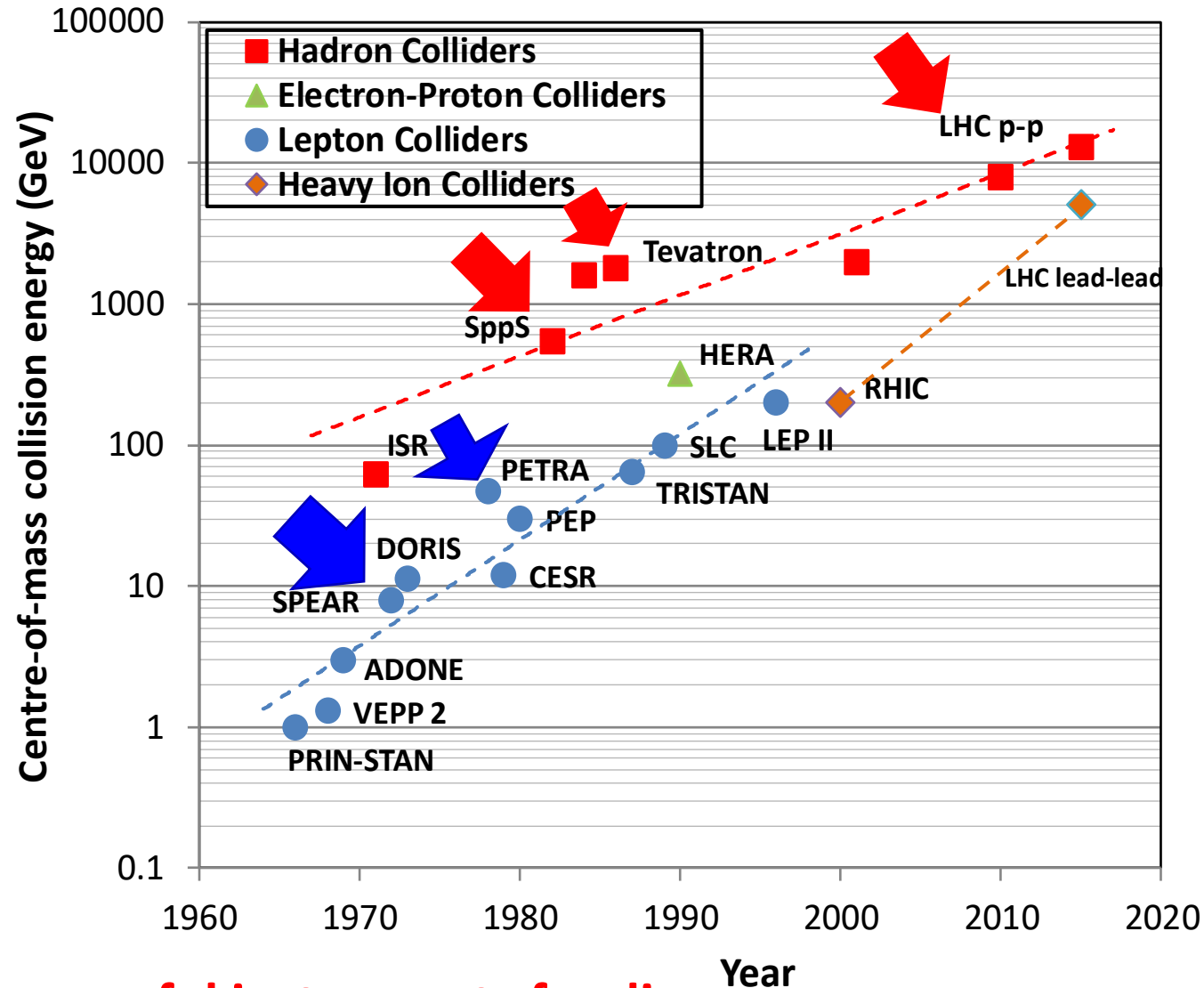


$$E_{\text{c.m.}} = 2 E_{\text{beam}} \quad \text{two beams collide}$$



colliding two beams against each other can provide
much higher centre-of-mass energies than fixed target!

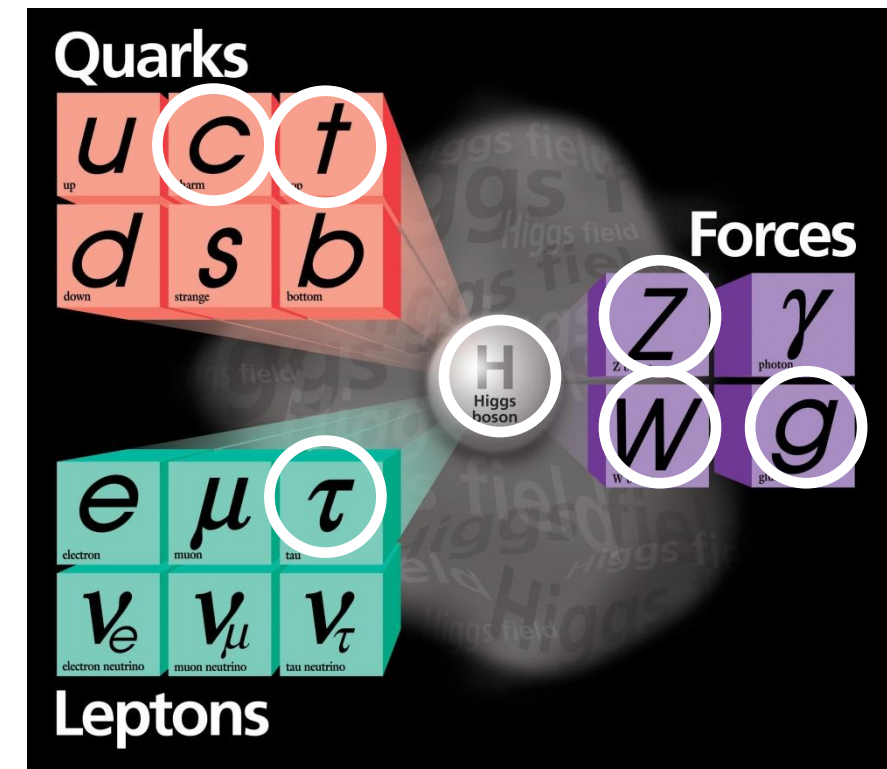
colliders and discoveries



powerful instruments for discovery
and precision measurement

Standard Model Particles and forces

A. Ballarino



**“An e^+e^- storage ring in the range of a few hundred GeV
in the centre of mass can be built with present
technology...” “...the most useful project on the horizon.”**



B. Richter, 1976

LEP/LEP2: highest energy so far

circumference 27 km

in operation from 1989 to 2000

maximum c.m. energy 209 GeV

maximum synchrotron radiation power 23 MW



FCC-ee physics requirements

- **beam energy range from 35 GeV to ≈ 200 GeV**

- **highest possible luminosities** at all working points

- physics programs / energies:

Z (45.5 GeV) Z pole, 'TeraZ' and high precision M_Z & Γ_Z

W (80 GeV) W pair production threshold, high precision M_W

H (120 GeV) ZH production (maximum rate of H's)

t (175 GeV): $t\bar{t}$ threshold, H studies

more (α_{QED} etc.)

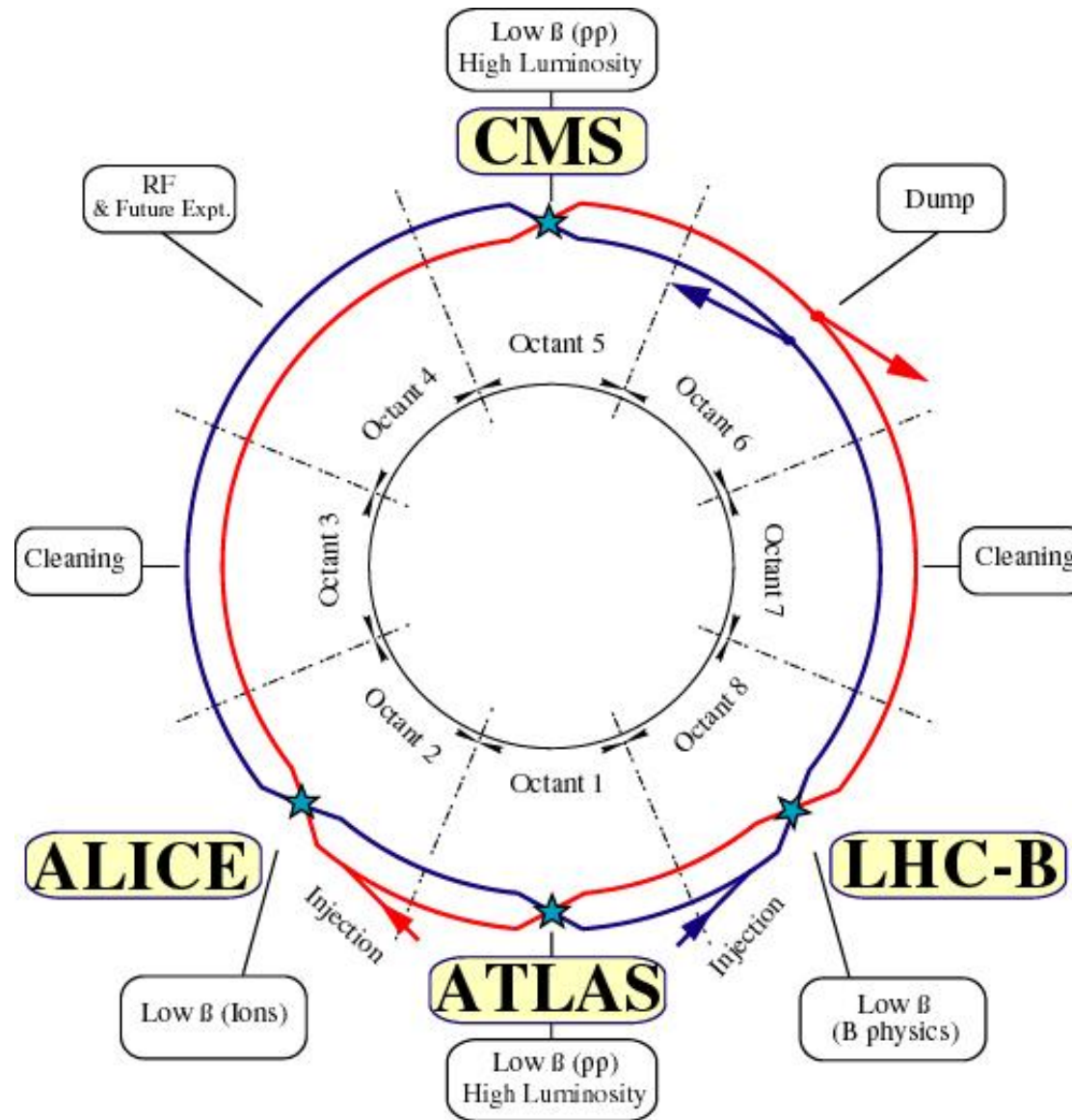
- possibly H (63 GeV) direct s-channel production w.

monochromatization

- **some polarization up to ≥ 80 GeV** for beam energy calibration

present flagship: Large Hadron Collider (LHC)

world's highest
energy p-p collider
at CERN/Geneva



installed in the
LEP tunnel ! –
circumference
27 km

*running
extremely well*

4 July, 13 years ago

LHC has already produced
> 30 million Higgs bosons

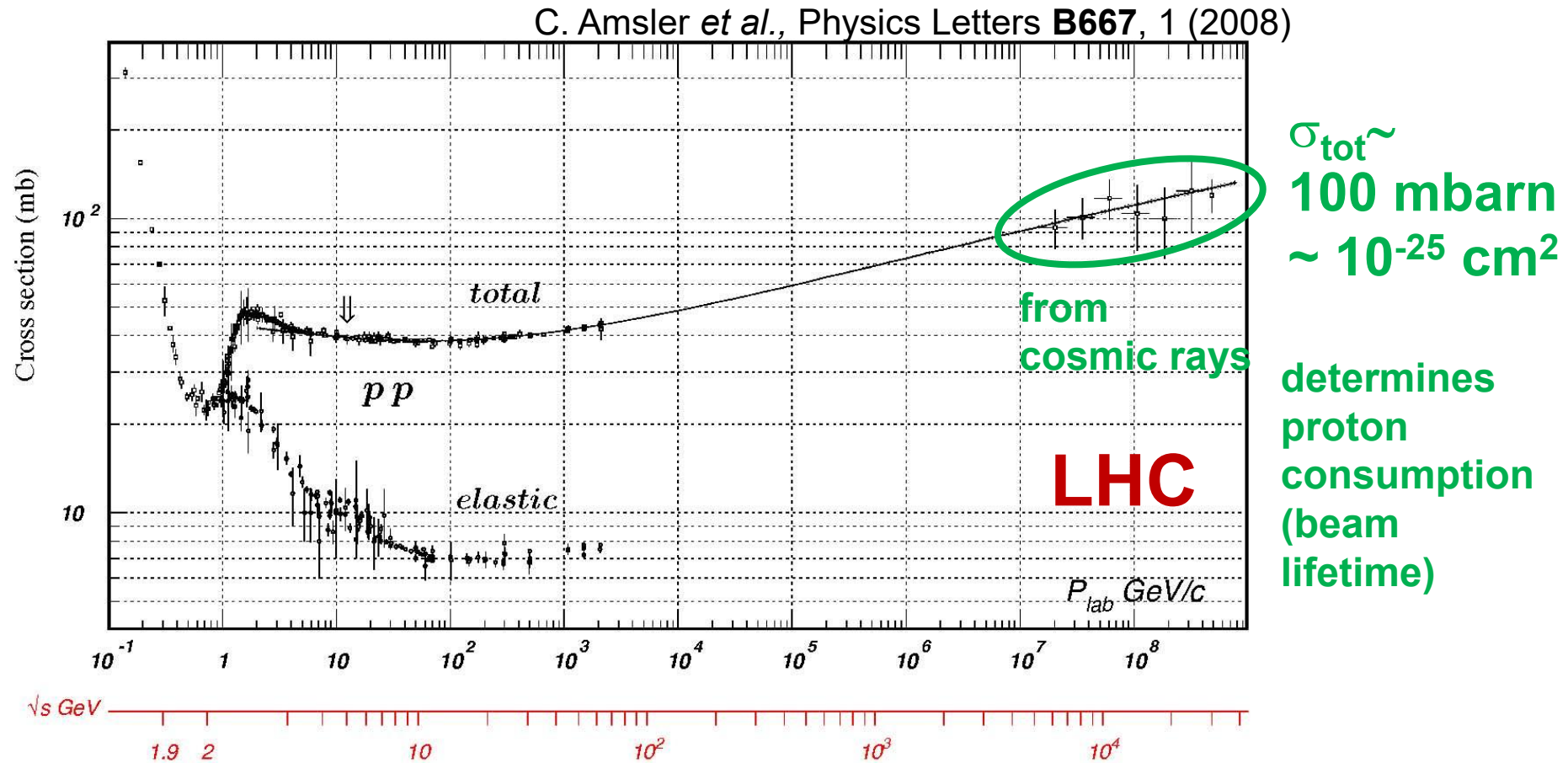


C. Grojean, SFP2025 Troyes

1.1 luminosity

$$R = \sigma L$$

reaction rate cross section luminosity

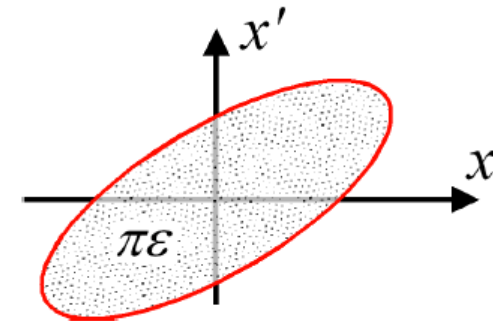


$\sigma_{\text{Higgs,p-p}} \sim 30 \text{ pb} \rightarrow \text{with } 300 \text{ fb}^{-1} \text{ LHC produces } \sim 10 \text{ million Higgs}$

luminosity for collision of flat beams

$$L = \frac{n_b N_b^2 f_{\text{rev}}}{4\pi \sigma_x^* \sigma_y^*} F = \frac{n_b N_b^2 f_{\text{rev}}}{4\pi \varepsilon_x \sqrt{\beta_x^* \beta_y^*} \kappa} F$$

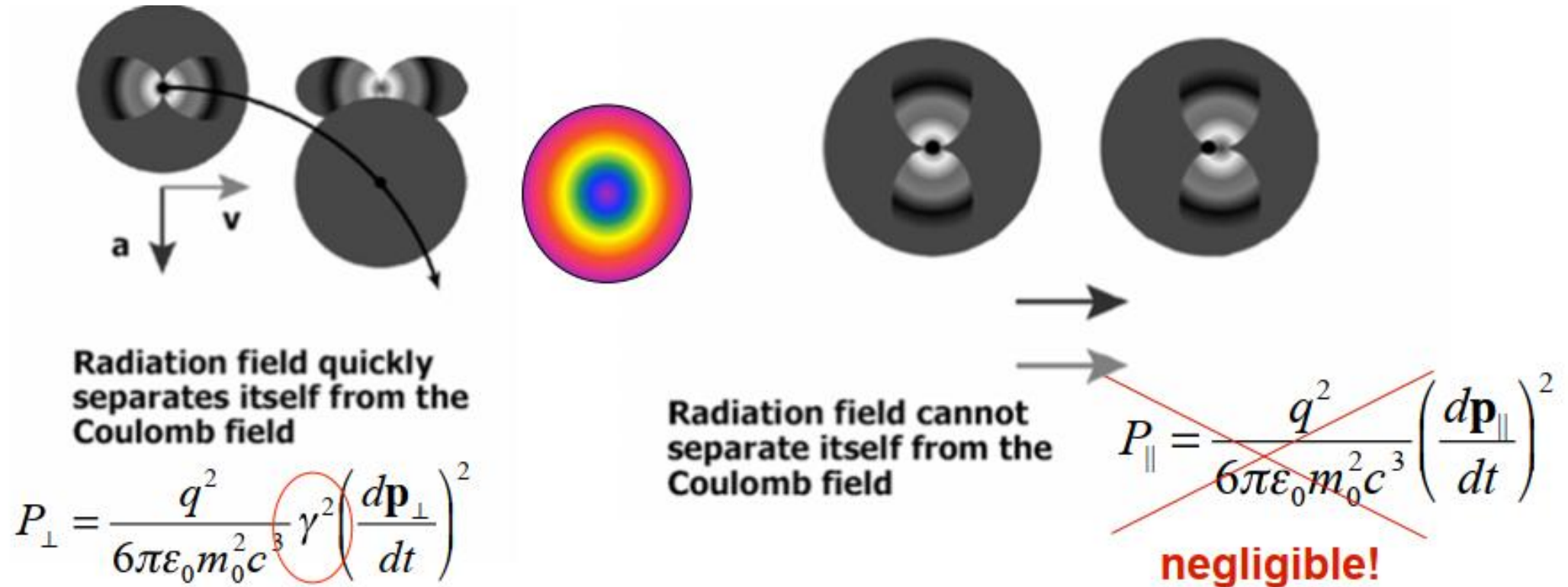
N_b	number of particles per bunch
n_b	number of bunches per beam
f_{rev}	revolution frequency
$\sigma_{x,y}^*$	hor./vert. beam size at interaction point
F	reduction factor due to crossing angle and hourglass effect
ε_x	hor. emittance (from optics)
κ	emittance coupling $\kappa = \varepsilon_y / \varepsilon_x$
β^*	hor./vert, beta function at IP



$$\sigma^* = \sqrt{\beta^* \varepsilon}$$

1.2 Synchrotron Radiation (SR)

transverse and longitudinal acceleration

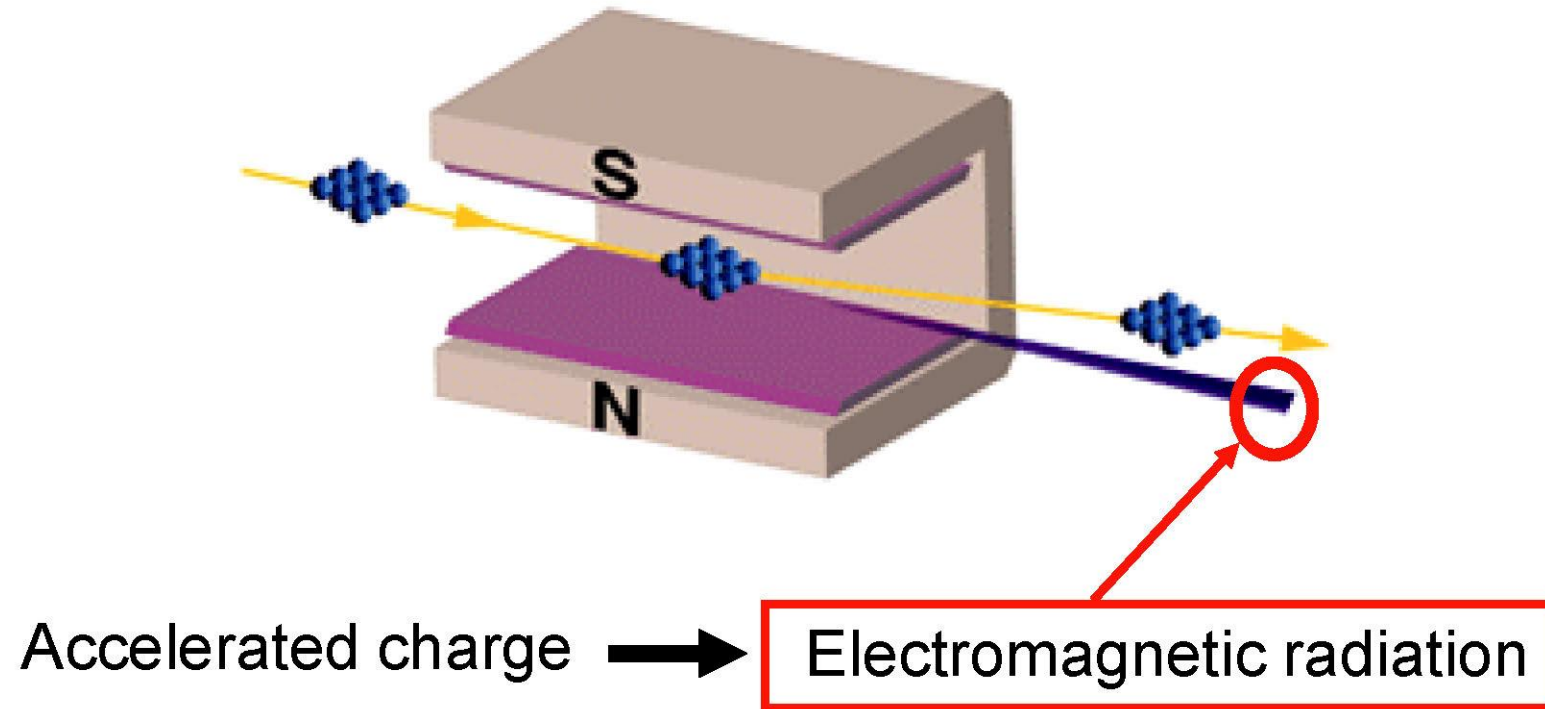


$$P_{\perp} = \frac{c}{6\pi\epsilon_0} q^2 \frac{(\beta\gamma)^4}{\rho^2} \quad \rho = \text{curvature radius}$$

W. Barletta, USPAS

excellent news for high-gradient acceleration !

curved orbit of e^- in magnetic field

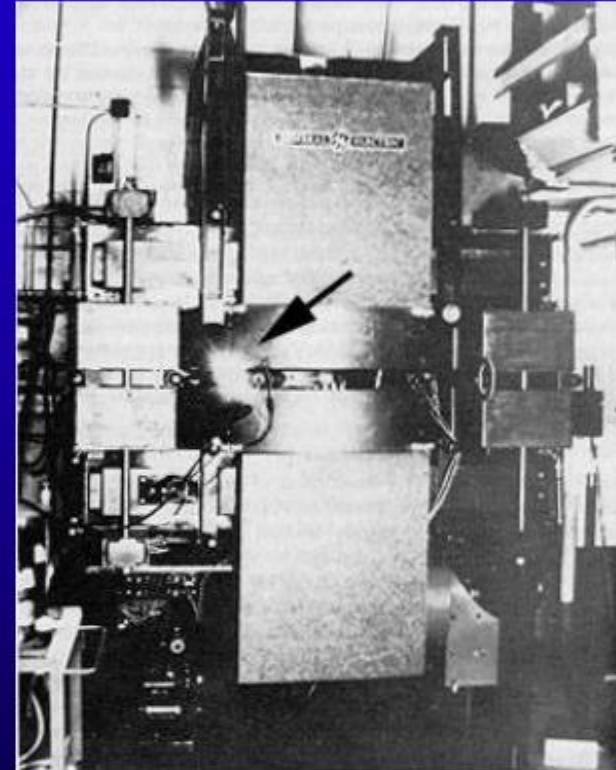


**Crab Nebula
6000 light years away**



**First light observed
1054 AD**

**GE Synchrotron
New York State**



**First light observed
1947**

classical theory of electromagnetic radiation

Liénard-Wiechert potentials

$$\varphi(t) = \frac{1}{4\pi\epsilon_0} \frac{q}{\left[r(1 - \vec{n} \cdot \vec{\beta}) \right]_{ret}}$$

$$\vec{A}(t) = \frac{q}{4\pi\epsilon_0 c^2} \left[\frac{\vec{v}}{r(1 - \vec{n} \cdot \vec{\beta})} \right]_{ret}$$

Lorentz gauge

$$\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \frac{\partial \varphi}{\partial t} = 0$$

retarded time defined by implicit equation

$$t_{ret} = t - \frac{1}{v} |\vec{r} - \vec{r}_s(t_{ret})|$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$



$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}$$

energy loss for a particle in a ring

Rate of energy loss (GeV/s) for single particle:

$$P_\gamma = \frac{e^2 c^3}{2\pi} C_\gamma E^2 B^2 = \frac{c C_\gamma E^4}{2\pi \rho^2}$$

with $C_\gamma = \frac{4\pi}{3} \frac{r_e}{(mc^2)^3} \approx 8.85 \times 10^{-5} \text{ m GeV}^{-3}$

$\rho = \frac{p}{eB}$ bending radius ;

E particle energy ; m particle mass ;

r_e classical particle radius ($2.8 \times 10^{-15} \text{ m}$ for electrons)

Energy lost in a ring over one turn (GeV):

$$U_0 = \frac{C_\gamma E_0^4}{2\pi} \oint G^2(s) ds \quad \text{with } G \equiv 1/\rho$$

With a constant guide field $G_0 \equiv 1/\rho_0$ along the curved path of the trajectory and zero elsewhere :

Energy
radiated
per turn

$$U_0 = \frac{C_\gamma E_0^4}{\rho_0}$$

Average
power

$$\langle P_\gamma \rangle = \frac{U_0}{T_0} = \frac{c C_\gamma E_0^4}{L \rho_0}$$

$T_0 = L/c$ with L circumference

An electron that is not on the ideal orbit radiates at a different rate. But if magnetic field varies linearly with position, **radiated power averaged over a betatron oscillation cycle is the same as that for an electron on the design orbit.** The same is not true for an electron with an energy different from E_0 (next slide).

For ultra-relativistic electrons the radiation is emitted primarily along the direction of motion. Most of the radiation is emitted within the angle $1/\gamma$. The accompanying momentum change is nearly exactly opposite to the direction of motion. **Then only radiation effect is to decrease e^- energy w/o changing its direction of motion.**

is this an issue for high energy accelerators?

energy loss per particle per turn $U_0 = \frac{C_\gamma E_0^4}{\rho_0} \longrightarrow$ SR power $P_{SR} = \frac{I_{beam}}{e} U_0$

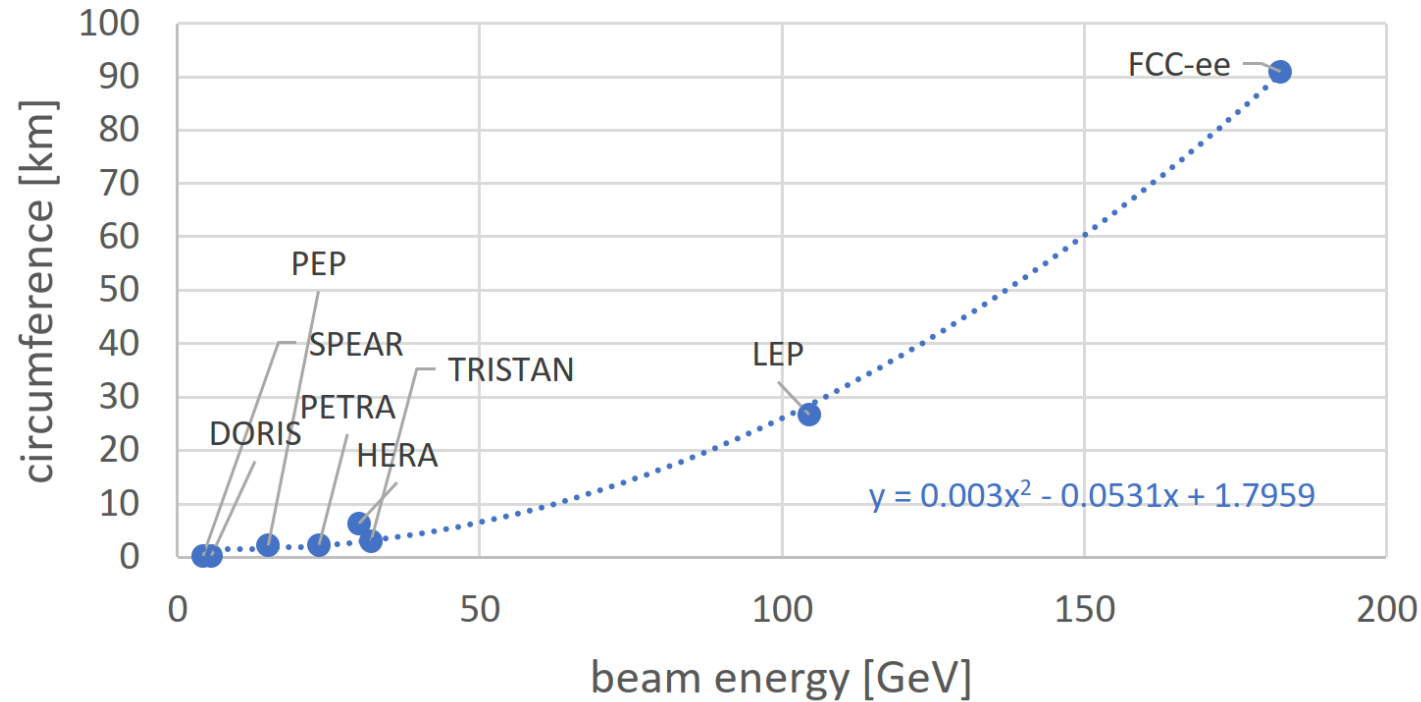
$$C_\gamma = \frac{4\pi}{3} \frac{r_e}{(mc^2)^3} \approx 8.85 \times 10^{-5} \text{ m GeV}^{-3}$$

e[±]: $P_{SR} = 23$ MW for LEP (former e⁺e⁻ collider in the 27 km LHC tunnel),
100 MW for FCC-ee (new ~90 km ring, imposed as design constraint),

protons: $P_{SR} = 0.01$ MW for LHC,
up to 5 MW for FCC-hh (new ~90 km ring, ~10x collision energy of the LHC)
– this may **require >100 MW cryoplant power** (FCC-hh CDR, 2018)

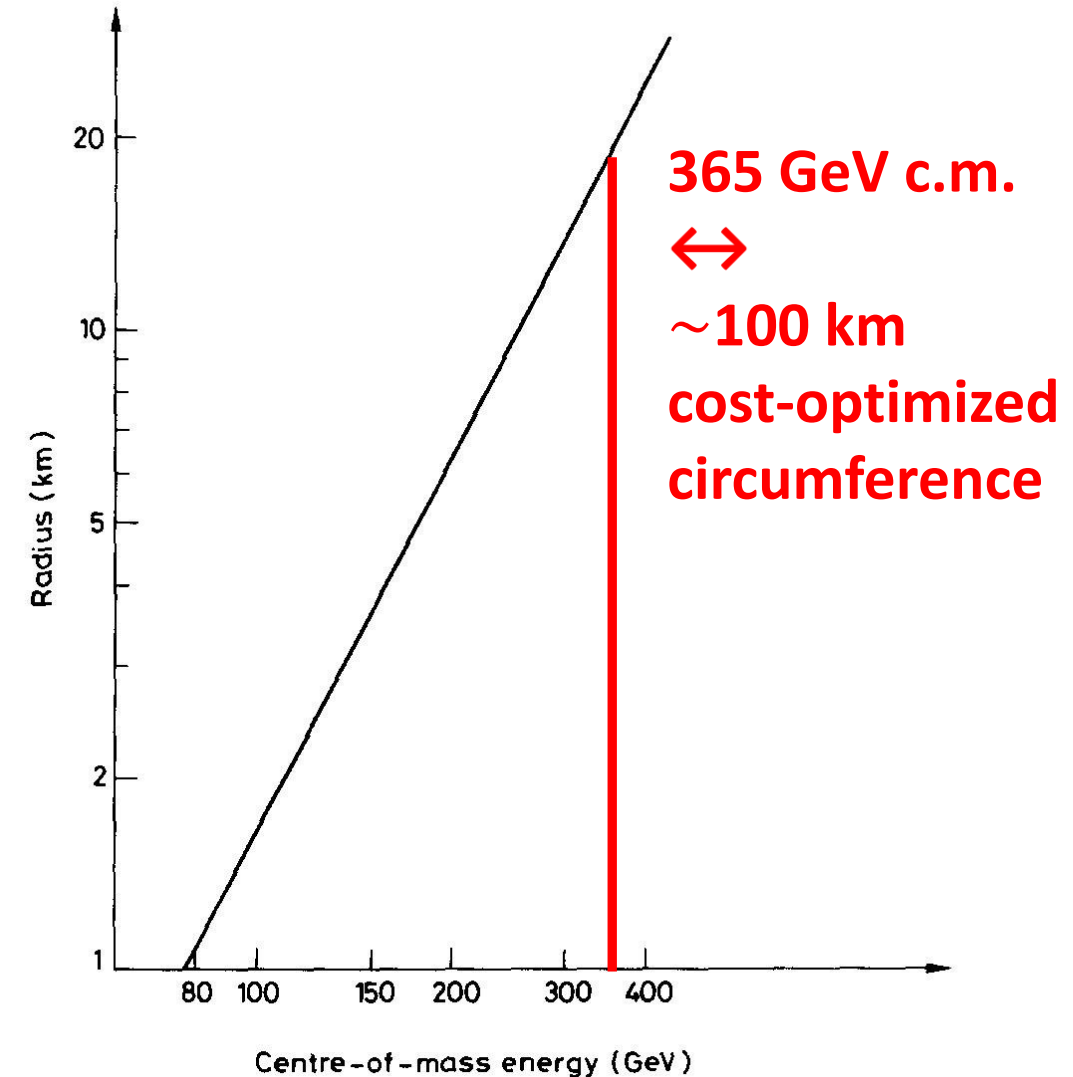
SR → size of circular e^+e^- colliders

lepton ring circumference versus beam energy



Data points from S. Myers, “FCC - Building on the Shoulders of Giants”, Eur. Phys. J. Plus (2021) **136**: 1076

Serendipitously, 90-100 km is exactly the size required for a 100 TeV hadron collider and optimum tunnel size in the Lake Geneva basin !



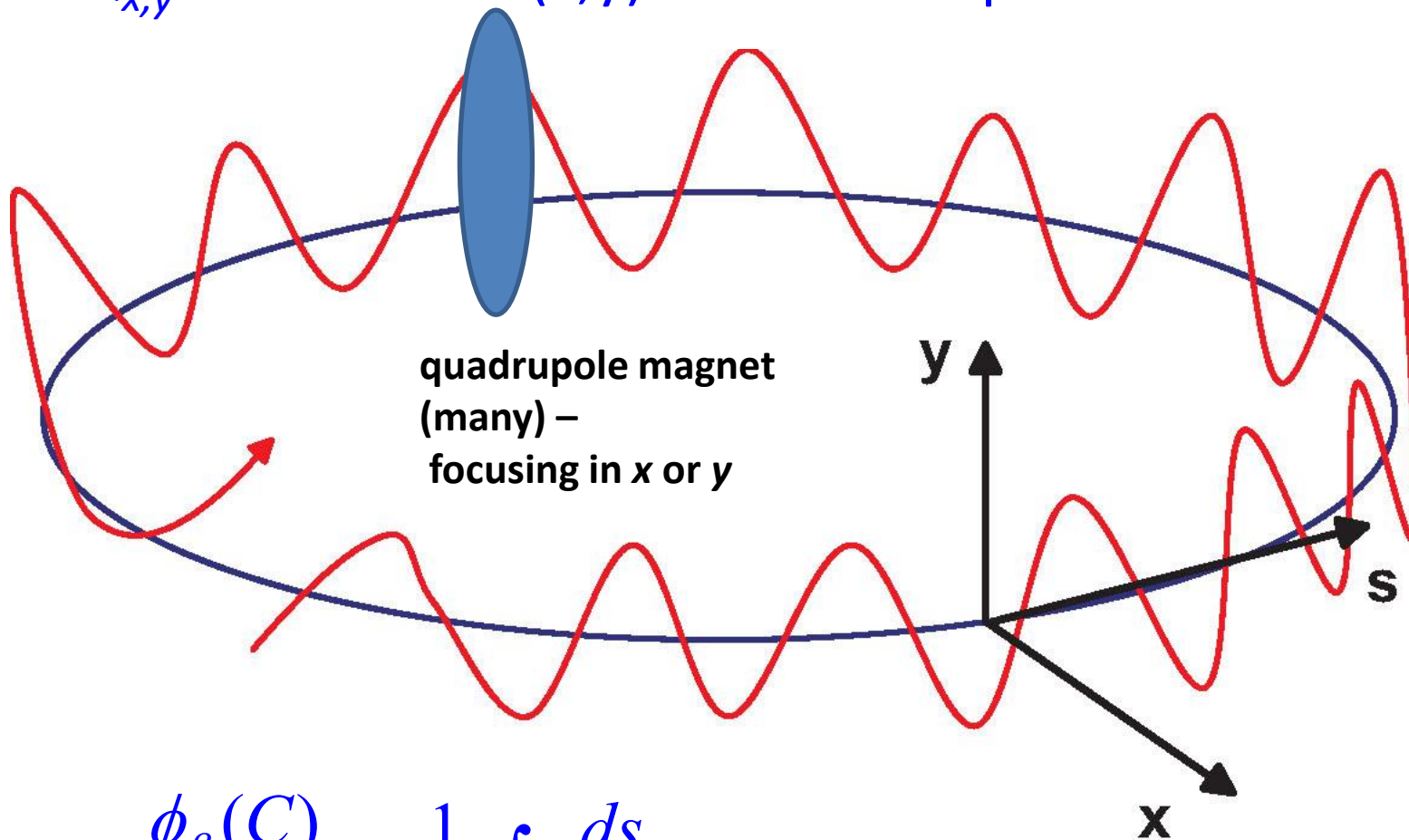
B. Richter, “Very High Energy Electron-Positron Colliding Beams for the Study of Weak Interactions”, NIM 136 (1976) 47-60

circular colliders

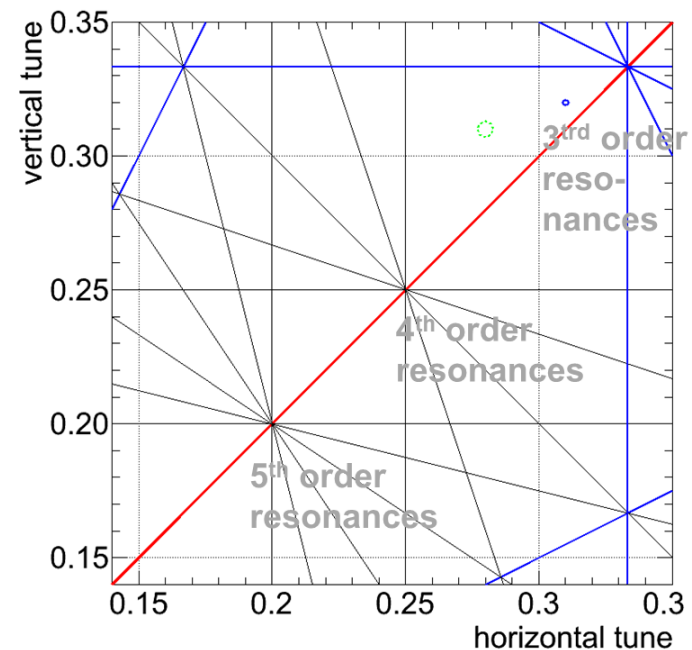
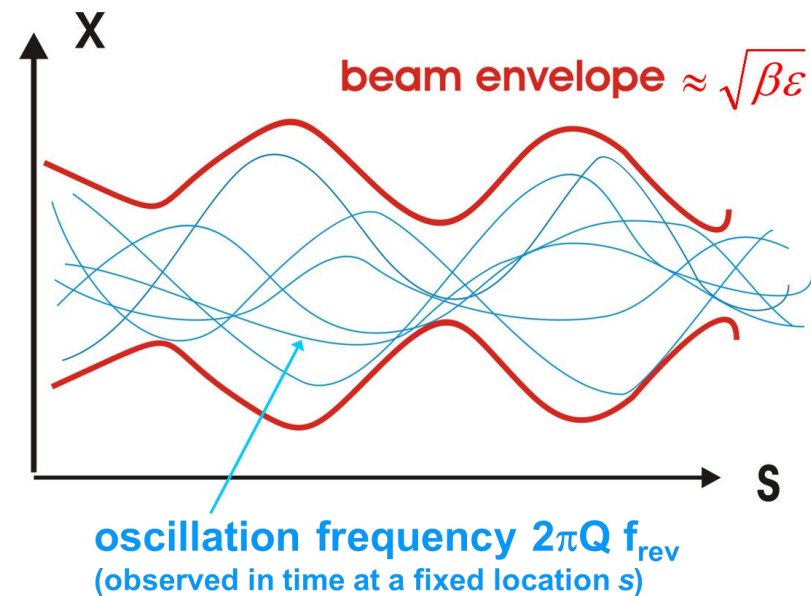
betatron motion

schematic of betatron oscillation around storage ring

tune $Q_{x,y}$ = number of (x,y) oscillations per turn



$$Q = \frac{\phi_\beta(C)}{2\pi} = \frac{1}{2\pi} \oint_C \frac{ds}{\beta(s)}$$



quadrupole strength

$$\frac{d^2 x}{ds^2} = -k(s)x$$

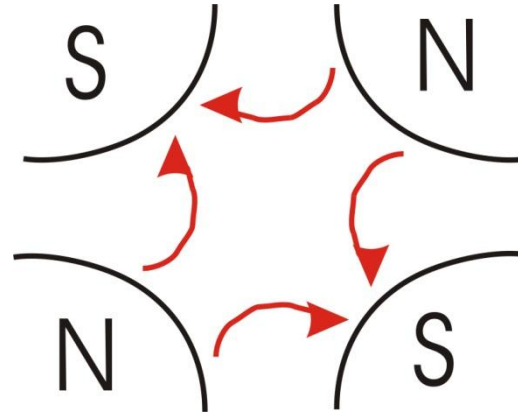
$$\Delta x' = -K x \quad \text{"kick approximation"}$$

$$k = \frac{B_T}{(B\rho)a}$$

B_T : pole-tip field
 a : pole-tip radius

$$(B\rho) = p/e = 3.356 \text{ T m } p [\text{GeV}/c]$$

$$K = k l_Q \quad \leftarrow \text{quadrupole length}$$



*relation between
beta function,
quadrupole strength,
and betatron tune
 $\Delta K \rightarrow \Delta Q$ - perhaps
the oldest technique,
documented in 1975*

ISR-TH-AH-BZ-amb

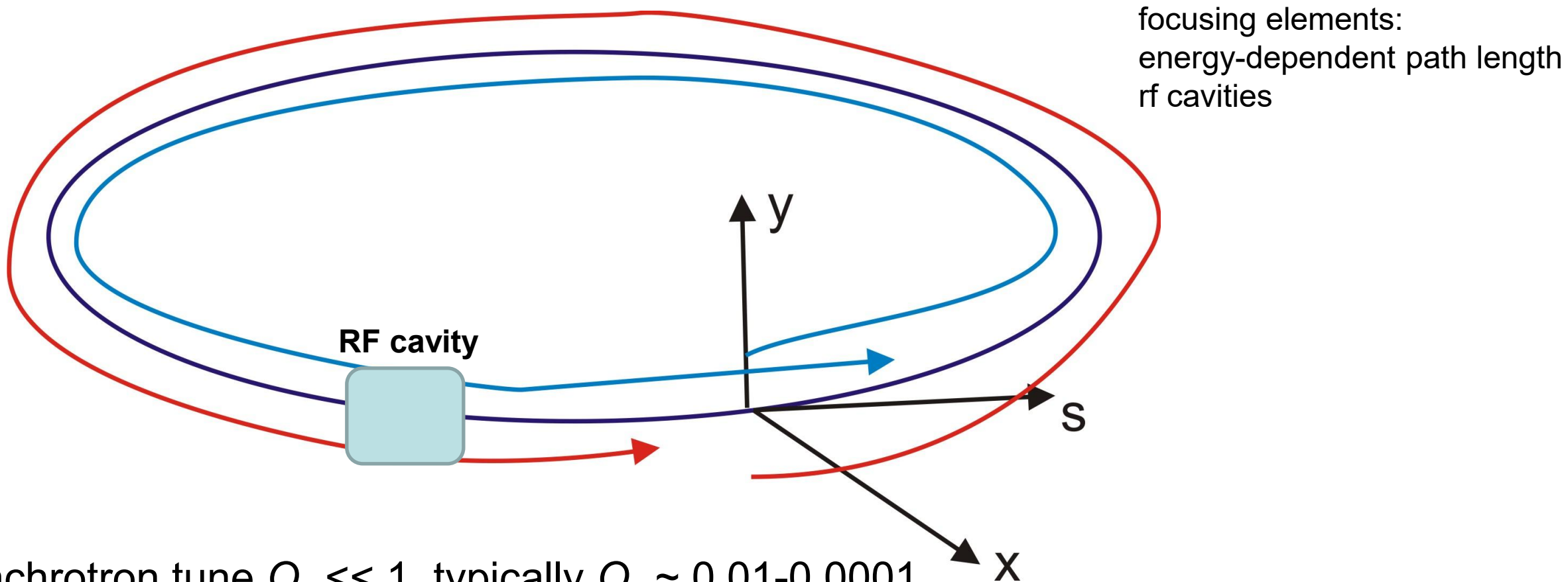
change quadrupole
strength by ΔK ;
and detect tune shift

$$\Delta Q \approx \frac{\beta_Q \Delta K}{4\pi}$$

synchrotron motion

schematic of “longitudinal oscillation” around storage ring

tune Q_s = number of synchrotron oscillations per turn



synchrotron tune $Q_s \ll 1$, typically $Q_s \sim 0.01-0.0001$

betatron tune $Q_{x,y} > 1$, typically $Q_{x,y} \sim 2 - 70$

radiation damping of synchrotron oscillations

$$\frac{d^2\epsilon}{dt^2} + 2\alpha_\epsilon \frac{d\epsilon}{dt} + \Omega^2\epsilon = 0$$

ϵ small energy deviation

synchrotron motion
with damping

synchrotron
radiation:

$$U_{\text{rad}} = U_0 + \epsilon D \quad \text{with} \quad D = \left(\frac{dU_{\text{rad}}}{dE} \right)_0$$

$$\alpha_\epsilon = \frac{D}{2T_0}$$

general
damping
decrement

$$\Omega^2 = \frac{\alpha_c e V_{\text{rf},0}}{T_0 E_0}$$

angular
synchrotron
frequency

When energy of an electron deviates from E_0 , the energy radiated in one revolution changes because

- (1) SR on nominal orbit
- (2) traveling through different magnetic field
- (3) different path length

U_{rad} from integrating P_γ around one complete off-energy orbit:

$$U_{\text{rad}} = \frac{1}{c} \oint \left(1 + \frac{D_x}{\rho} \frac{\epsilon}{E_0} \right) P_\gamma ds$$

$$\alpha_\epsilon = \frac{U_0}{2T_0 E_0} (2 + \mathcal{D})$$

$$\text{with} \quad \mathcal{D} = \frac{\oint D_x G (G^2 + 2K_1) ds}{\oint G^2 ds}$$

$$J_\epsilon = 2 + \mathcal{D}$$

partition number

typically $\mathcal{D} \ll 1$ and

$$\frac{dU_{\text{rad}}}{dE} = \frac{1}{c} \oint \left\{ \underbrace{2 \frac{P_\gamma}{E}}_{(1)} + \underbrace{2 \frac{P_\gamma D_x}{B E_0} \frac{dB}{dx}}_{(2)} + \underbrace{\frac{P_\gamma D_x}{E \rho}}_{(3)} \right\} ds$$

(1)

(2)

(3)

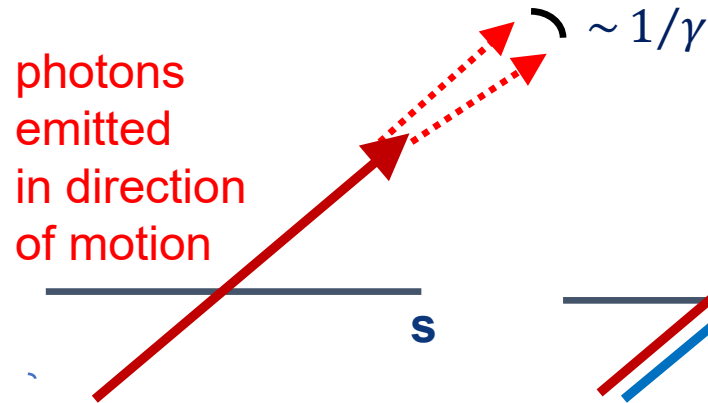
$$\alpha_\epsilon \approx \frac{U_0}{T_0 E_0} = \frac{\langle P_\gamma \rangle}{E_0}$$

damping time for energy oscillations
= the time it takes an electron to radiate
away its total energy !

Sands, 1970

radiation damping of betatron oscillations (sketch)

synchrotron
radiation
in magnetic field



photons
emitted
in direction
of motion

both transverse
and longitudinal momentum
of emitting electron reduced

longitudinal
acceleration
in radiofrequency
cavities



only longitudinal
momentum is restored
by RF cavities

→ net damping

effect of horizontal dispersion

$$\alpha_y = \frac{U_0}{2T_0 E_0}$$

$$J_y = 1$$

$$\alpha_x = \frac{U_0}{2T_0 E_0} (1 - \mathcal{D})$$

$$J_x = 1 - \mathcal{D}$$

Note: Robinson sum rule

$$\alpha_x + \alpha_y + \alpha_\epsilon = 4 \frac{U_0}{2T_0 E_0}$$

$$\alpha_x + \alpha_y + \alpha_\epsilon = 4$$

Robinson, 1956-58

theorem for arbitrary
dissipative force:

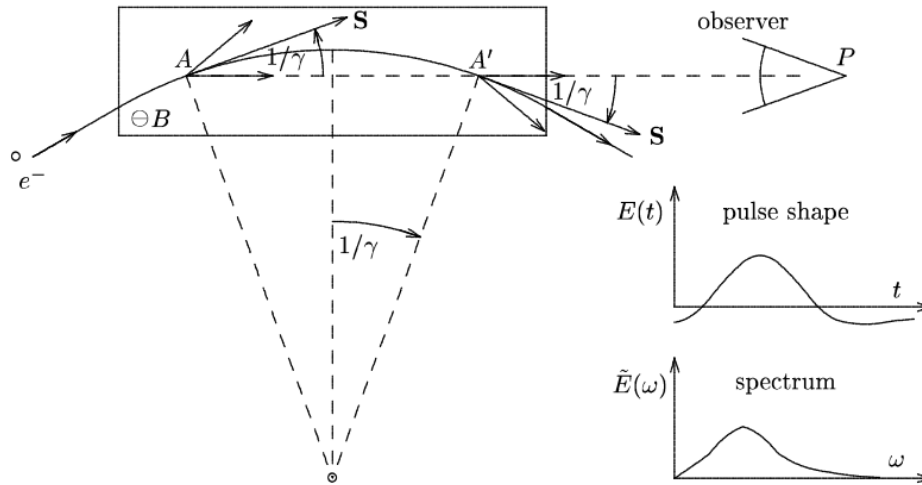
$$\alpha_x + \alpha_y + \alpha_\epsilon = \frac{1}{2} \frac{W}{pc} \left[2 + \frac{\partial \ln W}{\partial \ln p} \right]$$

W : rate of energy loss
 p : particle momentum

Pestrikov

SR spectrum

typical frequency of the spectrum emitted in long magnets



arc dipole magnets in electron storage rings

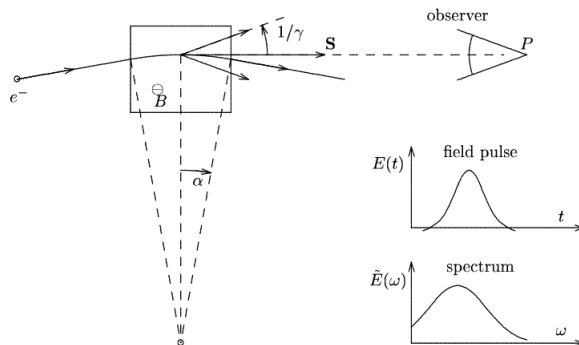
length of the light pulse seen by an observer

$$\Delta t = t_e - t_\gamma = \frac{2\rho}{\beta\gamma c} - \frac{2\rho \sin(1/\gamma)}{c} \approx \frac{4}{3} \frac{\rho}{c\gamma^3}$$

typical frequency

$$\omega_{typ} \sim \frac{2\pi}{\Delta t} \sim \frac{3\pi c\gamma^3}{2\rho}$$

typical frequency of the spectrum emitted in a short magnet



length of the light pulse seen by an observer

$$\Delta t_{sm} = \frac{L}{\beta c} - \frac{L}{c} \approx \frac{L}{2c\gamma^2}$$

typical frequency

$$\omega_{sm} \sim \frac{2\pi}{\Delta t_{sm}} \sim \frac{4\pi c\gamma^2}{L}$$


Hofmann, CAS 1996

this could be important for future hadron colliders

Zimmermann, IPAC2022

SR spectrum & photon emission

$$P_\gamma = \int_0^\infty \wp(\omega) d\omega$$



power spectrum

$$\wp(\omega) = \frac{P_\gamma}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho}$$

critical frequency
(half of the power each is emitted
at lower or higher frequencies)

$$S(\xi) = \frac{9\sqrt{3}}{8\pi} \xi \int_\xi^\infty K_{5/3}(\bar{\xi}) d\bar{\xi}$$

$$\int_0^\infty S(\xi) d\xi = 1$$

first obtained by Schwinger

radiation is emitted in the form of quanta
(photons) of energy

$$u = \hbar\omega$$

$$u n(u) du = \wp(u/\hbar) du/\hbar$$

$$n(u) = \frac{P_\gamma}{u_c^2} F\left(\frac{u}{u_c}\right)$$

quantum
distribution
function

$$u_c = \hbar\omega_c = \frac{3\hbar c\gamma^3}{2\rho}$$

critical photon energy

photon emission rate

$$\mathcal{N} = \frac{15\sqrt{3}}{8} \frac{P_\gamma}{u_c}$$

$$\langle u \rangle = \frac{8}{15\sqrt{3}} u_c$$

$$\langle u^2 \rangle = \frac{11}{27} u_c^2$$

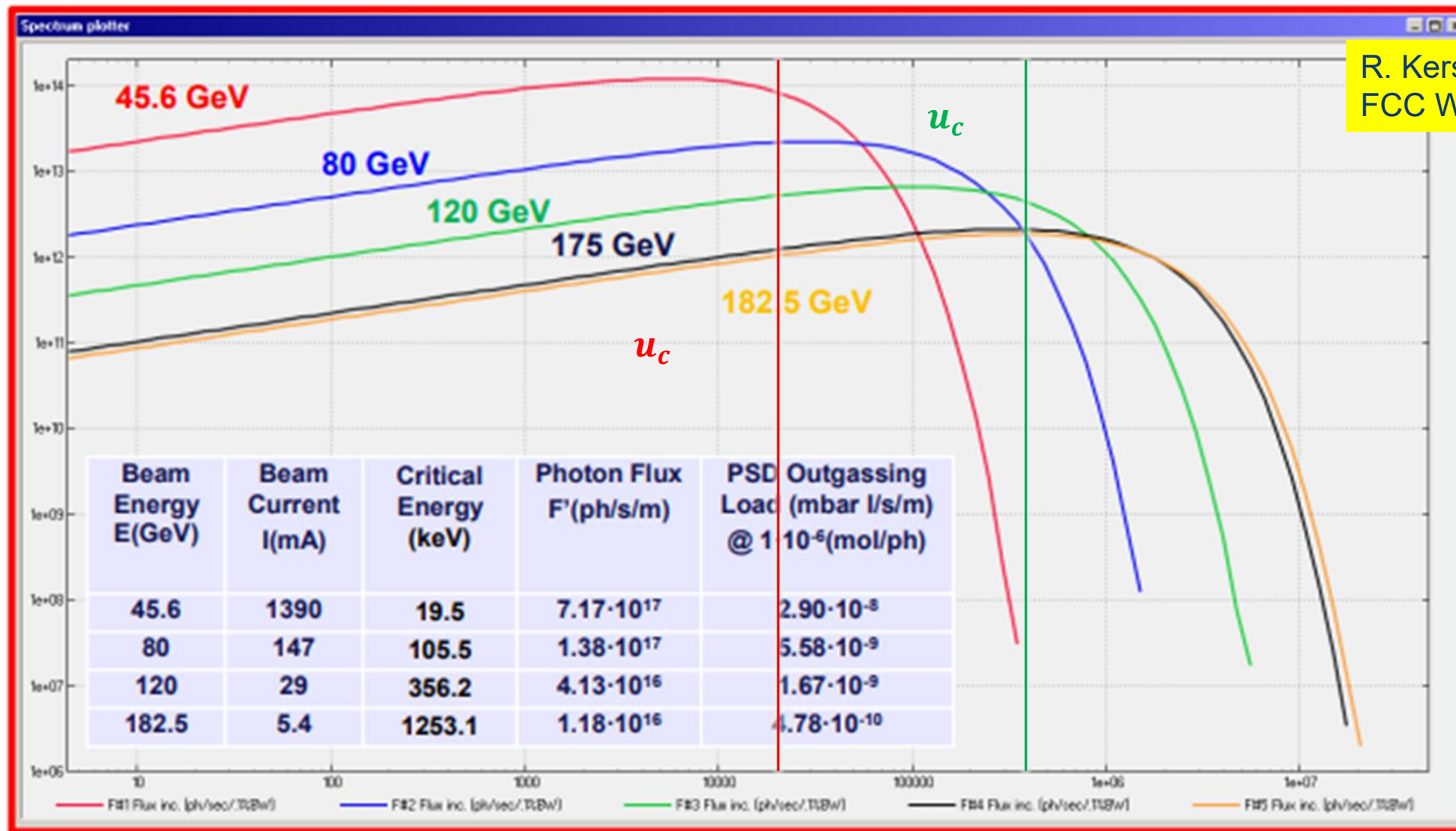
mean number of quanta per radian deflection:

$$\boxed{5/(2\sqrt{3})\alpha\gamma} \text{ with } \alpha \approx 1/137 \text{ fine-structure constant}$$

Sands, 1970

example SR spectra for FCC-ee

R. Kersevan,
FCC Week 2021



Units: Vertical: photons/s/(0.1% bandwidth)/m; Range $[10^6 - 2 \cdot 10^{14}]$
Horizontal eV; Range $[4 - 5 \cdot 10^6]$

SR in collision: beamstrahlung

synchrotron radiation in the strong field of the opposing beam (“**beamstrahlung**”)

collision of Gaussian bunches:

Chen & Yokoya, 1992 and before

average Upsilon

$$\Upsilon_{av} \approx \frac{5}{6} \frac{N r_e^2 \gamma}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

where $\Upsilon = \frac{2}{3} \frac{\hbar \omega_c}{E_0} = \gamma \frac{|B| + |E/c|}{B_c}$

linear colliders:

$$0.2 \leq \Upsilon \leq 100$$

circular colliders:

$$\Upsilon \sim 10^{-4}$$

$$n_\gamma \approx \frac{2.16 \alpha r_e N}{\sigma_x + \sigma_y} \frac{1}{(1 + \Upsilon_{av}^{2/3})^{1/2}}$$

photons emitted per electron

historical design constraint for linear colliders:

$$n_\gamma < 1$$

$$\delta_E \approx 2.09 \frac{r_e^3 N^2 \gamma}{\sigma_z} \left(\frac{2}{\sigma_x + \sigma_y} \right)^2 \frac{1}{(1 + (1.5 \Upsilon_{av})^{2/3})^2}$$

average relative energy loss

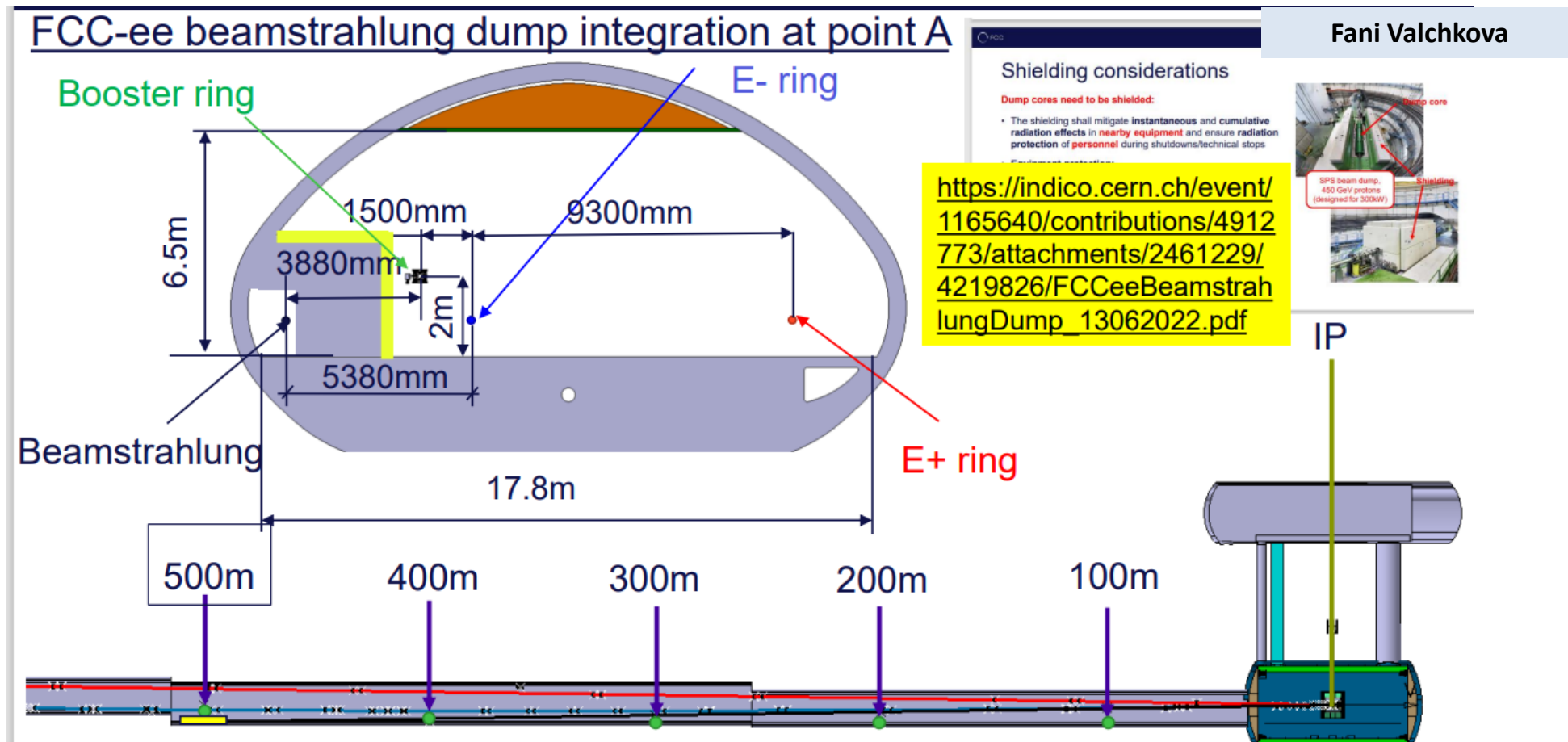
how to reduce beamstrahlung: (1) flat beams $\sigma_x \gg \sigma_y$ (!), and/or (2) very short bunches ?

**circular
colliders**

beamstrahlung increases equilibrium beam energy spread and/or limits the beam lifetime

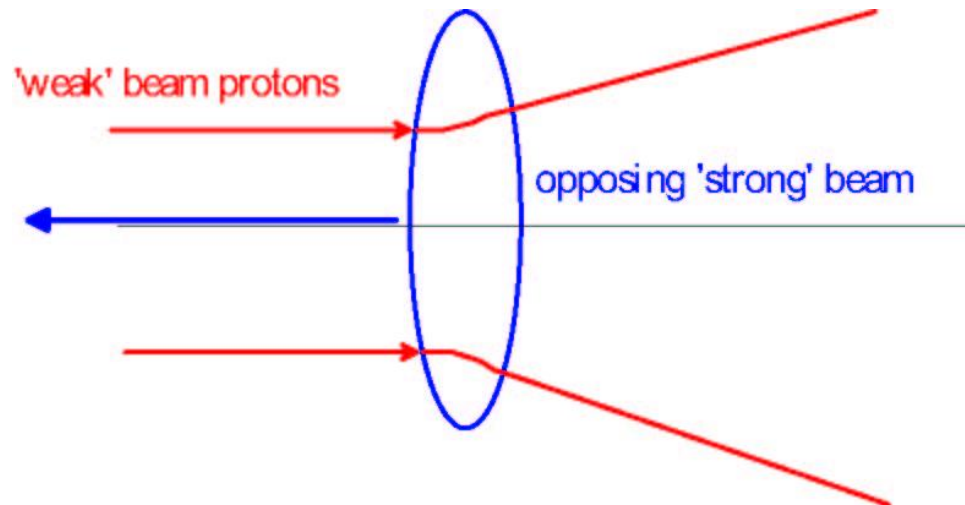
handling of beamstrahlung at FCC-ee

up to 0.5 MW beamstrahlung photon power per beam per IP,
requires dedicated shielded photon beam dumps



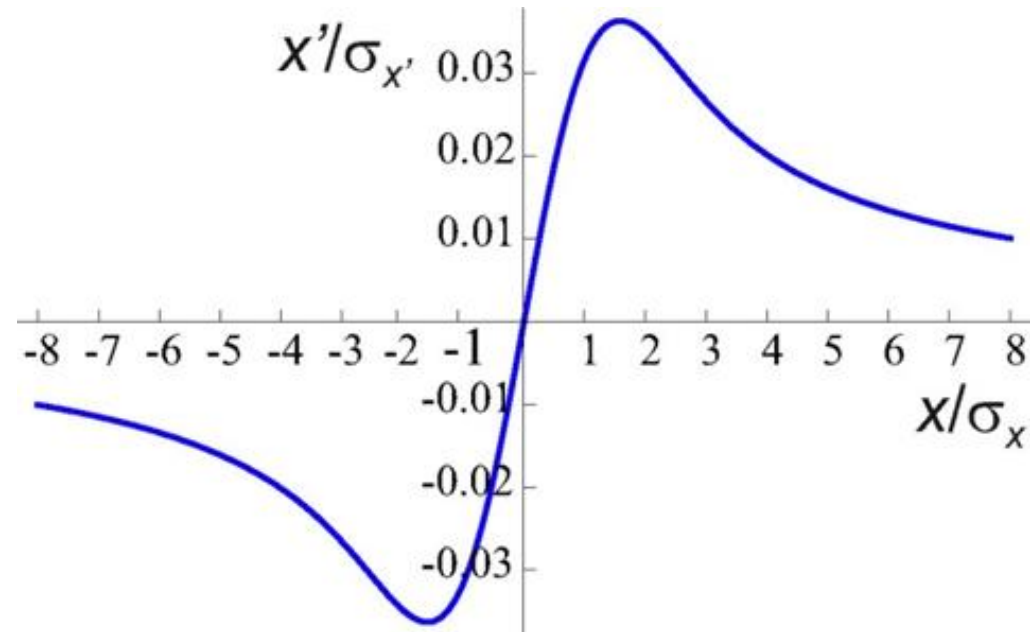
1.3 Beam-beam effects

collider figure of merit: beam-beam tune shift



head-on beam-beam collision in the LHC

(nonlinear) beam-beam force



at small amplitude similar to effect of defocusing quadrupole

for pure head-on collision

$$\Delta Q_{x,y;\max} = \xi_{x,y} = \frac{2N_b r_0 \beta^*}{4\pi\gamma(2\sigma^{*2})} = \frac{N_b}{\varepsilon_N} \frac{r_0}{4\pi}$$

for single
collision
(nominal
LHC ~0.0033)

beam-beam tune spread

vertical
tune Q_y

maximum
acceptable
tune
spread
is limited
by resonances

$$nQ_x + mQ_y = p$$

up to resonance
order $|n| + |m| \sim 13$

tune spread
 ΔQ_y

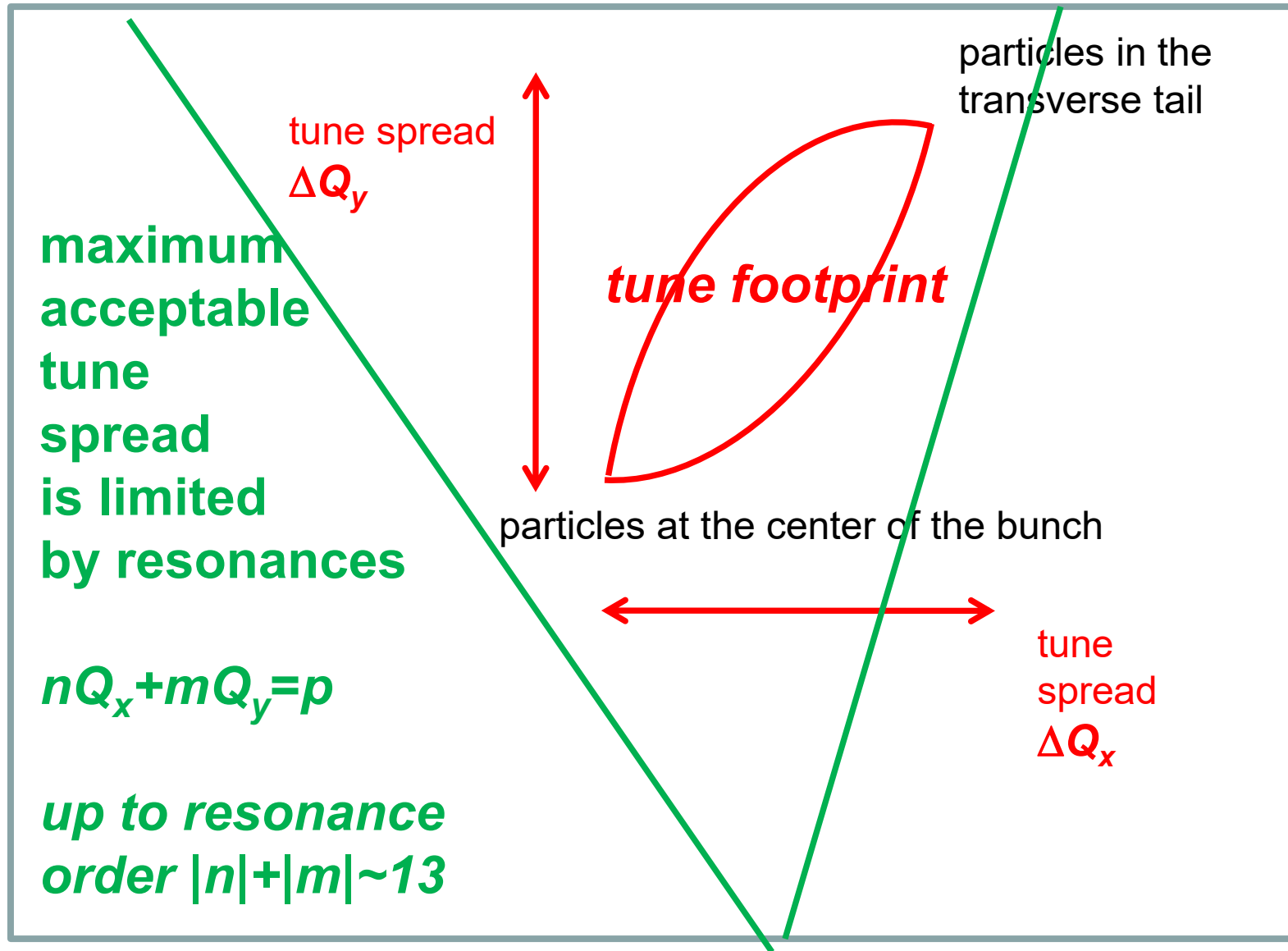
tune footprint

particles in the
transverse tail

particles at the center of the bunch

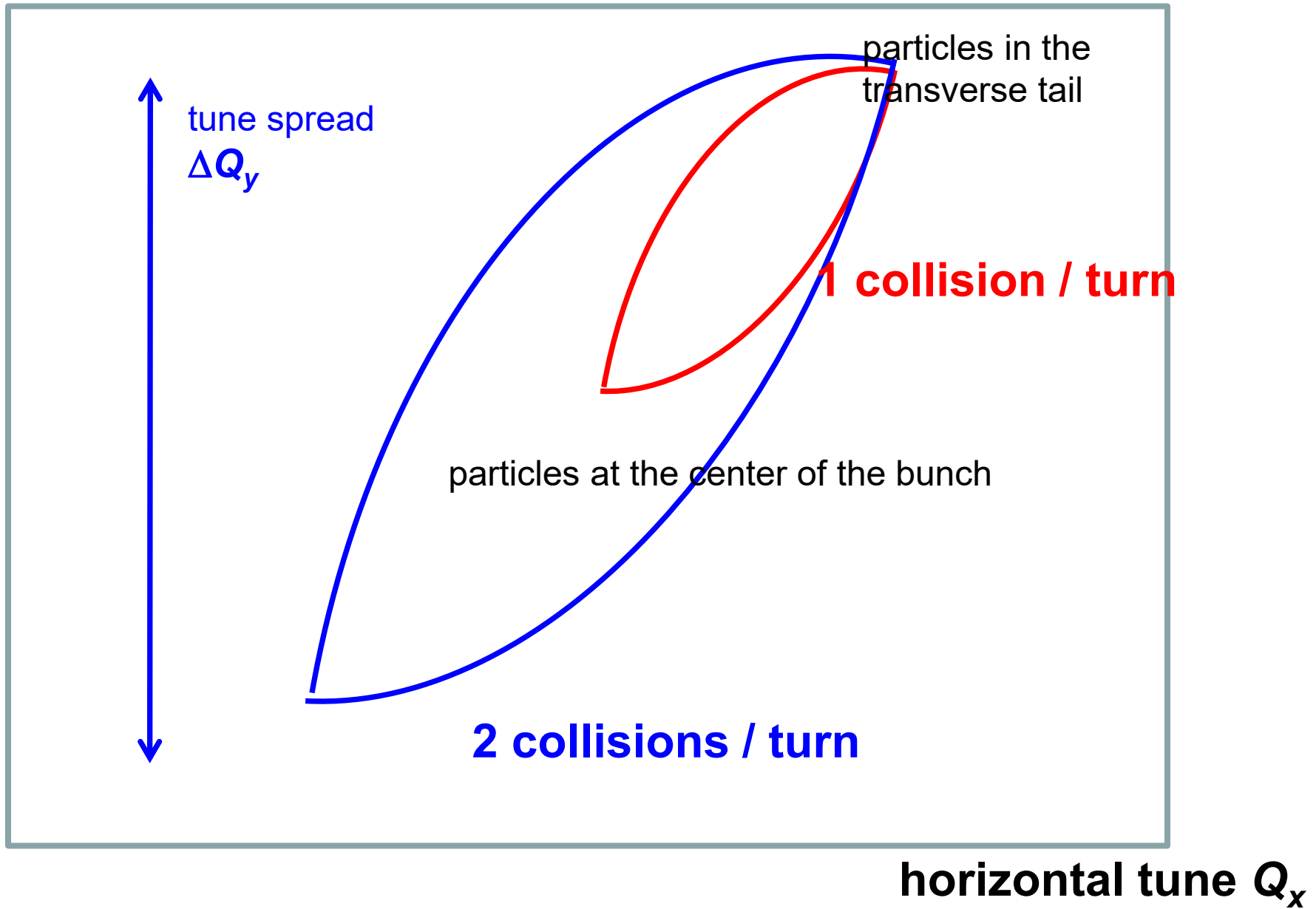
tune
spread
 ΔQ_x

horizontal tune Q_x



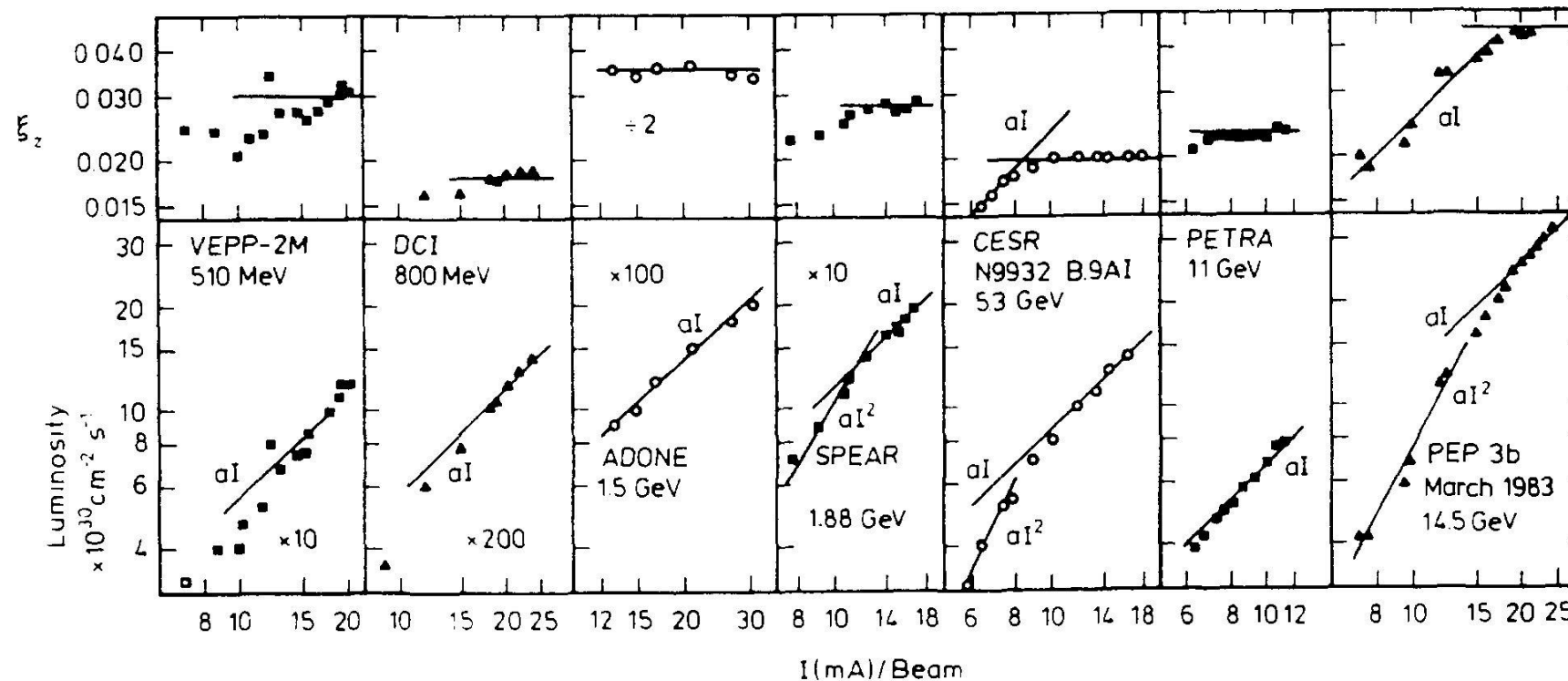
multiple interaction points

vertical
tune Q_y



beam-beam limit in e^+e^- colliders

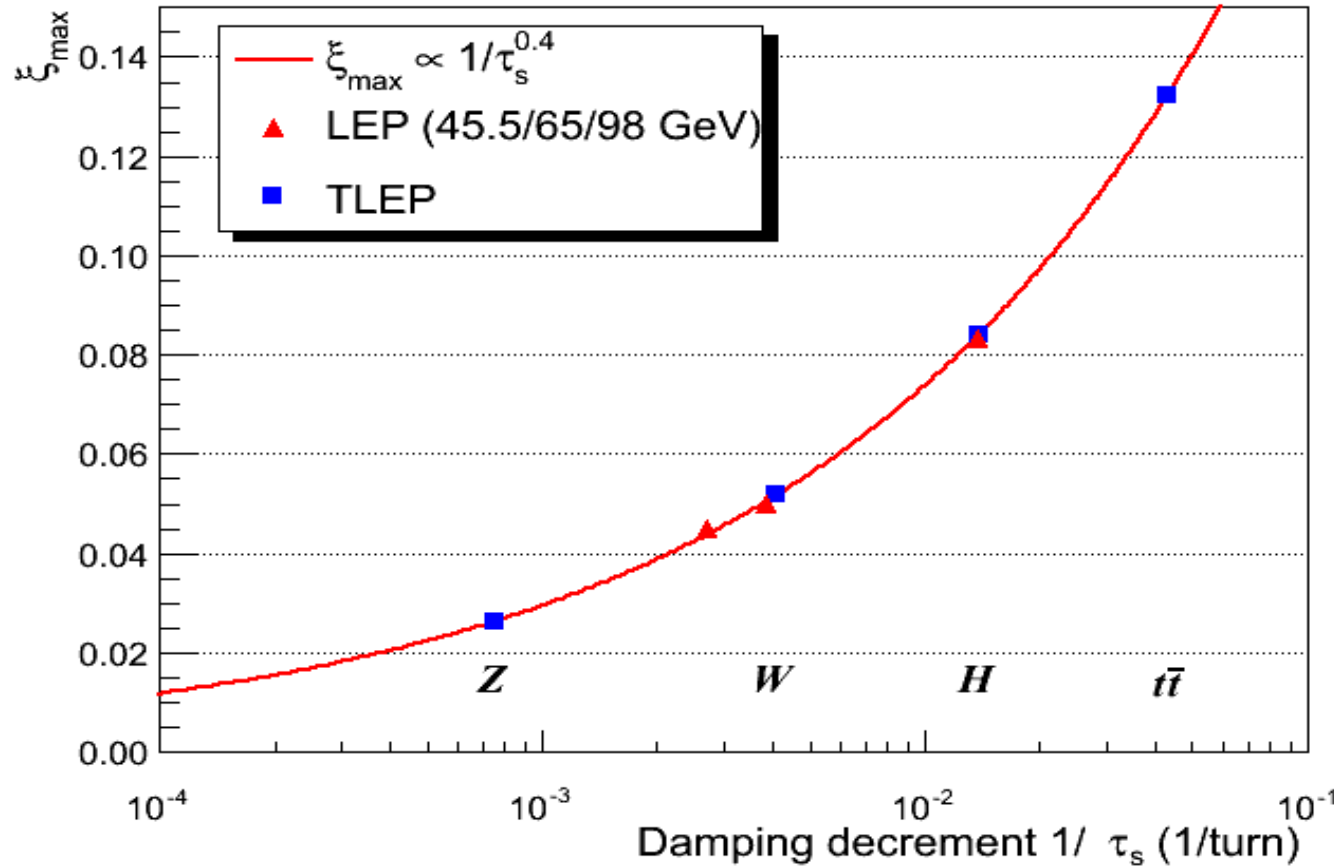
J. Seeman



luminosity and vertical tune-shift parameter versus beam current for various electron-positron colliders; the tune shift saturates at some current value, above which the luminosity grows linearly

beam-beam limit w strong SR damping

R. Assmann

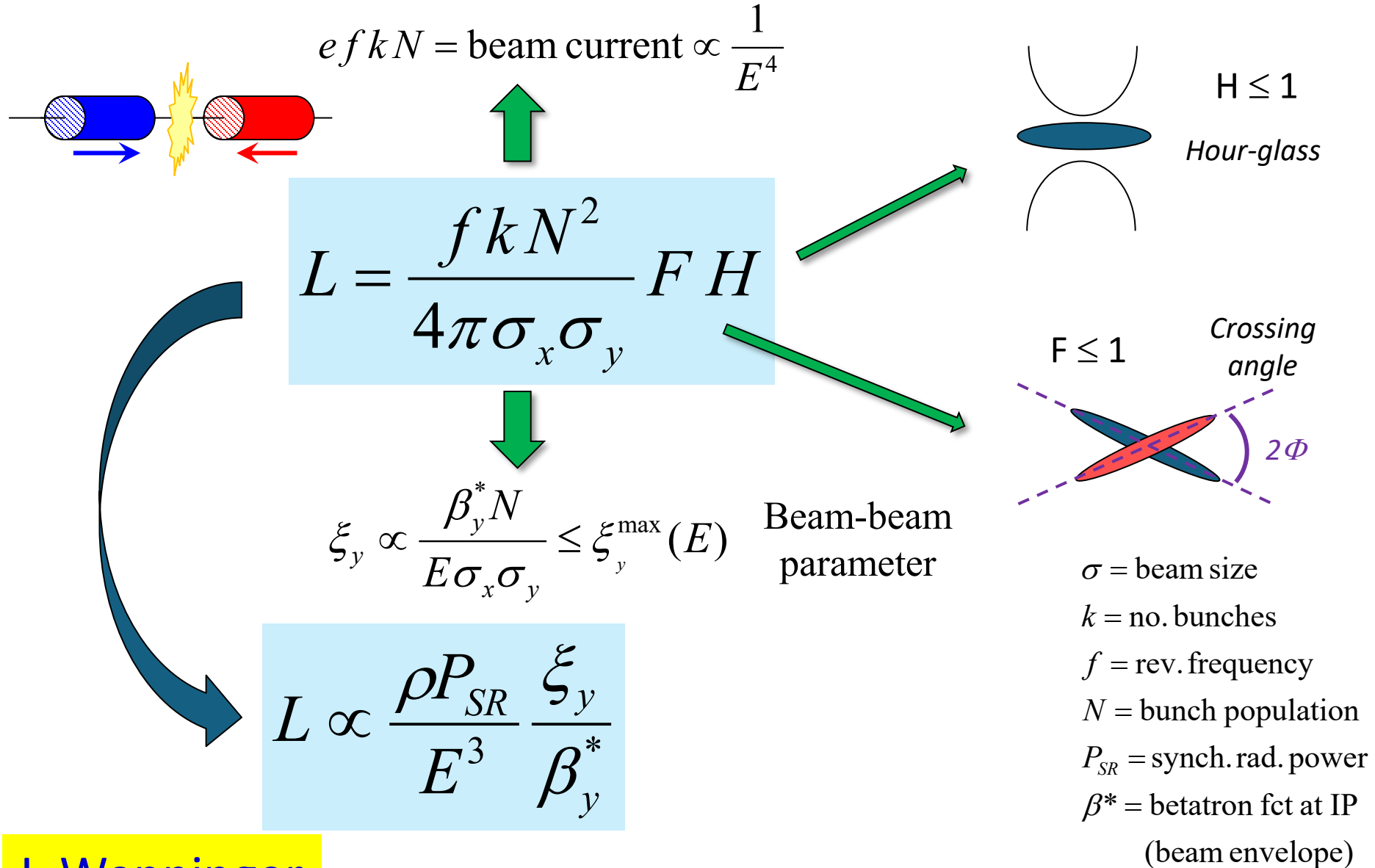


$$\lambda_d = 1/(f_{rev} \cdot \tau \cdot n_{ip}) \quad \xi_y^\infty \propto (\lambda_d)^{0.4}$$

damping decrement per IP

1.4 Luminosity revisited

scaling: larger E & ρ



luminosity scaling: damping

- beam-beam parameter ξ measures strength of field sensed by the particles in a collision
- beam-beam parameter limits can be scaled from LEP data (4 IPs); crab waist allows for higher ξ

$$\xi_y \propto \frac{\beta_y^* N}{E \sigma_x \sigma_y} \leq \xi_y^{\max}(E)$$

$$\xi_y^{\max}(E) \propto \frac{1}{\tau_s^{0.4}} \propto E^{1.2}$$

FCC-ee
vs LEP



x3.7

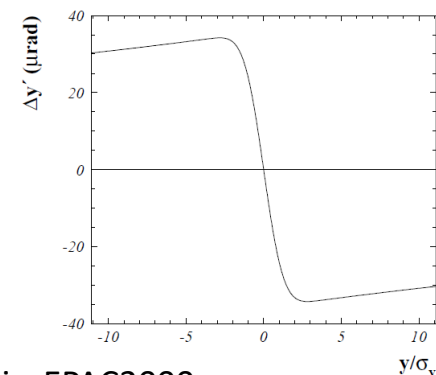
x4.5

<x2

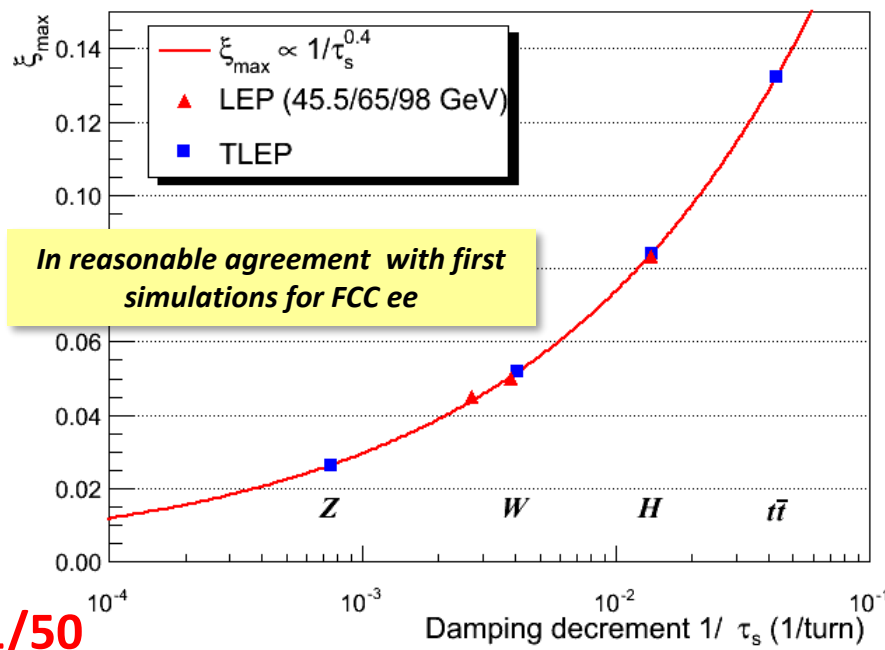
$$L \propto \frac{\rho P_{SR}}{E^{1.8}} \frac{1}{\beta_y^*}$$

x1/25-1/50

→ extremely high luminosity

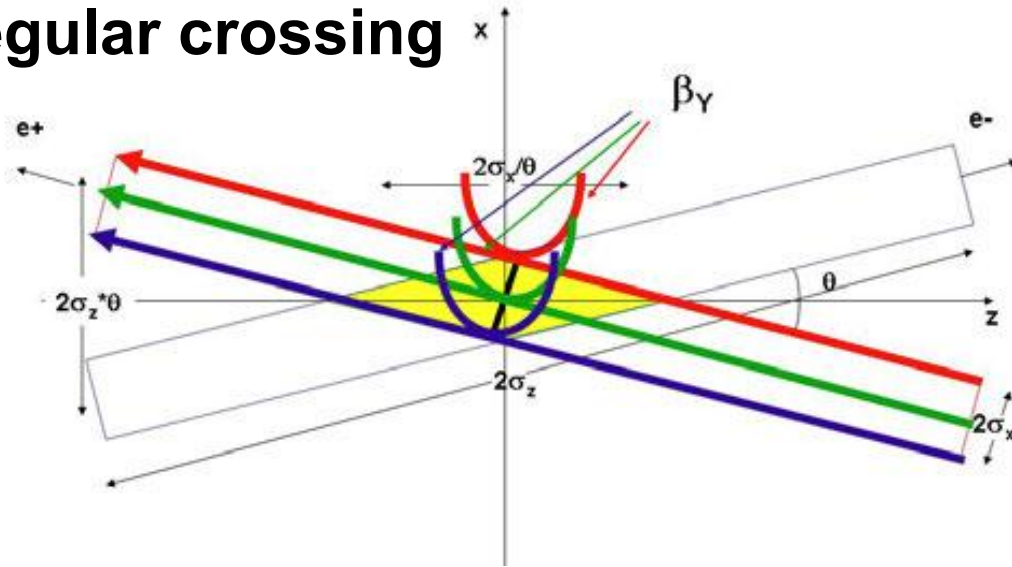


R. Assmann & K. Cornelis, EPAC2000

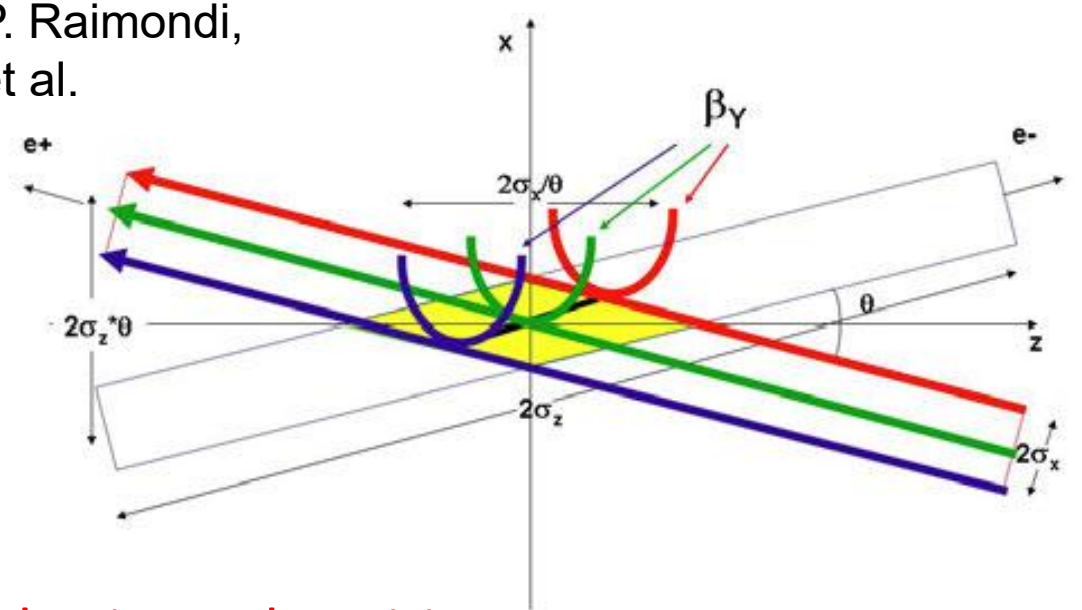


crab-waist crossing for flat beams

regular crossing



P. Raimondi,
et al.



crab waist - vertical waist position in s varies with horizontal position x

- allows for small β_y^* and for small $\epsilon_{x,y}$
- and avoids betatron resonances (\rightarrow higher beam-beam tune shift)

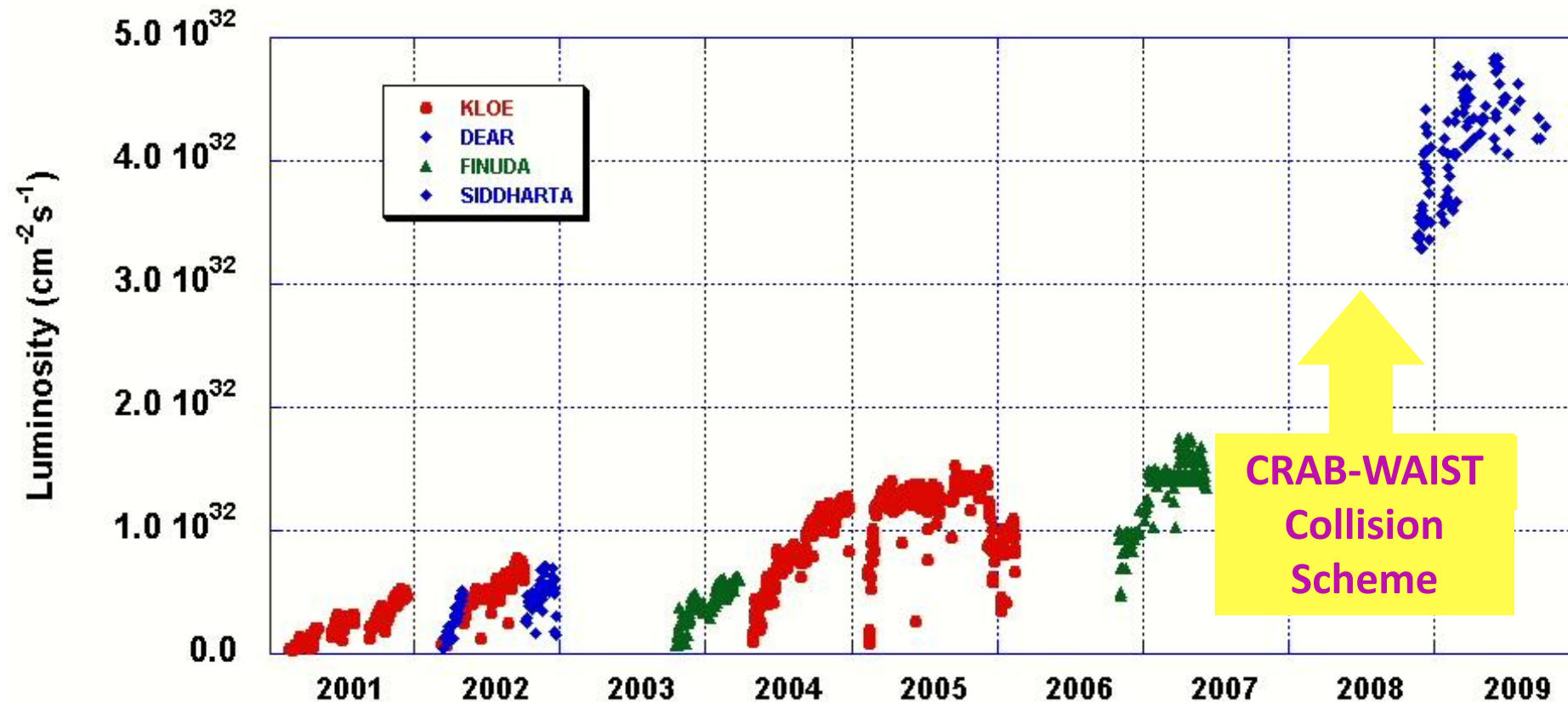
$$\Phi \equiv \frac{\sigma_z \theta}{2\sigma_x^*} \quad \text{"Piwinski angle"}$$

$$L = \frac{N_b f_0}{4\pi\sigma_x\sigma_y} \left[\frac{N^2}{\sqrt{1+\Phi^2}} \right]; \quad \xi_y = \frac{r_e \beta_y}{2\pi\gamma\sigma_x\sigma_y} \left[\frac{N}{\sqrt{1+\Phi^2}} \right]; \quad \xi_x = \frac{r_e \beta_x}{2\pi\gamma\sigma_x^2} \left[\frac{N}{1+\Phi^2} \right]$$

$$\rightarrow L = \frac{I_{beam}}{e} \frac{\gamma}{2r_e \beta_y^*} \xi_y$$

DAΦNE: “crab waist” collisions

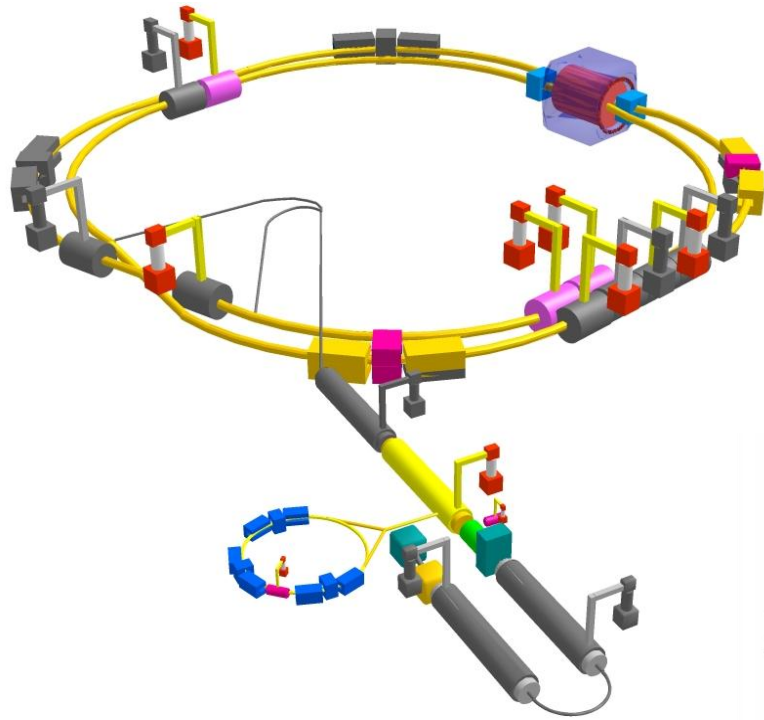
DAΦNE Peak Luminosity



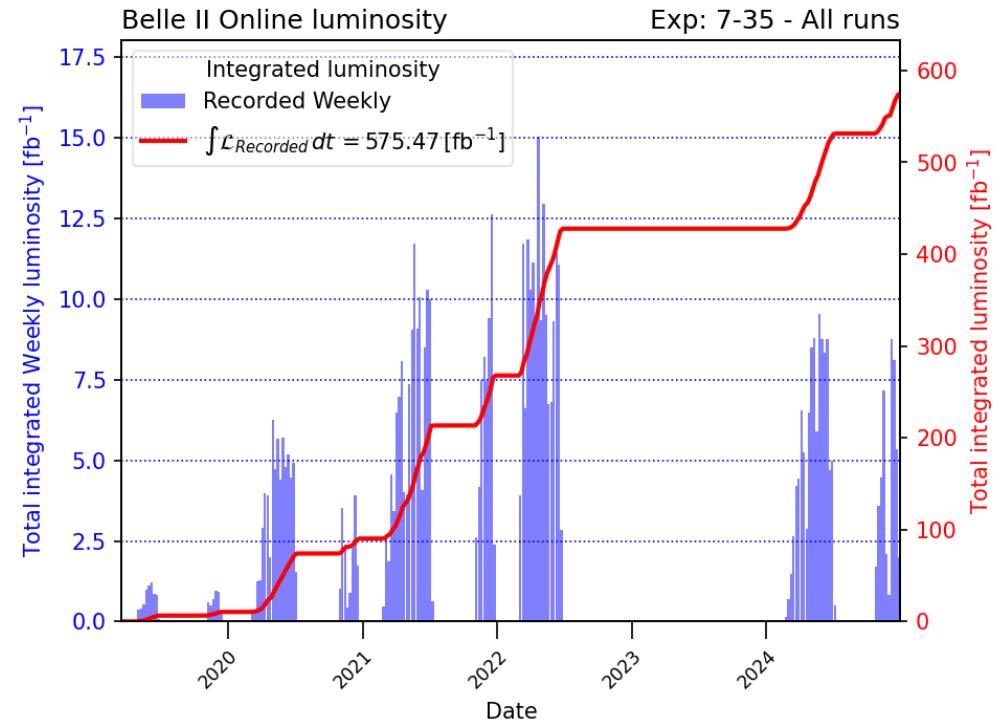
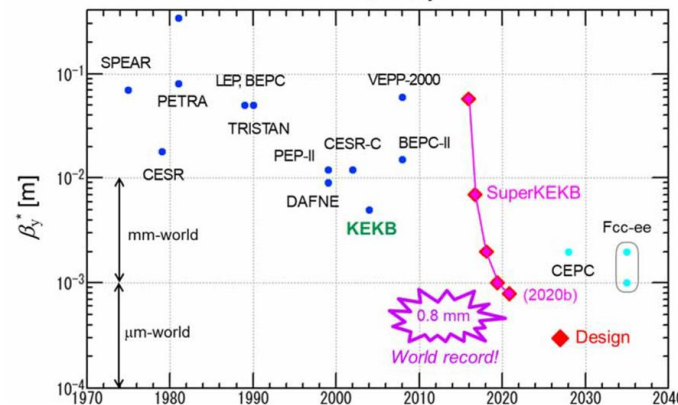
Design Goal

SuperKEKB 4 GeV e^+ vs 7 GeV e^- in Japan

circumference 3 km



world's highest
luminosity &
lowest β^* e^+e^-
collider at
KEK/Tsukuba



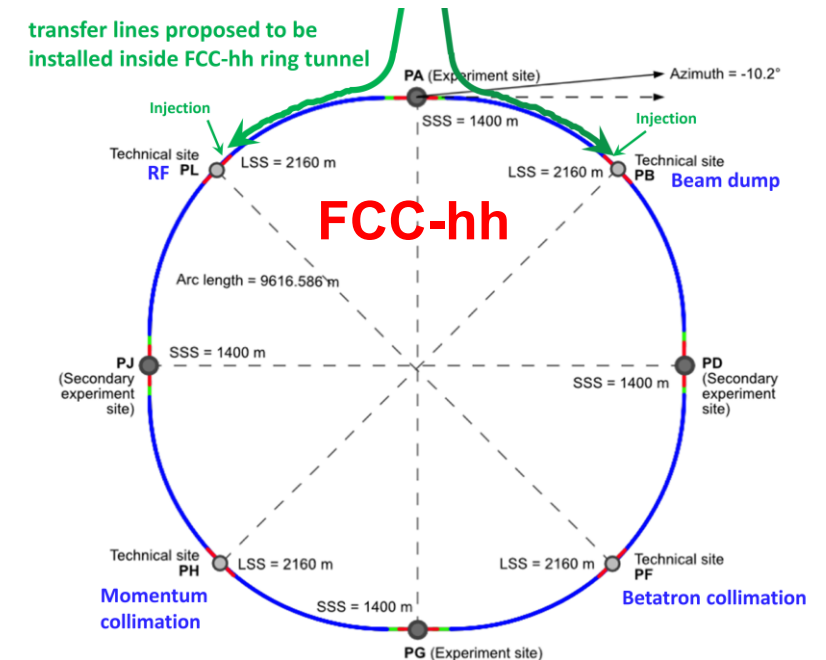
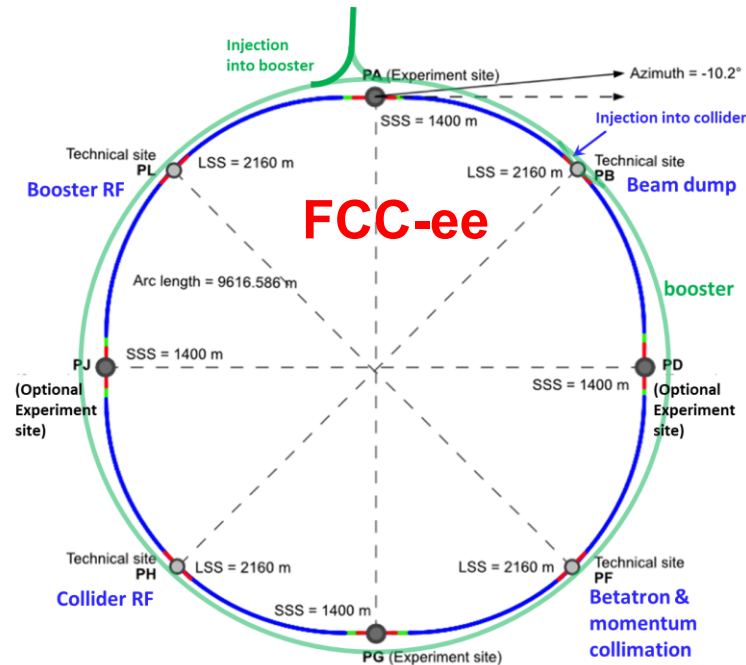
total integrated luminosity so far
 $\sim 575 \text{ fb}^{-1}$ over ~ 6 years

world record luminosity of $4.71 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$, $\beta_y^* = 1.0 \text{ mm}$ routinely, also $\beta_y^* = 0.8 \text{ mm}$ shown
– with “virtual” crab-waist collision scheme originally developed for FCC-ee (K. Oide)

1.5 FCC concepts

FCC integrated program

- inspired by the successful LEP/LHC programs at CERN
- **stage 1: FCC-ee (Z, W, H, $t\bar{t}$) as Higgs factory, electroweak & top factory at highest luminosities**
- **stage 2: FCC-hh (~100 TeV) as natural continuation at energy frontier, pp & AA collisions; e-h option**
- highly synergetic and complementary programme maximising the physics opportunities
- common civil engineering and technical infrastructures, building on and reusing CERN's existing infrastructure
- FCC integrated project allows the start of a new, major facility at CERN within a few years of the end of HL-LHC



2020 - 2045

2045 - 2065

2070 -

FCC-ee key design concepts

double ring collider

- many bunches, high current, like LHC and B factories, different from LEP

crab-waist collision scheme

- successfully demonstrated at DAΦNE (Italy) and SuperKEKB (Japan)

top-up injection

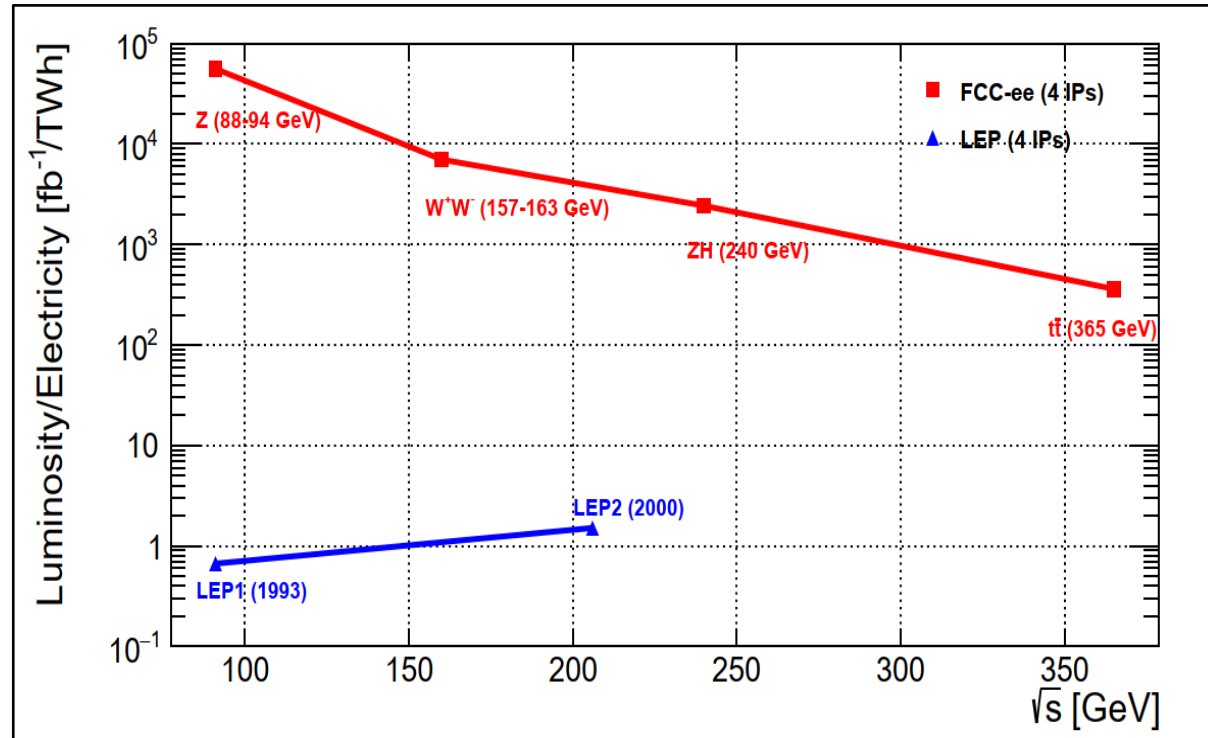
- standard at modern light sources, like SLS
- used at recent e^+e^- colliders, PEP-II (USA), KEKB (Japan), BEPCII (China)

SC radiofrequency system

- Nb/Cu 400 MHz SC cavities pioneered at former CERN LEP
- bulk Nb 800 MHz SC cavities similar to ESS (Sweden), EuXFEL (Germany)
- revolutionary highly efficient RF power sources
- new operation scheme for flexible energy switching & reduced complexity

Combining concepts from past and present lepton colliders yields giant step in efficiency:

→ $10^4 - 10^5 \times$ luminosity/energy of LEP
→ sustainable physics



FCC-ee design parameters

parameter	Z	WW	H (ZH)	tt̄	
Collision energy \sqrt{s} [GeV]	88, 91, 94	157, 163	240	340-350	365
synchrotron radiation/beam [MW]	50	50	50	50	50
beam current [mA]	1294	135	26.8	6.0	5.1
number bunches / beam	11200	1852	300	70	64
total RF voltage 400 / 800 MHz [GV]	0.08 / 0	1.0 / 0	2.1 / 0	2.1 / 7.4	2.1 / 9.2
luminosity / IP [$10^{34} \text{ cm}^{-2}\text{s}^{-1}$]	144	20	7.5	1.8	1.4
luminosity / year [ab^{-1}]	68	9.6	3.6	0.83	0.67
run time (including lumi ramp-up) [years]	4	2	3	1	4
total integrated luminosity [ab^{-1}]	205	19.2	10.8	0.4	2.7
total number of events	$6 \cdot 10^{12} \text{ Z}$	$2.4 \cdot 10^8 \text{ WW}$ (incl. WW at higher \sqrt{s})	$2.2 \cdot 10^6 \text{ ZH}$ $65\text{k WW} \rightarrow \text{H}$	$2 \cdot 10^6 \text{ tt̄} + 370\text{k ZH}$ $+ 92\text{k WW} \rightarrow \text{H}$	

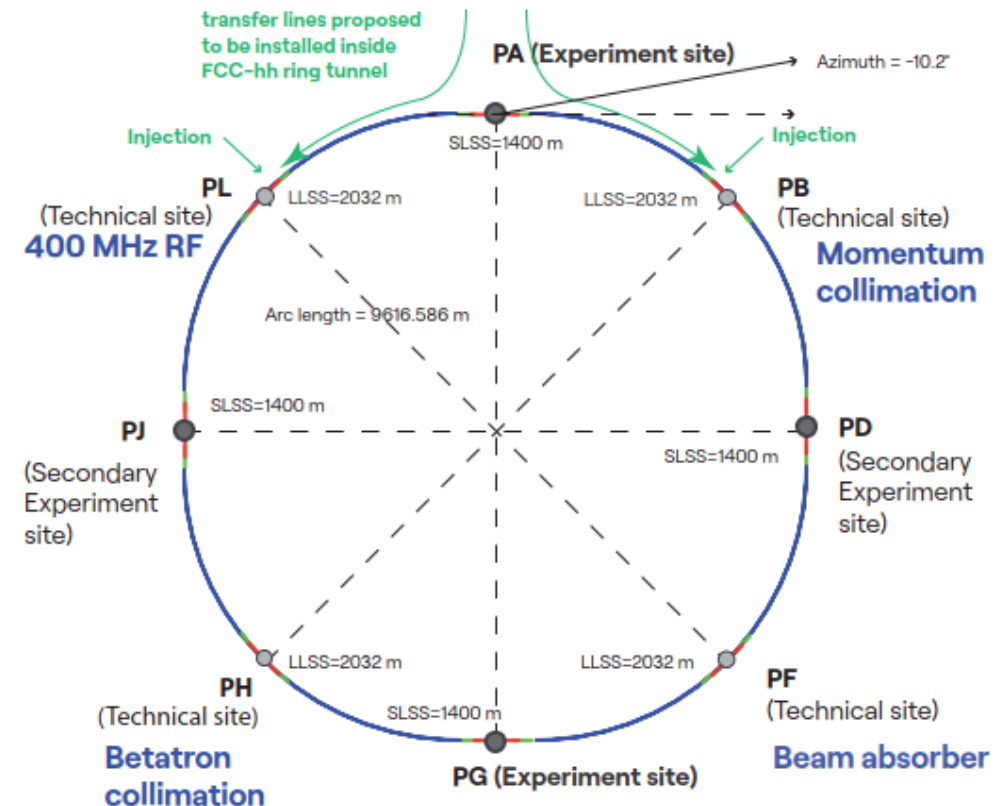
Stage 2: hadron collider FCC-hh

- Parameter optimization to lower electricity consumption (~max. consumption of FCC-ee)
- Magnetic field considered realistic with today's technologies (Nb_3Sn , ~14T; alternative: HTS)

Main parameters 2025

parameter	FCC-hh	HL-LHC
collision energy cms [TeV]	85	14
dipole field [T]	14	8.33
circumference [km]	90.7	26.7
beam current [A]	0.5	1.1
synchr. rad. per ring [kW]	1200	7.3
peak luminos. [$10^{34} \text{ cm}^{-2}\text{s}^{-1}$]	30	5 (lev.)
events/bunch crossing	1000	132
stored energy/beam [GJ]	6.5	0.7
integr. luminosity / IP [fb^{-1}]	20000	3000

FCC-hh functional layout



Lecture 2 – FCC-ee optics

2.1 arc emittance

2.2 final focus

2.3 full ring optics

2.4 errors, DA, MA, BBA, etc.

2.5 beam-beam performance

2.1 arc emittance

equilibrium energy spread and bunch length:

$$\sigma_E \approx \sqrt{E_0 u_c}$$

$$\sigma_z \approx \frac{c \alpha_c}{\Omega_s} \frac{\sigma_E}{E_0}$$

Sands, 1970

equilibrium excitation & damping

horizontal equilibrium emittance
for bending angle θ per cell:

$$\varepsilon_x = \frac{C_q}{1 - \mathcal{D}} F_T \gamma^2 \theta^3$$

$$C_q = \frac{55 \hbar c}{32 \sqrt{3} m_e c^2} \approx 3.84 \times 10^{-13} \text{ m}$$

$$F_T = \frac{\rho^2}{l^3} \langle \mathcal{H} \rangle_{\text{dipole}}$$

$$\langle \mathcal{H} \rangle_{\text{dipole}} = \frac{1}{2\pi\rho} \oint (\gamma_x D_x^2 + 2\alpha_x D_x D'_x + \beta_x D_x'^2) ds$$

$$F_T \approx 2.5 L/l \text{ for 90 deg FODO cell}$$

$$\approx 0.1 \text{ for "useful \& realistic" TME cell}$$

vertical equilibrium emittance normally determined by spurious vertical dispersion and betatron coupling – intrinsic limit is set by $1/\gamma$ opening angle of the synchrotron radiation:

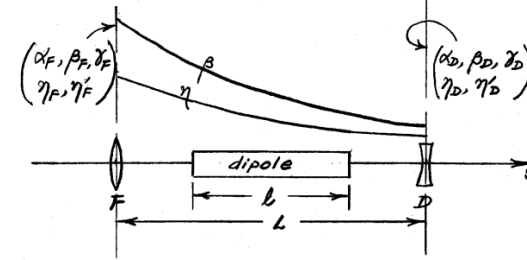
$$\varepsilon_y \approx C_q \beta_{\text{typical}} / \rho$$

$$\varepsilon_y = \frac{13}{55} C_q \frac{\oint \frac{\beta}{\rho^3} ds}{\oint \frac{1}{\rho^2} ds}$$

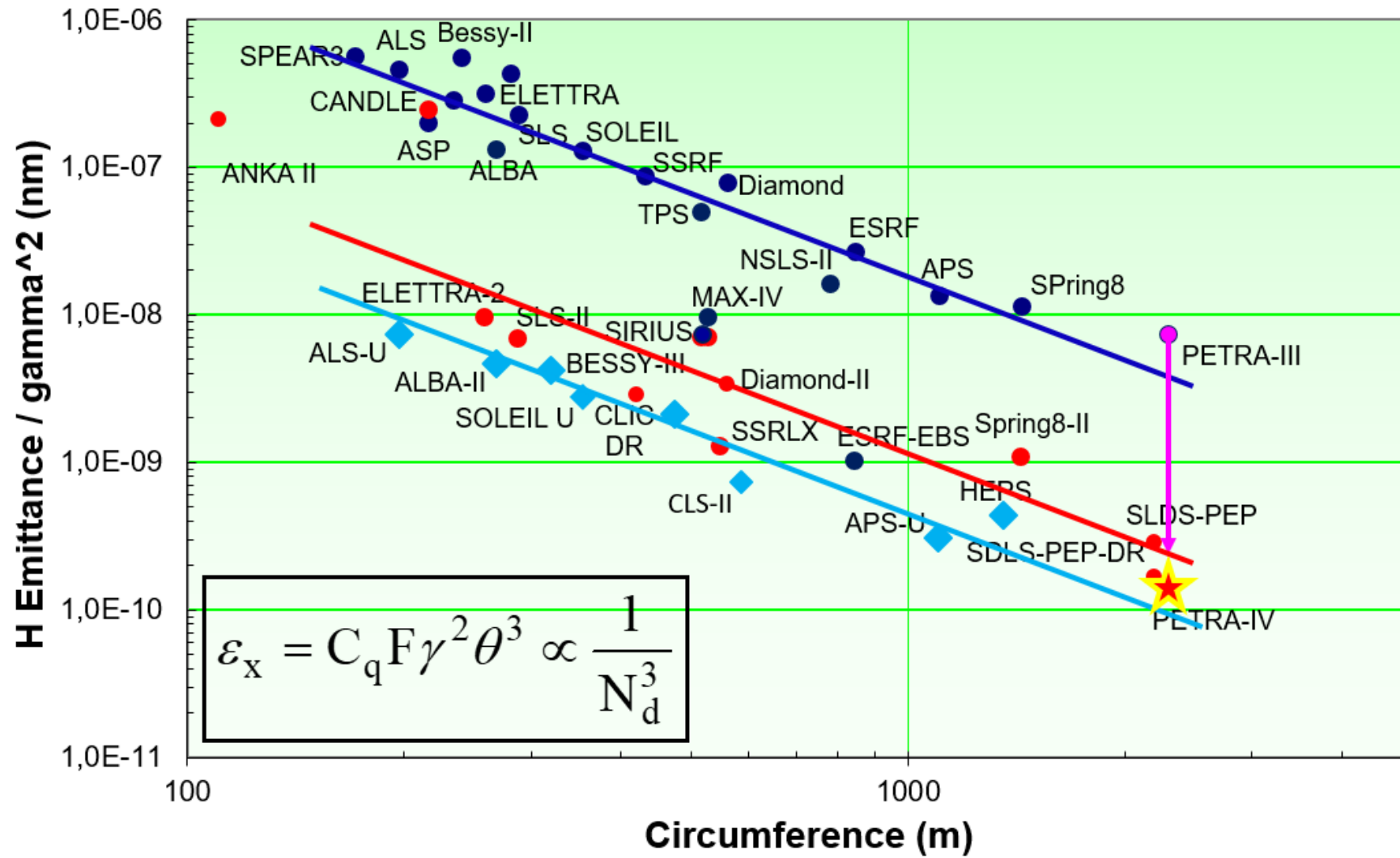
taking into account correlation
between photon emission
angle and energy

T. Raubenheimer, 1991
K. Hirata, 1993

Teng, TM-1269, 1984

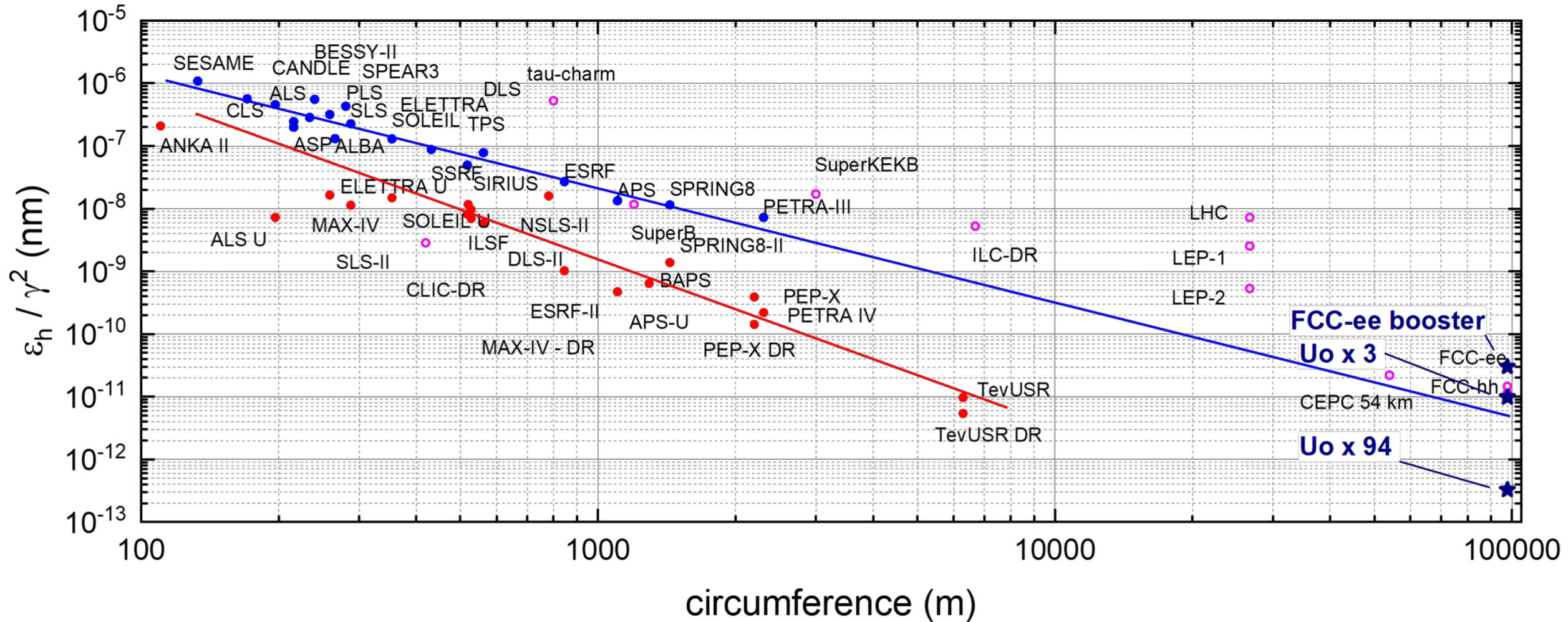


low-emittance rings – the state of the art



R. Bartolini --
iFAST Annual Meeting,
Trieste, April 2023

low-emittance rings – bring on the FCC-ee !



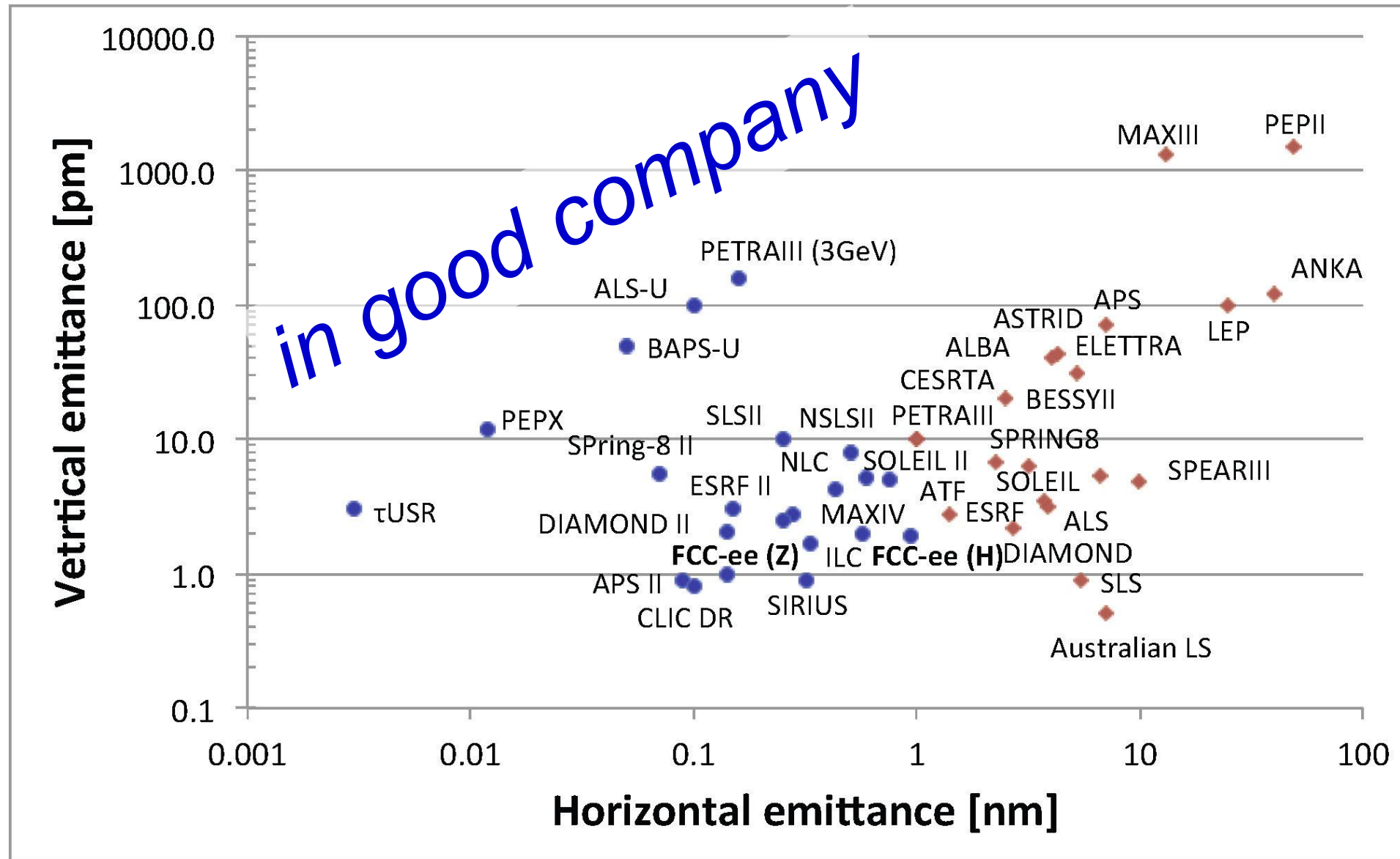
ring equilibrium emittance

$$\varepsilon_x \sim 10^{-12} \left(\frac{l_b}{\rho} \right)^3 \gamma^2 \text{ [m]}$$

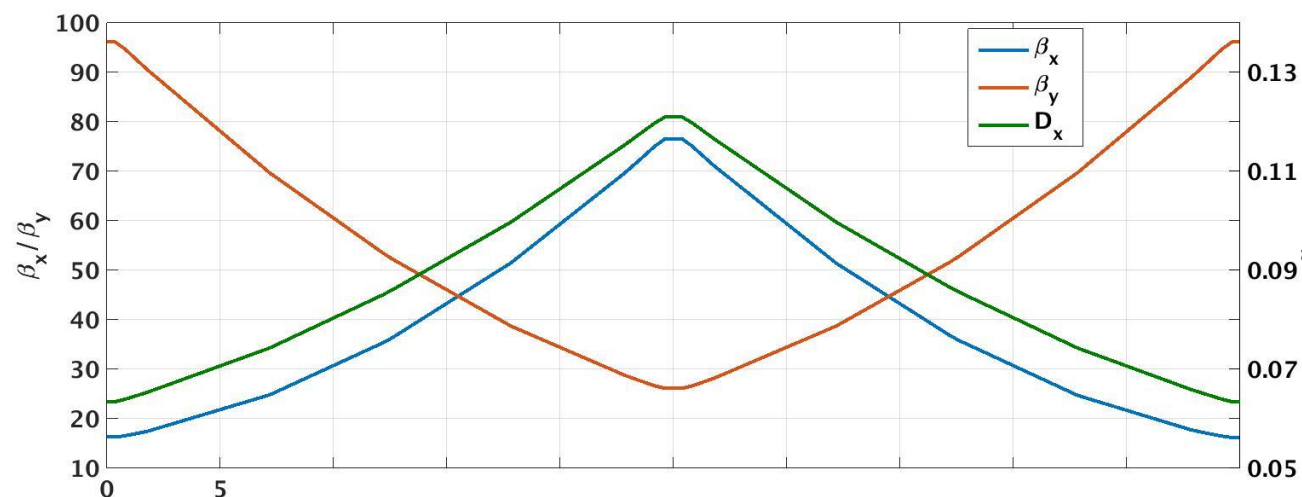
	bending radius ρ [km]	beam energy [GeV]	γ	l_b (~1/2 cell length)	ε_x [nm]
LEP2	3.1	104	2.0×10^5	39.5	22
FCC-ee-Z	10	45.6	8.9×10^4	~46	0.7
FCC-ee-H	10	120	2.4×10^5	~20	0.66
FCC-ee-t	10	182.5	3.6×10^5	~20	1.5

we can either use 90 degree FODO optics and half the cell length going to Higgs or higher energy,
or we can change the phase advance per cell

FCC-ee also needs a small vertical emittance ~ 1 pm



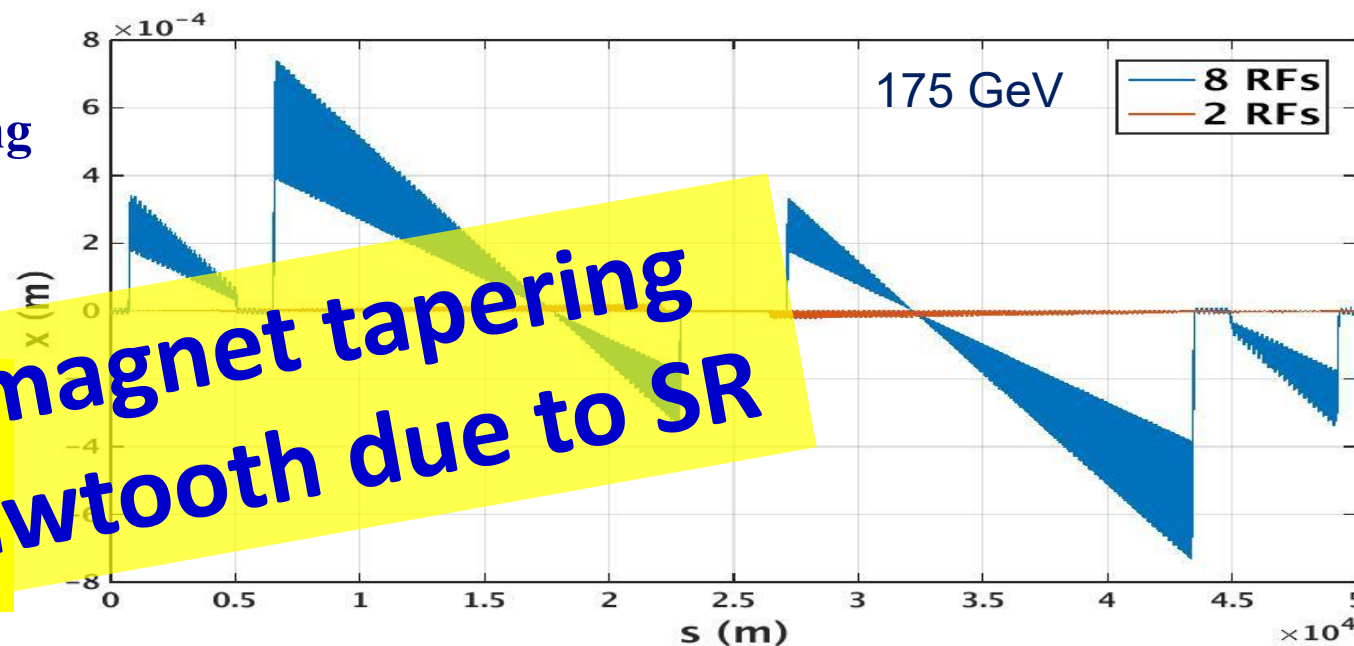
arc optics & sawtooth tapering



arc FODO
cell length
50 m

(LEP: 79 m)

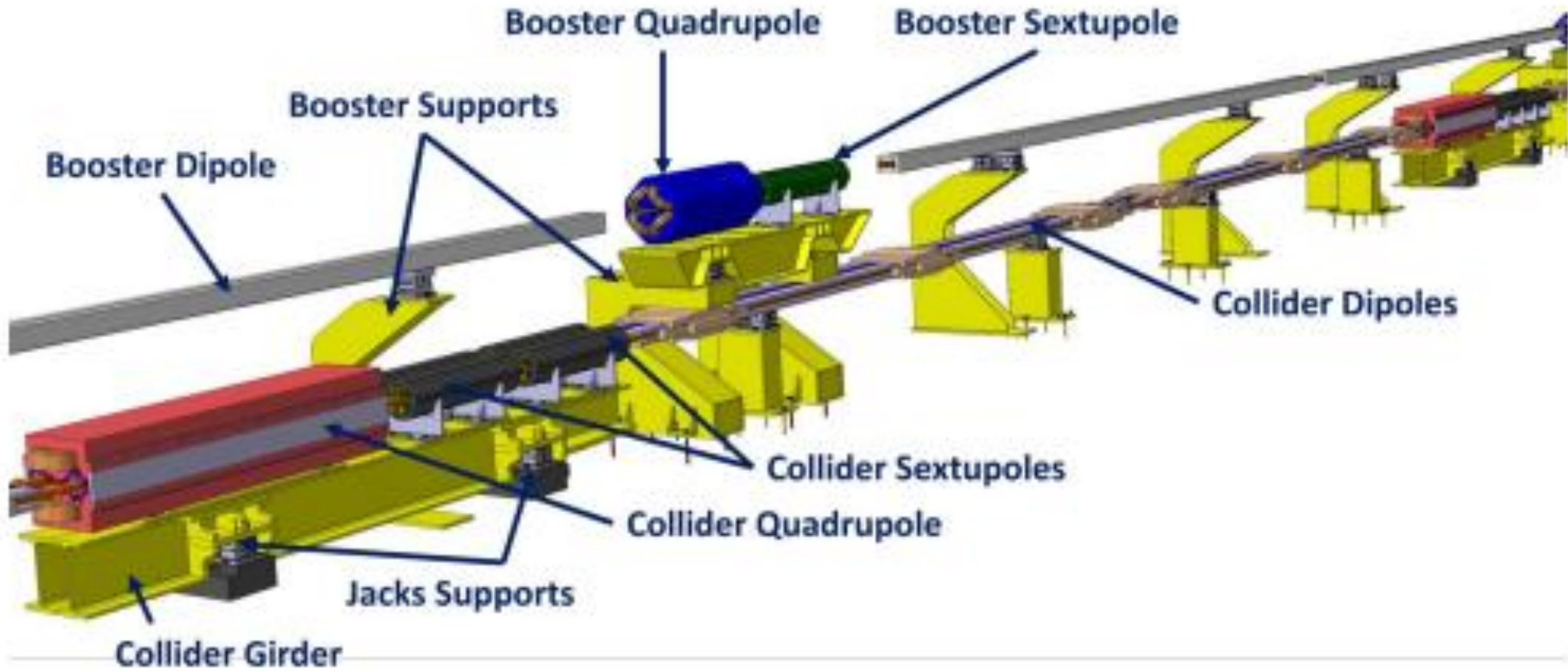
Comparison:
8 RF w/o tapering
2 RF with
tapered dipoles



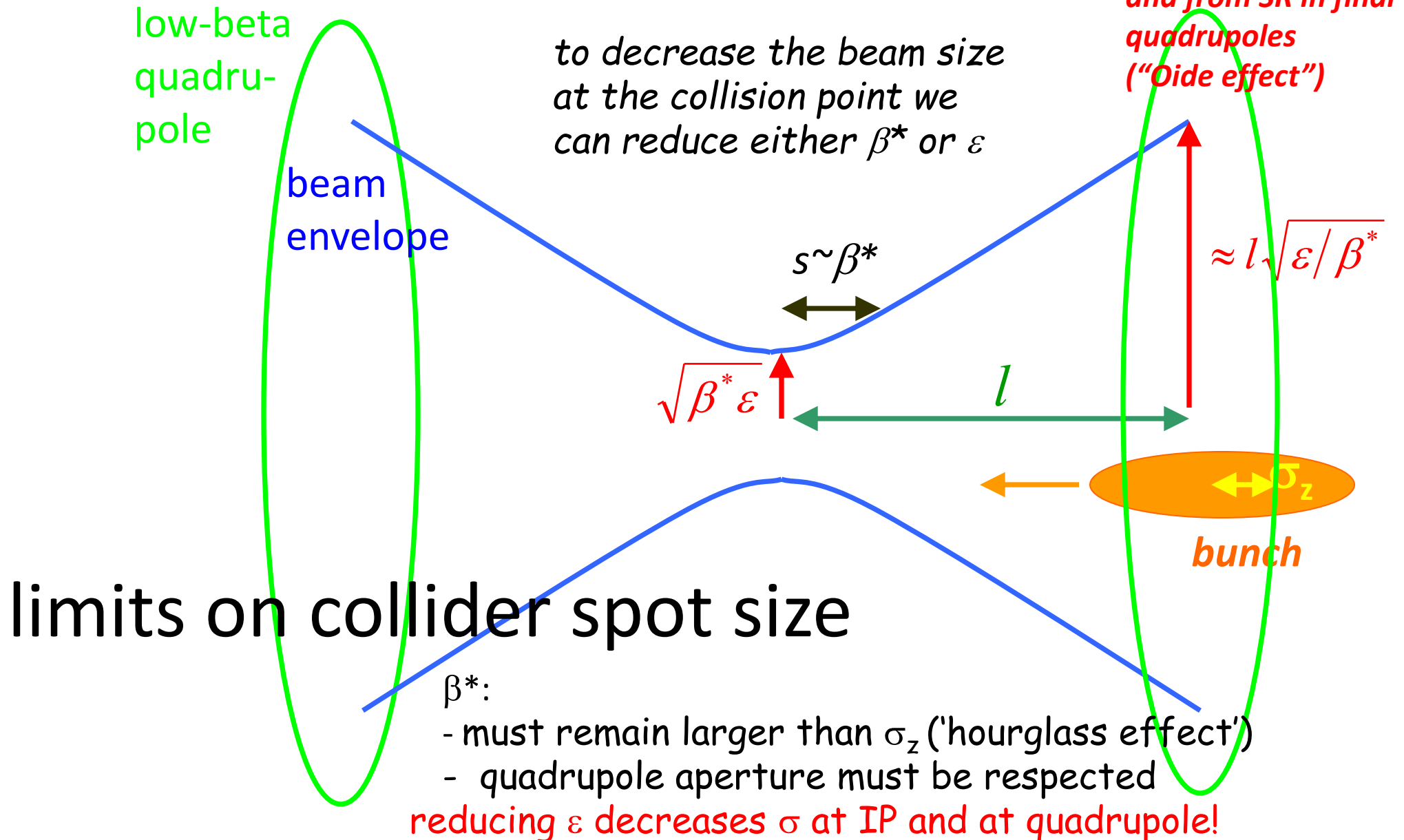
double ring and magnet tapering
remove energy sawtooth due to SR

Sandra Aumonier
Pascale Andrei
Andreas Doblhammer
Bernhard Wille

arc half cell



2.2 final focus



Machine-Detector Interface

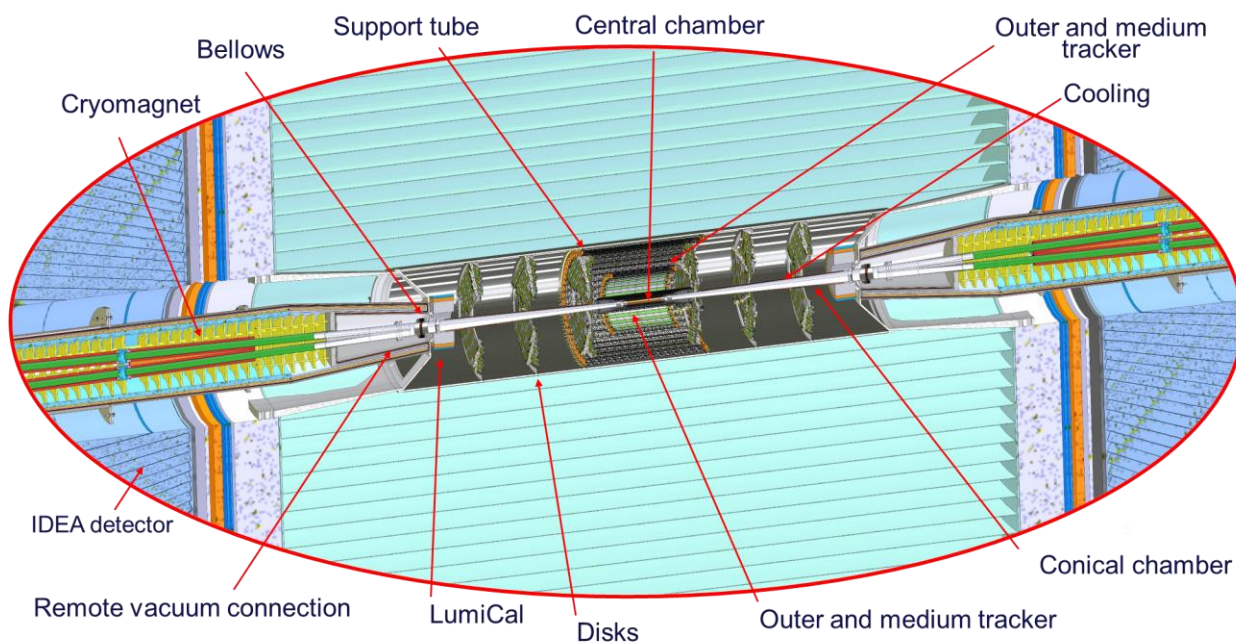
Key topics:

SC IR magnet system
& Cryostat design

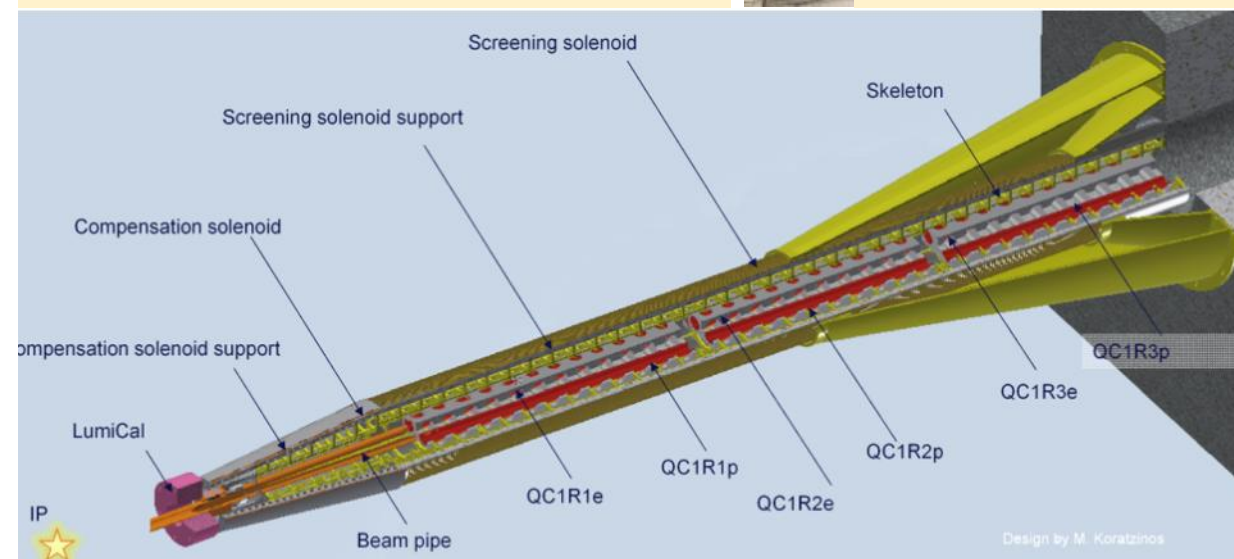
3D integration

IR mock-up / INFN

Machine		FCCee	CEPC	ILC	SuperKEKB
Crossing-angle	mrad	30	33	14	83
L*	m	2.2	1.9	3.5	0.935
Vertical β_y^* at IP	mm	0.7-1.6	0.9-2.7	0.4	0.3
Detector soln field	T	2/3	3	3.5/5	1.5
Detector stay clear	mrad	100	118/141	90	350/436
Two beam ΔX at L*	mm	66	62.7	49	77.6
He temperature	K	1.9	4.2	4.5	4.5

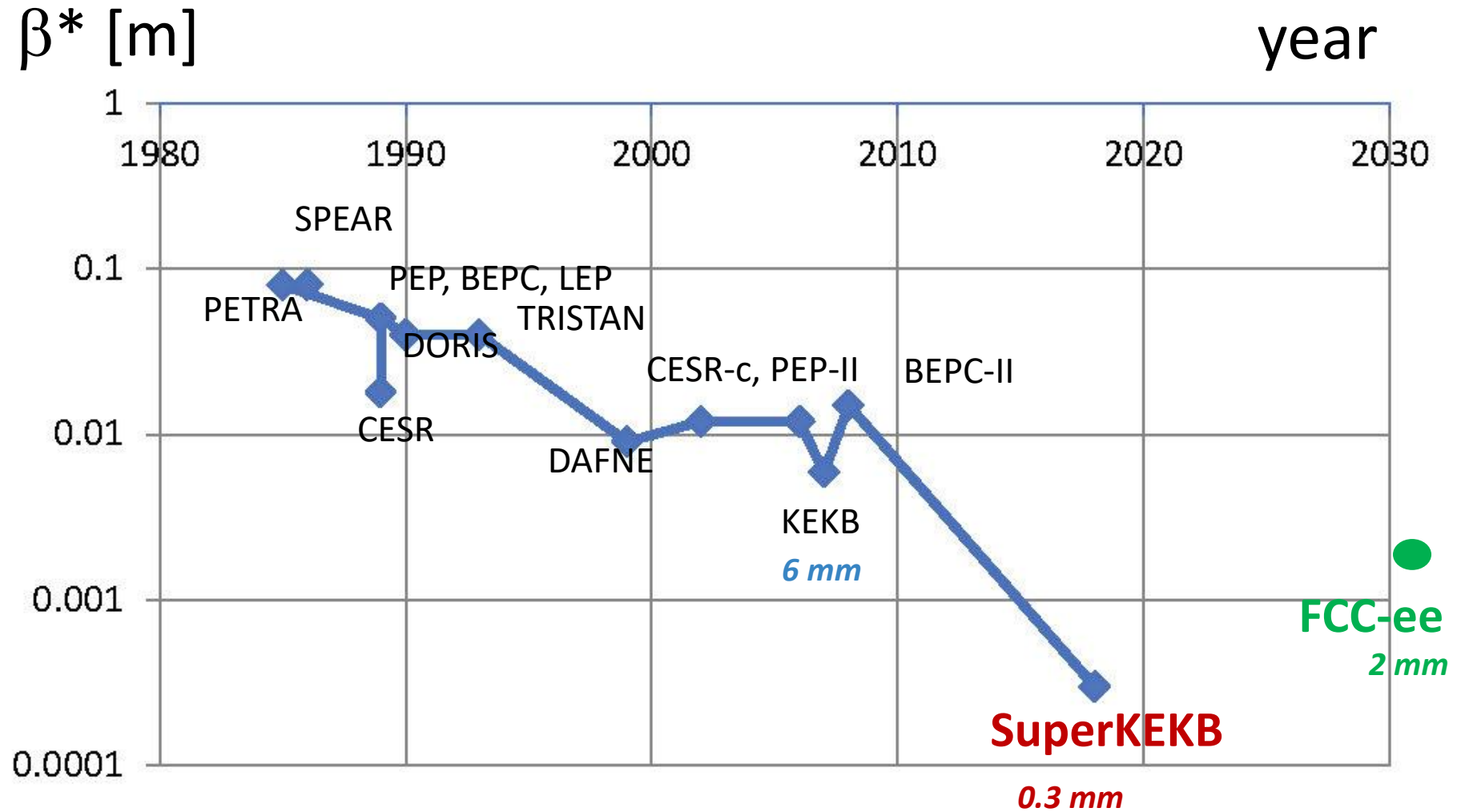


J. Seeman, A. Novokhatski, SLAC
B. Parker, Vikas Teotia, BNL
P. Tavares, CERN



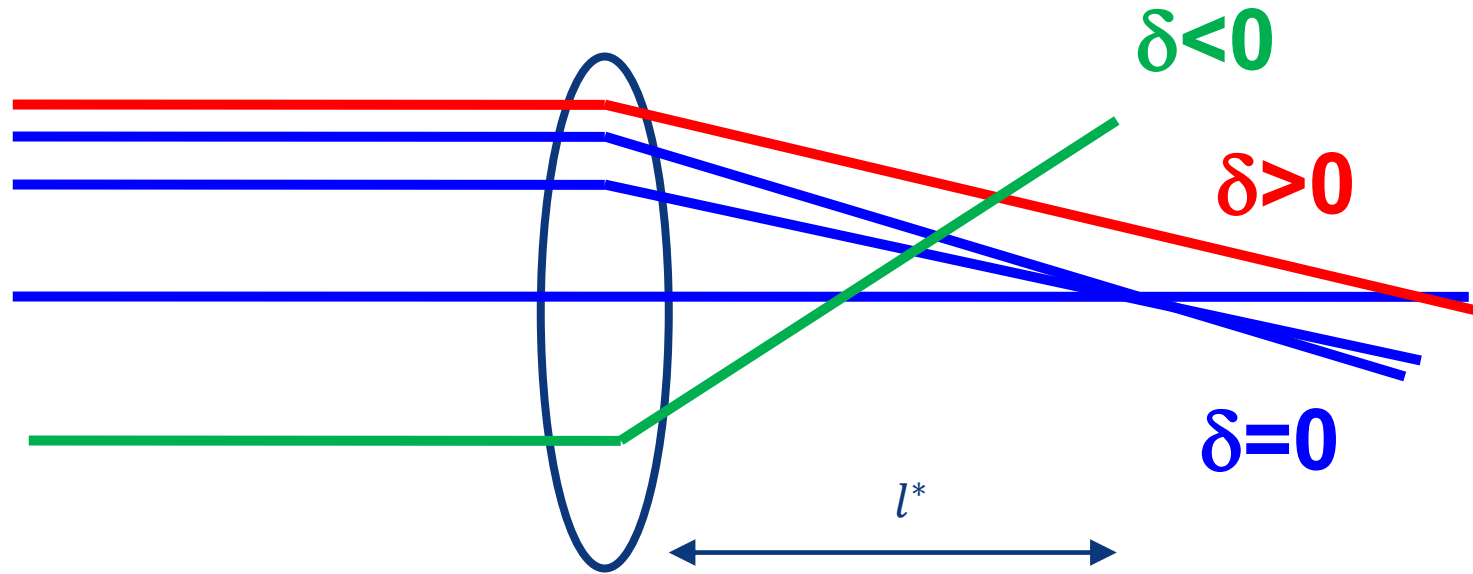
M. Boscolo, F. Palla, INFN

β_y^* evolution over 40 years



entering a new regime for ring colliders –
SuperKEKB will pave the way towards $\beta^* \leq 2$ mm

final focus chromaticity



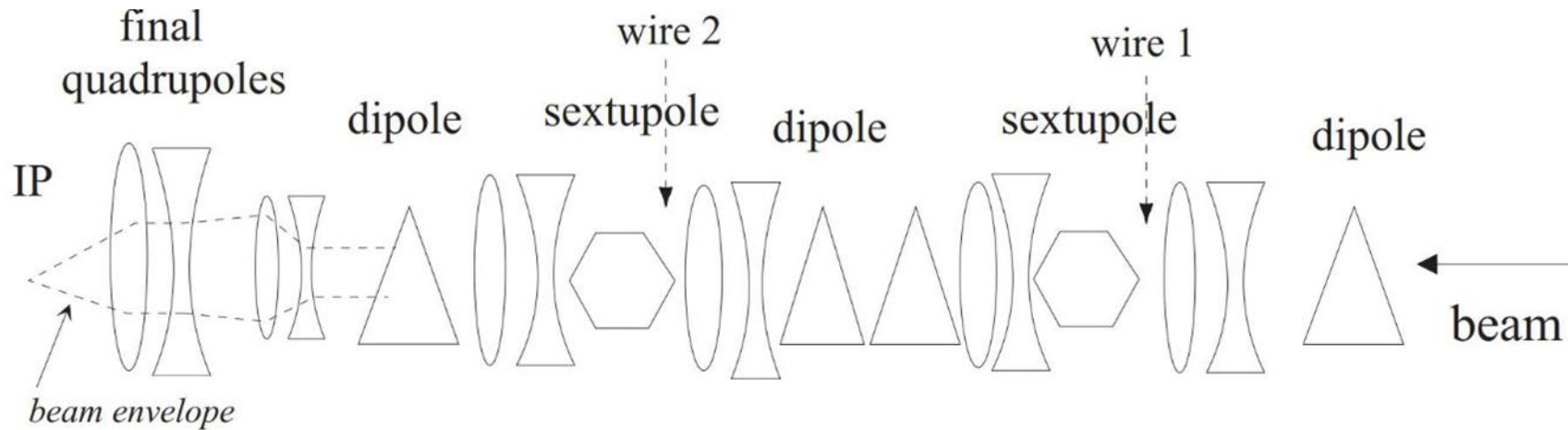
$$\frac{\Delta\sigma_y^*}{\sigma_{y0}^*} = \xi \delta_{rms}$$

$$\sigma_{y0}^* \equiv \sqrt{\beta_y^* \epsilon_y}, \quad \xi \approx \frac{l^*}{\beta^*}$$

spot size increase due to
(uncorrected) chromaticity,

schematic of a final focus system

sextupole magnets at location with nonzero dispersion
to correct the chromaticity of the final quadrupoles

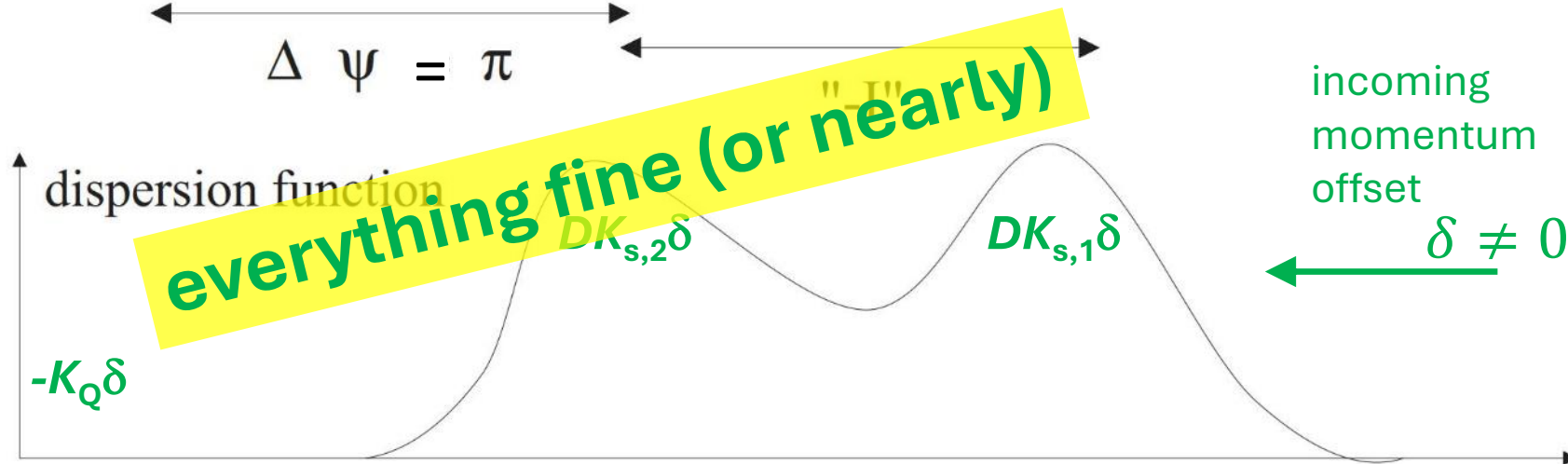
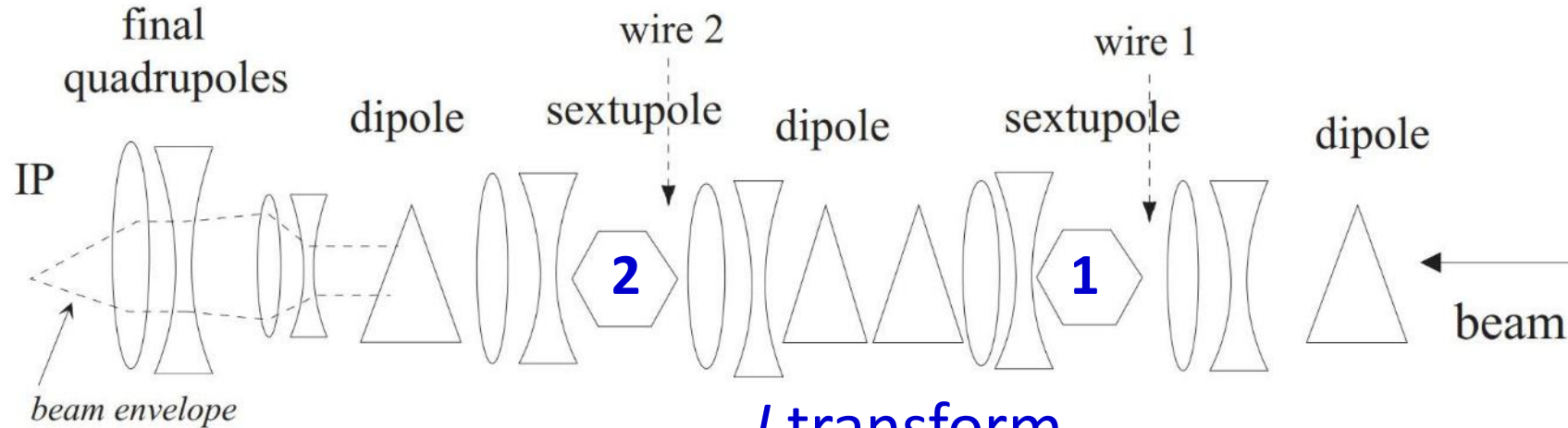


IP

final doublet

dipoles,
quadrupoles,
sextupoles, ...

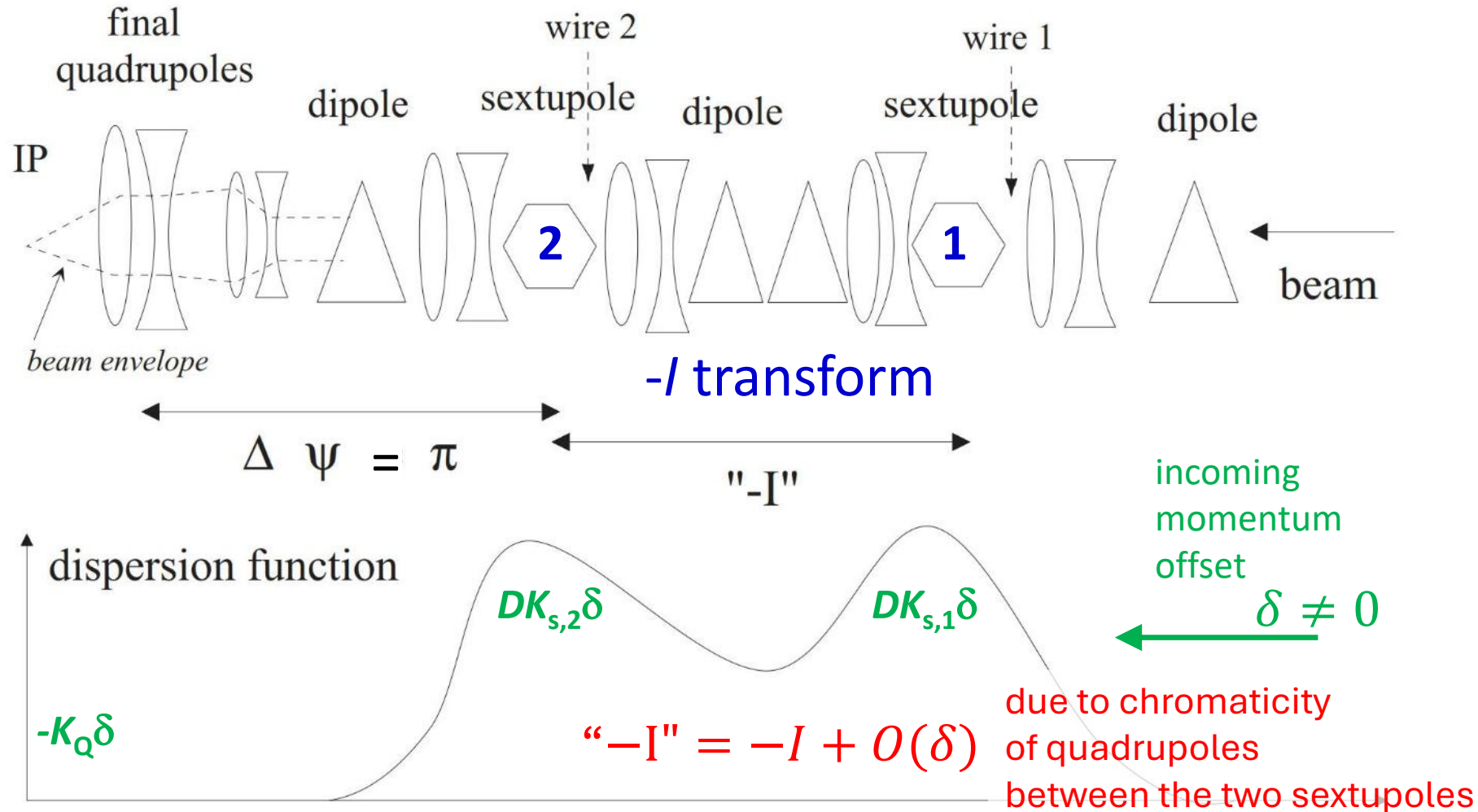
chromatic correction



$$x'_2 = -x'_\beta - \frac{1}{2}K_{s,1}(x_\beta^2 + 2x_\beta D_i \delta + D_i^2 \delta^2) + \frac{1}{2}K_{s,2}(x_\beta^2 - 2x_\beta D_i \delta + D_i^2 \delta^2)$$

$$K_{s,1} = K_{s,2}: \quad x'_2 = -x'_\beta - 2K_s x_\beta D_i \delta.$$

one problem: chromatic breakdown of -/



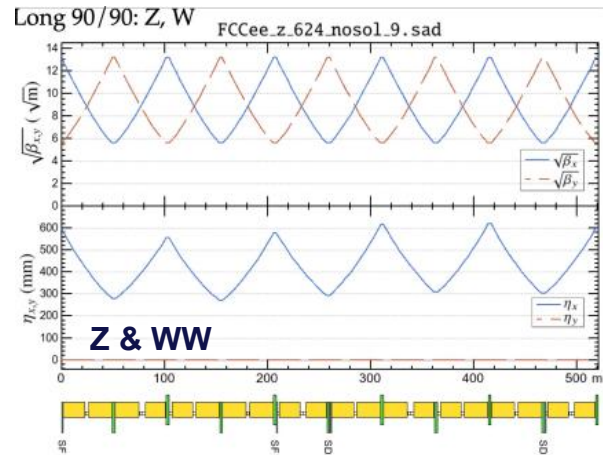
$$x'_2 = -x'_\beta - \frac{1}{2}K_{s,1}(x_\beta^2 + 2x_\beta D_i \delta + D_i^2 \delta^2) + \frac{1}{2}K_{s,2}(x_\beta^2 - 2x_\beta D_i \delta + D_i^2 \delta^2)$$

$$K_{s,1}=K_{s,2}: \quad x'_2 = -x'_\beta - 2K_s x_\beta D_i \delta + c_1 x x' \delta + c_2 x^3 \delta^2 + \dots$$

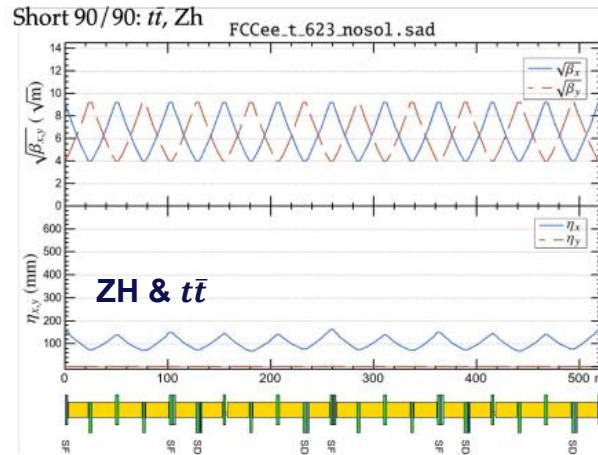
2.3 full ring optics

FCC-ee optics baseline **GHC**

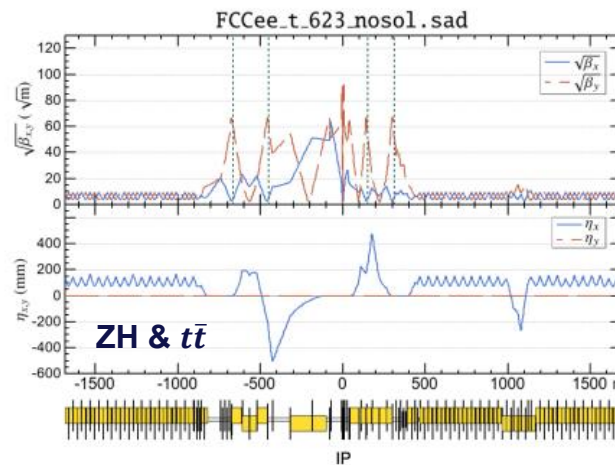
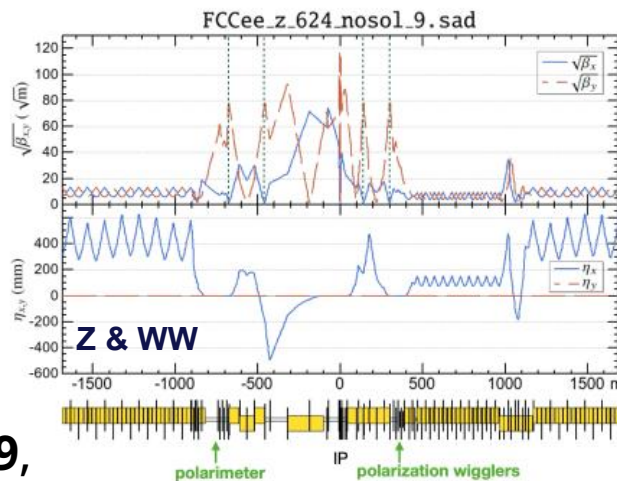
arc



K. Oide et al.



experimental straight

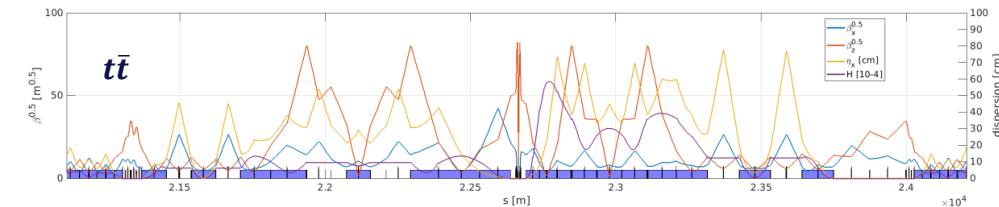
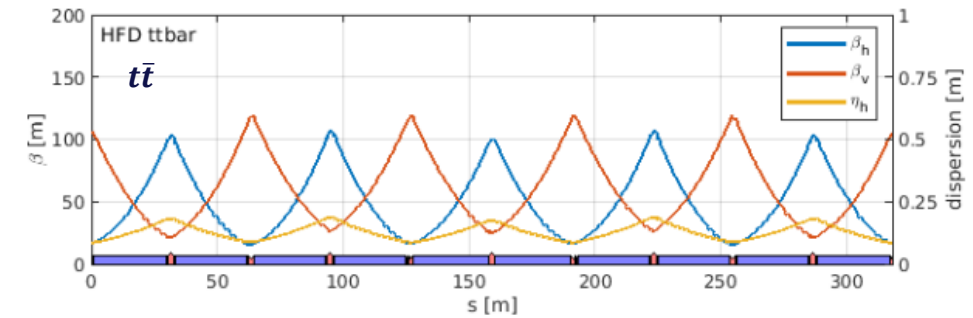


PRAB 19,
111005 (2016)

→ **March 2026: Decision on Optics**

alternative LCC

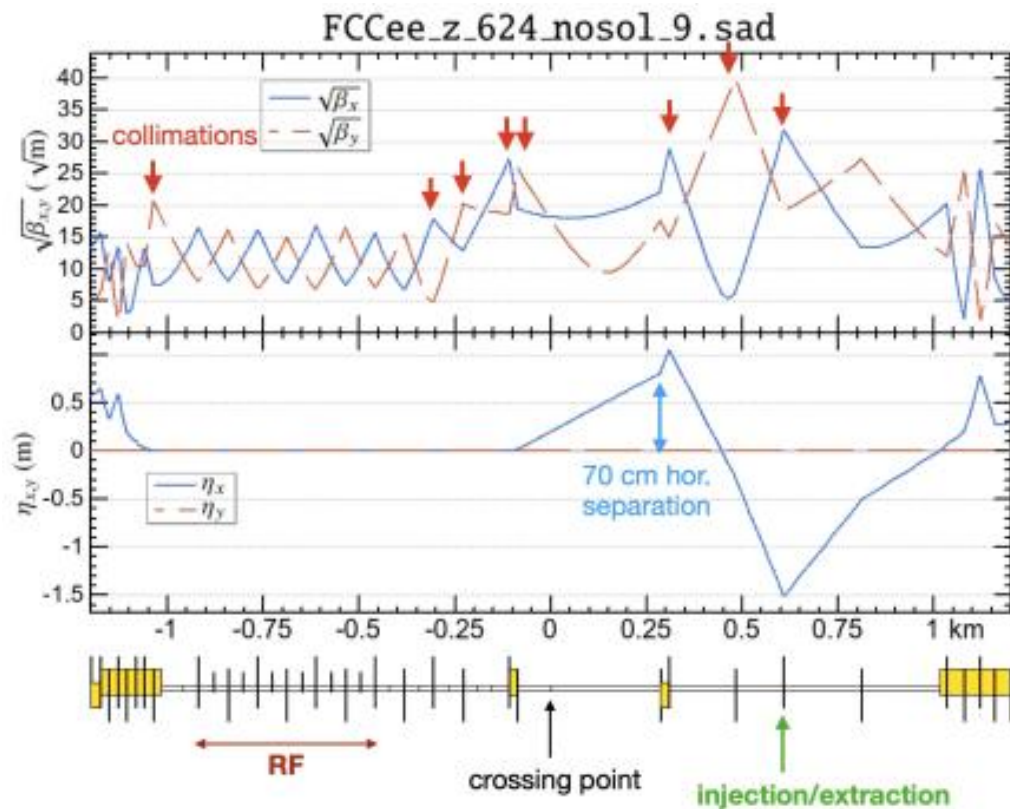
P. Raimondi, S. Liuzzo, et al.



PRAB 28,
021002 (2025)

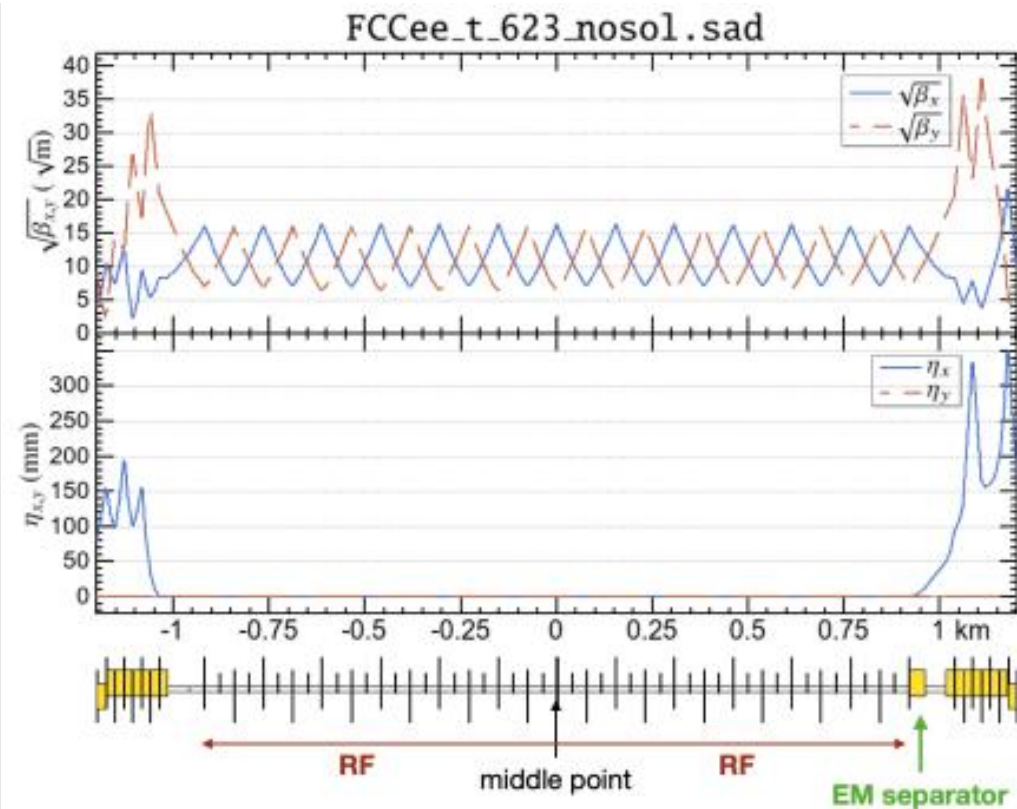
GHC baseline optics: technical straight

Universal optics for all technical insertions in Z/W operation: RF, injection/extraction, & collimation



Maintaining perfect superperiodicity

Optics for the common RF section in ZH/ttbar operation

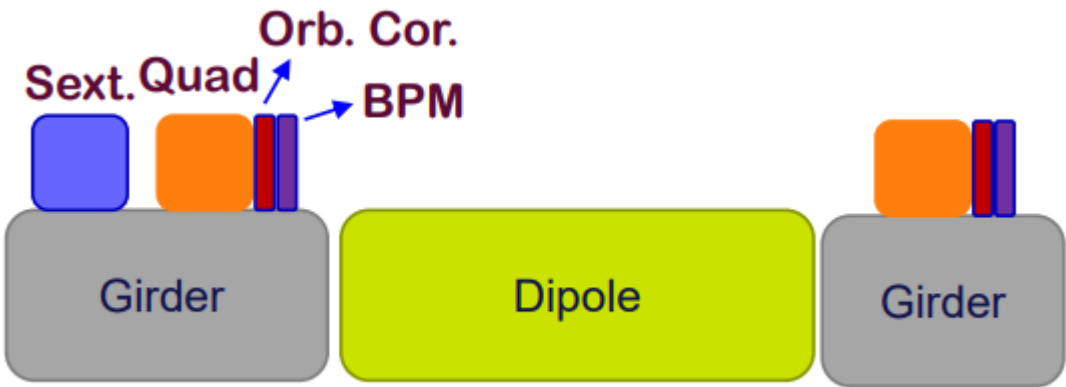


FCC design with Xsuite

<https://xsuite.readthedocs.io/en/latest/>



2.4 errors etc.



Arc alignment and strength tolerances. These values correspond to one sigma Gaussian distribution truncated at 2.5 sigma. Quadrupoles and sextupoles are placed on top of common girders.

Element	$\sigma_{x/y}$ [μm]	$\sigma_{\theta/\psi/\phi}$ [μrad]	$\Delta k/k$ [10^{-4}]
Arc quads & sext.	50	50	2
Dipoles	1000	1000	2
Girders	150	150	-
BPMs-to-quad	100	-	-

Arc BPM performance specifications.

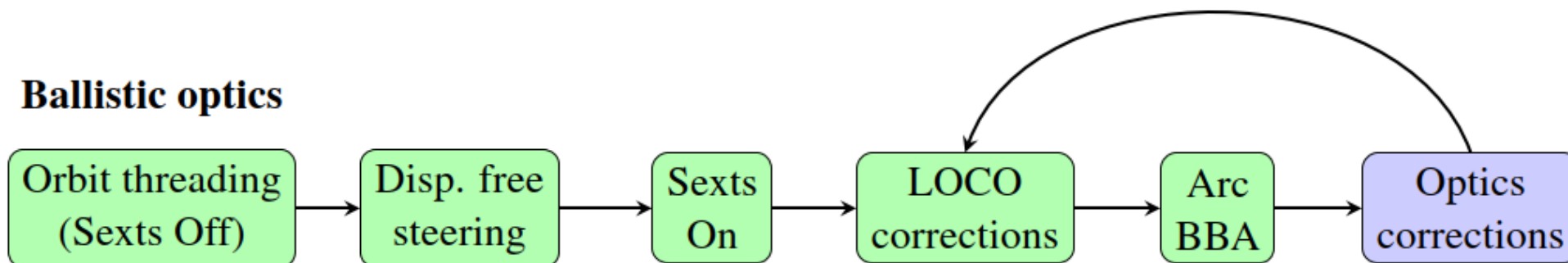
Closed orbit resolution	0.1 μm
Turn-by-turn (TbT) position resolution	1 μm
Number of turns in TbT mode	50000

Location of arc magnet correctors and BPMs.

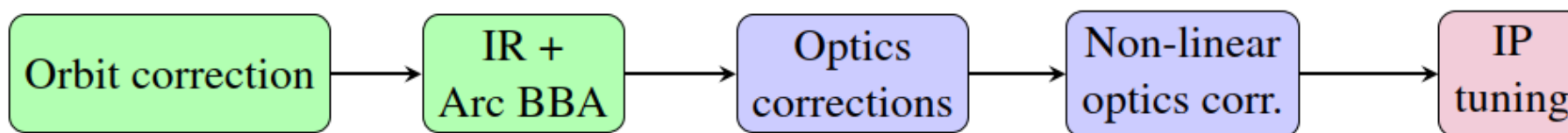
Device	Location
Horizontal orbit corrector	Embedded at the edge of main dipole next to main quadrupole
Vertical orbit corrector	Embedded at the sextupole or stand-alone
Quadrupolar corrector	Trim coil in all main quadrupoles
Skew quadrupole	Embedded at the sextupole
BPM (H & V)	Attached to the main quadrupole

Optics commissioning sequence

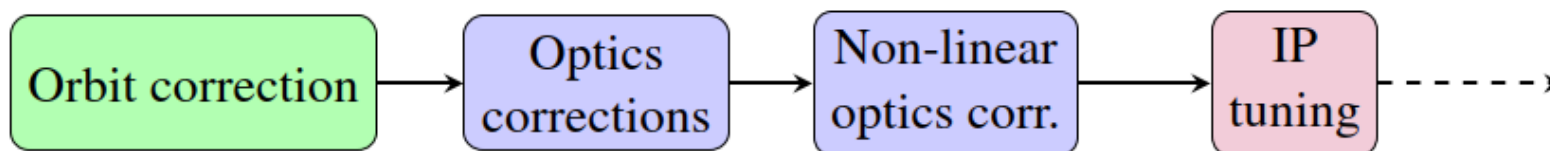
Ballistic optics



Relaxed optics (various iterations decreasing β^*)

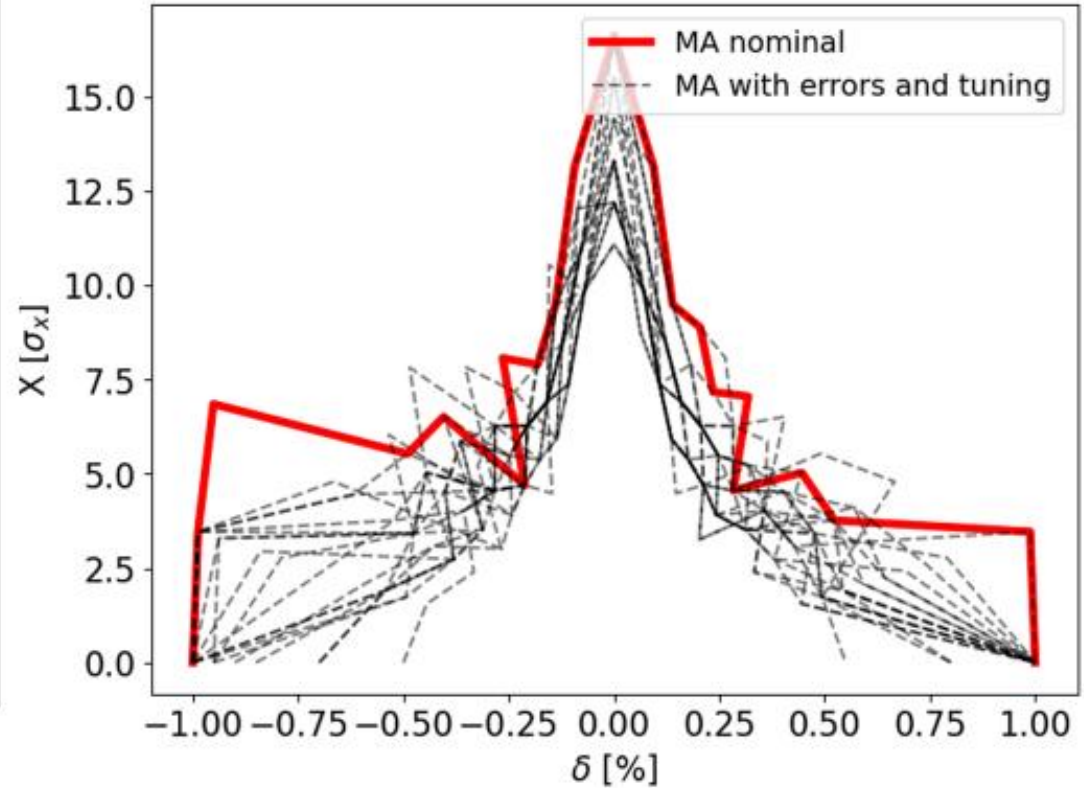
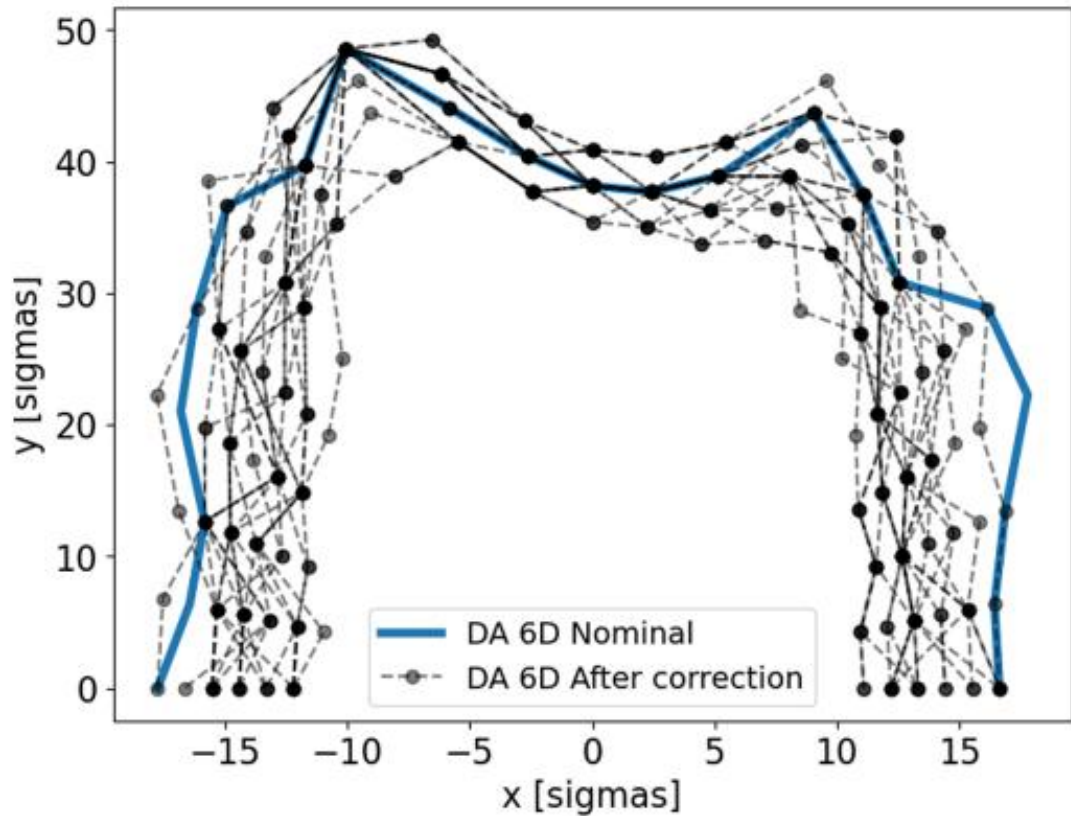


Baseline optics



Steps during the FCC-ee optics commissioning starting with the most relaxed optics, the ballistic optics.

Simulated performance of baseline optics with errors

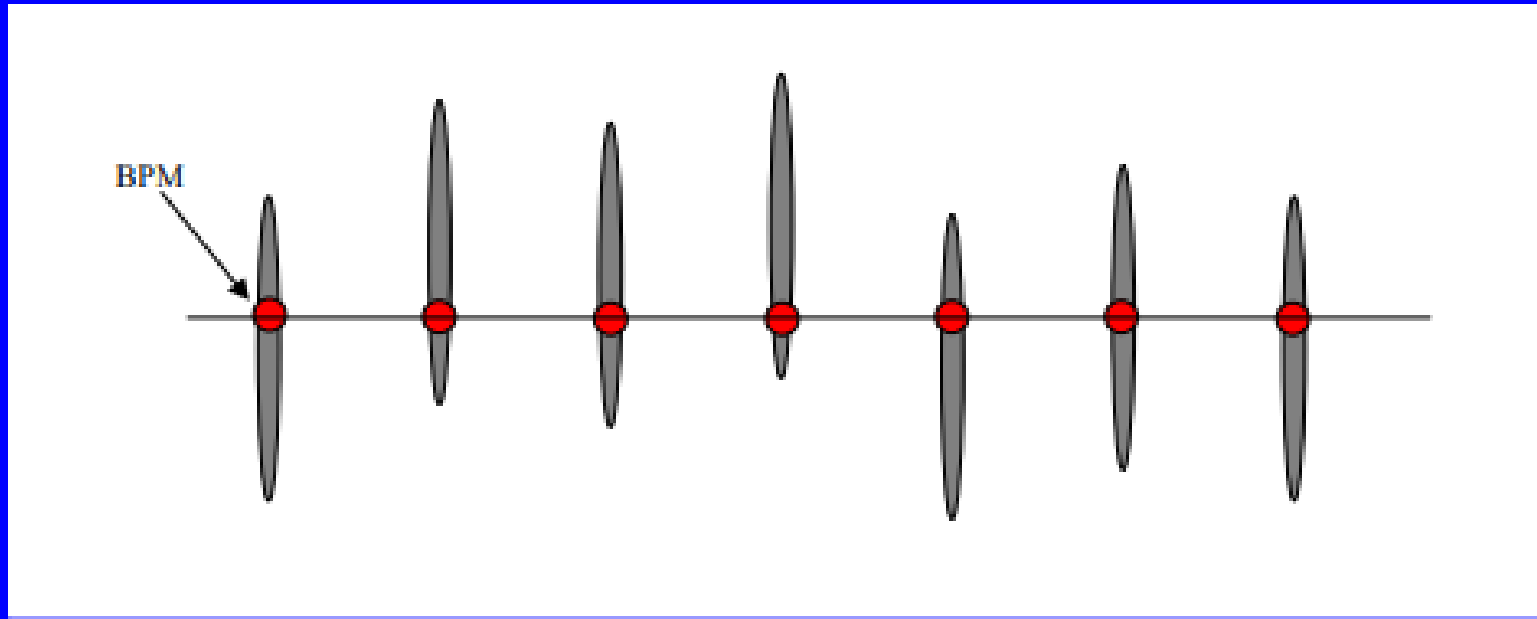


Dynamic Aperture (left) and Momentum Acceptance (right) after linear optics corrections having used the errors described before.

Median rms values of several optics parameters before (after sextupole ramping) and after linear optics correction (nominal lattice). $\Delta\tilde{E}$ stands for phase advance deviations between nearby BPMs.

Parameter	Before correction (rms)	After correction (rms)
horizontal orbit (μm)	120.2	120.5
vertical orbit (μm)	217.5	217.6
$\Delta\beta_x/\beta_x$ (%)	7.41	0.29
$\Delta\beta_y/\beta_y$ (%)	15.79	2.81
ΔD_x (mm)	57.79	0.28
ΔD_y (mm)	62.24	2.80
ε_h (nm)	0.72	0.71
ε_v (pm)	26.01	0.57
horiz. $\Delta\psi$ [2π]	1.1×10^{-2}	2.9×10^{-4}
vert. $\Delta\psi$ [2π]	1.9×10^{-2}	2.3×10^{-3}
Re f_{1001}	4.9×10^{-2}	1.7×10^{-4}
Im f_{1001}	4.4×10^{-2}	5.2×10^{-5}
Re f_{1010}	3.7×10^{-2}	1.3×10^{-4}
Im f_{1010}	3.7×10^{-2}	1.3×10^{-4}

Scenario 1: Quad offsets, but BPMs aligned



Assuming:

- a BPM adjacent to each quad

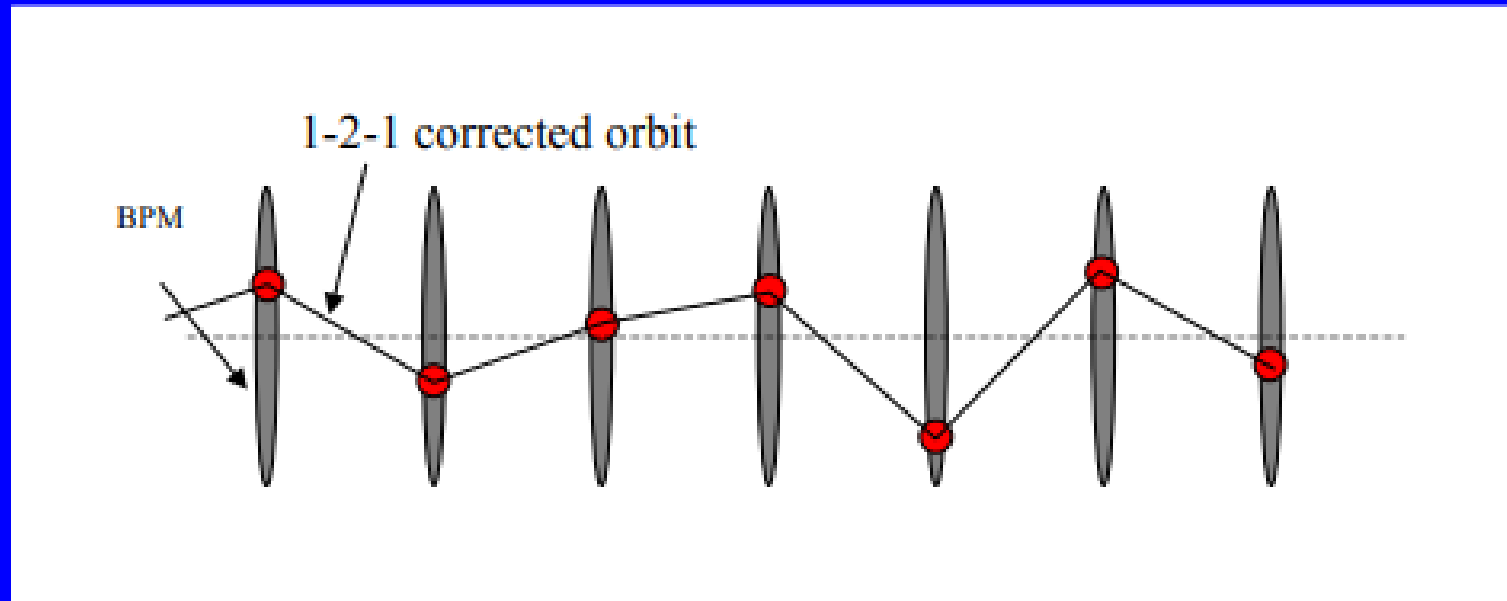
- a 'steerer' at each quad

simply apply one to one steering to orbit

steerer { quad mover
dipole corrector

N. Walker,
USPAS 2003,
Santa Barbara

Scenario 2: Quads aligned, BPMs offset



one-to-one correction BAD!

Resulting orbit not Dispersion Free \Rightarrow emittance growth

Need to find a steering algorithm which effectively puts
BPMs on (some) reference line **beam-based alignment**
followed by dispersion-free steering

real world scenario: some mix of scenarios 1 and 2

N. Walker,
USPAS 2003,
Santa Barbara

Beam-based alignment (BBA)

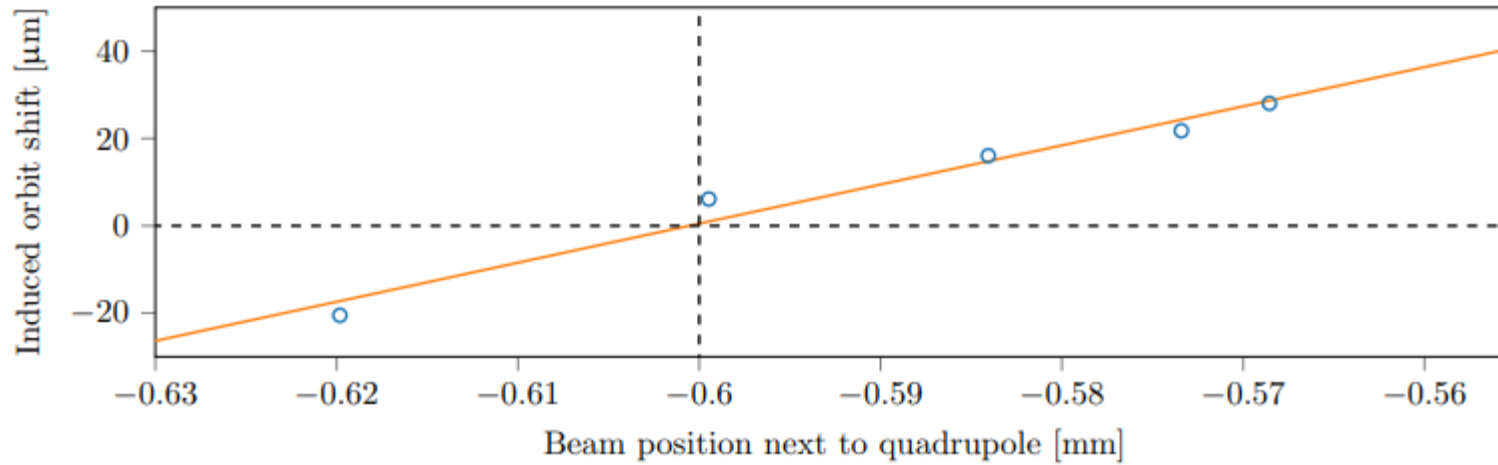
BBA can be performed after the sextupoles have been switched on

sheer size of the FCC → parallel approaches are explored, aimed at achieving approximately 10 to 20 μm effective alignment after BBA

Using Parallel Quadrupole Modulation System (PQMS), a technique which has already successfully been tested at SLAC/SPEAR, is applied to the FCC-ee.

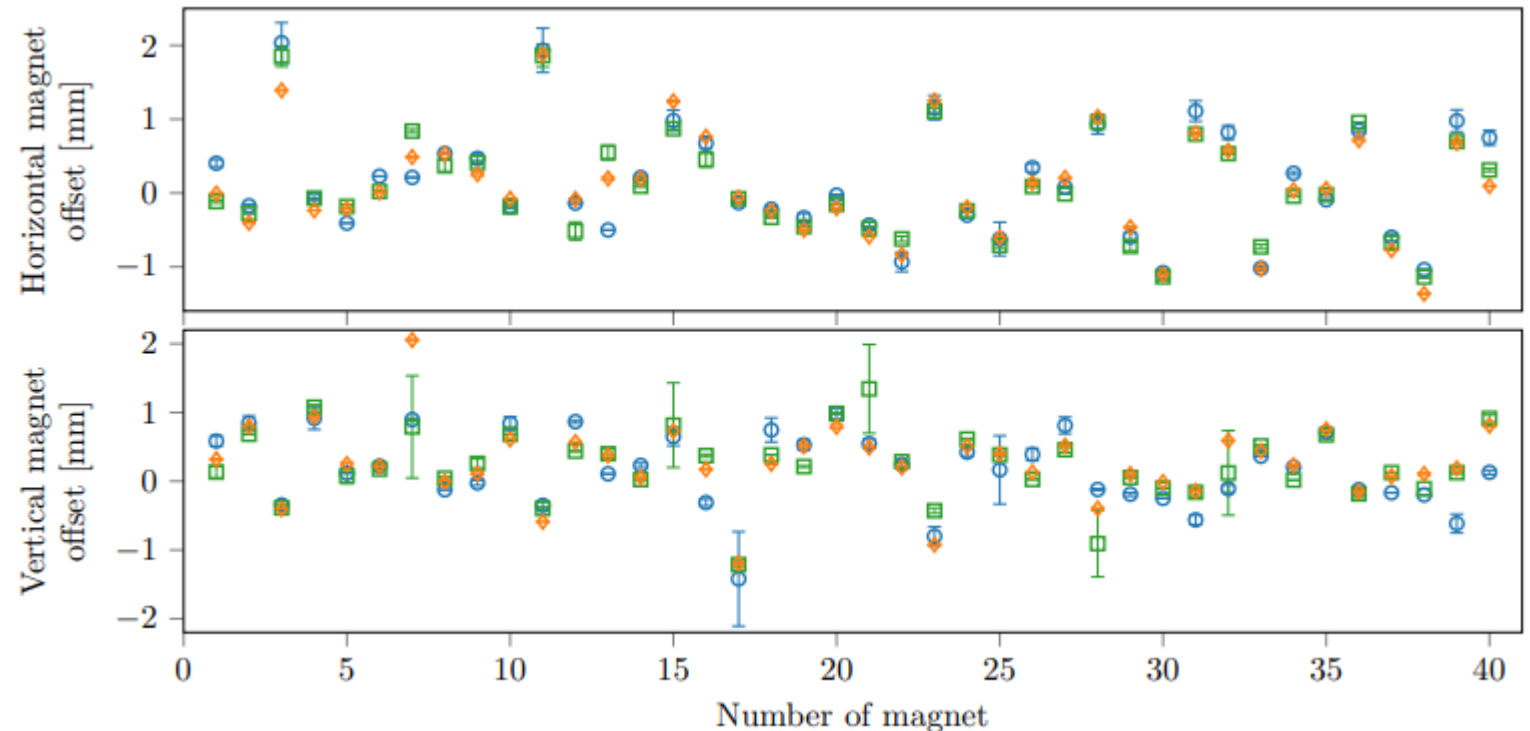
Modulating 10 quadrupoles in parallel with a $\Delta K/K$ of 2%, distributed equally over one arc, and using a calibrated lattice with 1 μm BPM resolution, an accuracy below 20 μm for vertical and horizontal arc quadrupole BBA is expected.

BBA tests for FCC-ee at KIT/KARA in Karlsruhe



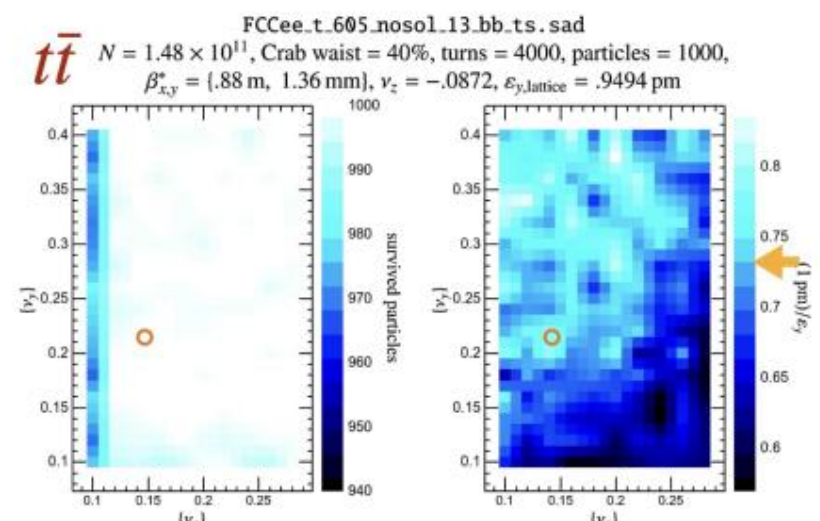
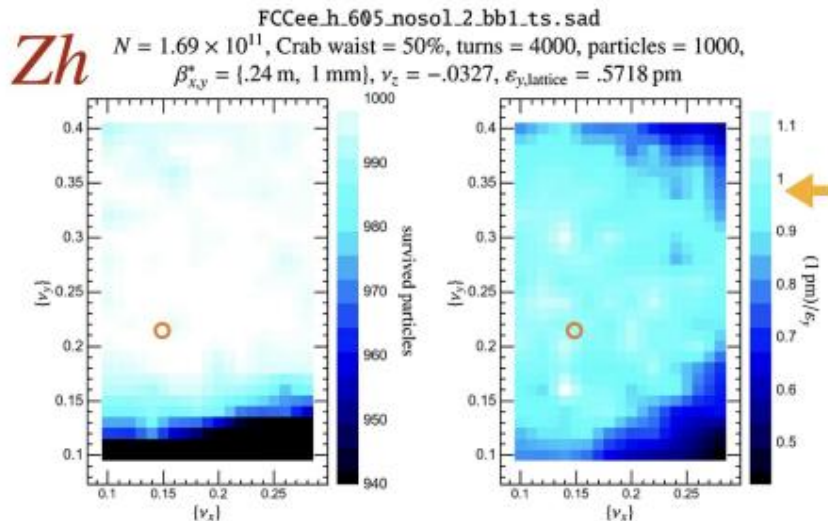
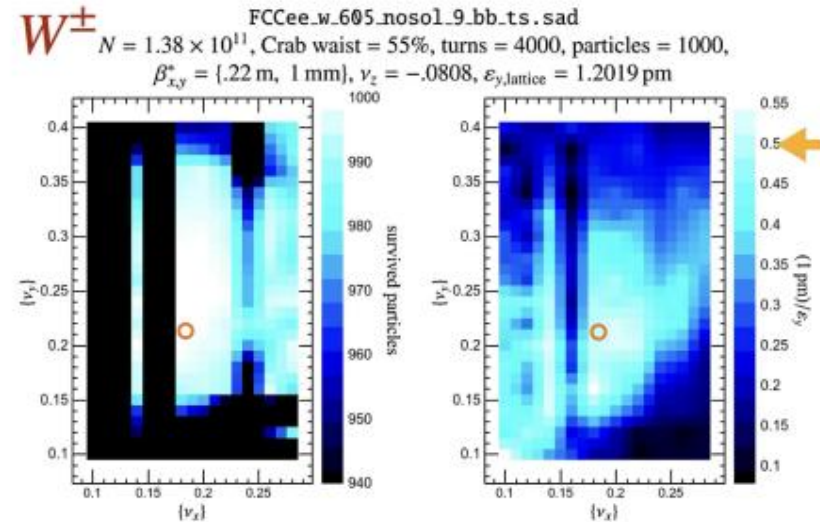
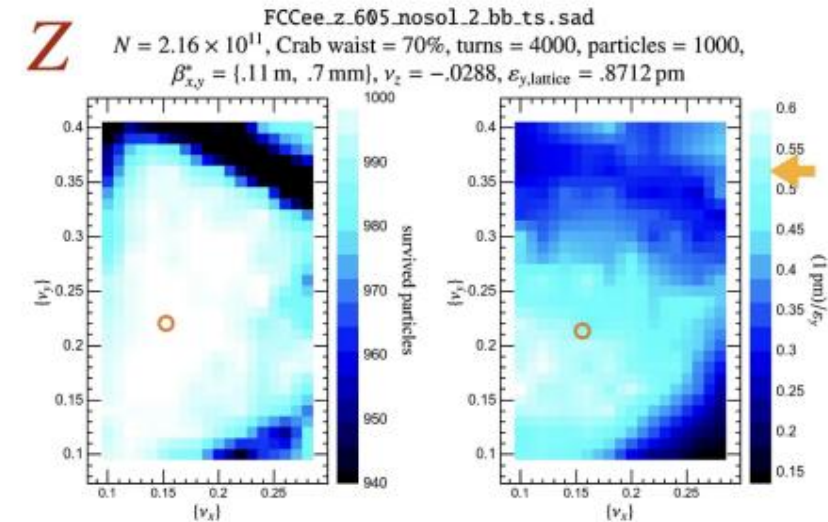
Dependence of measured induced orbit shift on beam position close to quadrupole.

Estimated magnet offsets with respect to closest BPM using linear (blue), rms (orange) and parallel BBA (green).



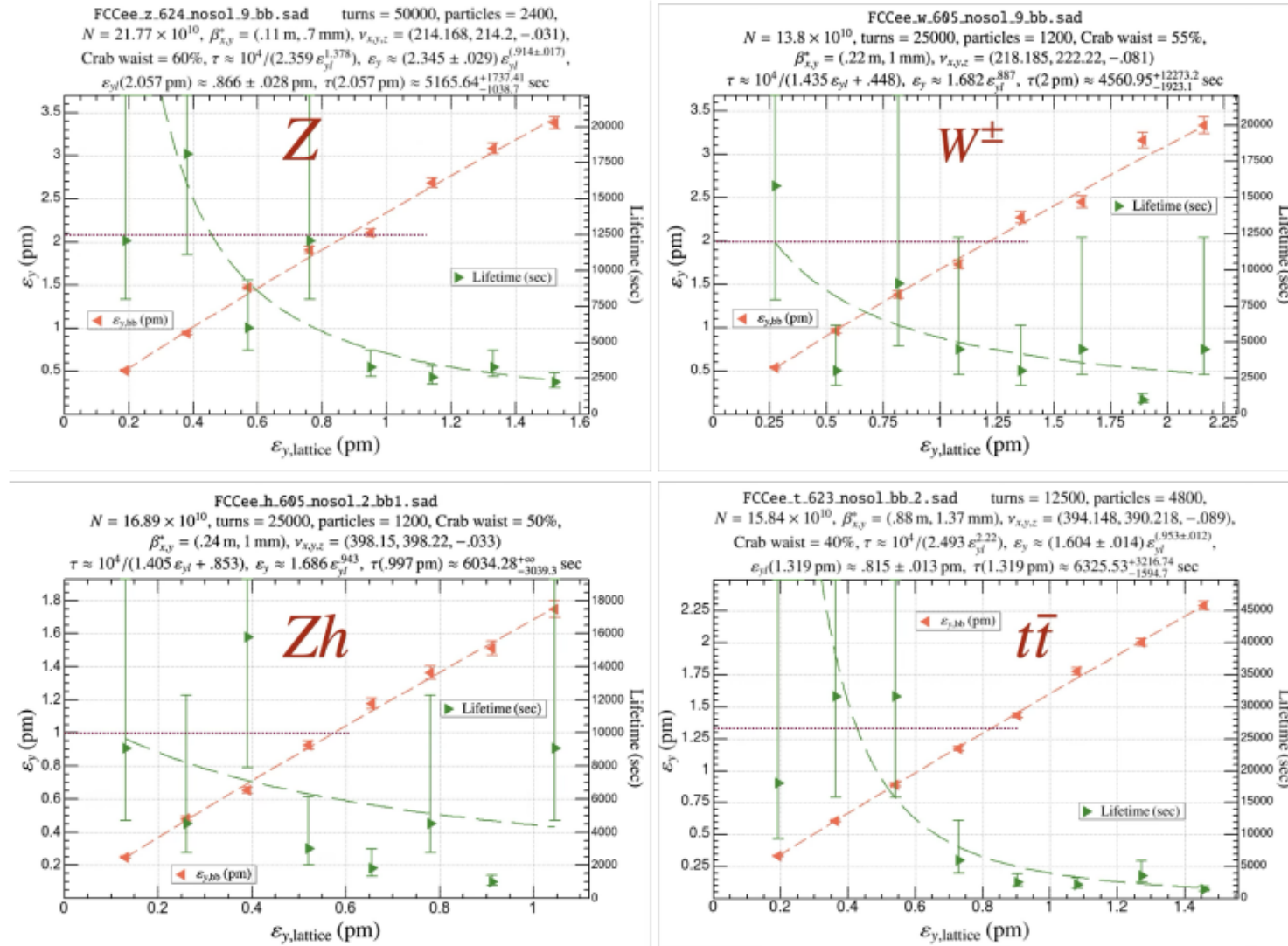
2.5 beam-beam performance

GHC optics



tune scan of the beam-beam effect with the full lattice

vertical bare lattice emittance required with beam-beam



results of beam-beam tracking with lattice and beamstrahlung for each energy

further reading

M. Sands, The Physics of Electron Storage Rings, SLAC Report 121 (1970)

Proc. CAS 1996 on Synchrotron Radiation and Free Electron Lasers, Grenoble,
CERN Yellow Report 98-04 (1998)

H. Wiedemann, Synchrotron Radiation, Springer-Verlag Berlin Heidelberg (2003)

K.-J. Kim, Z. Huang, and R. Lindberg, Synchrotron Radiation and Free-Electron
Lasers, Cambridge University Press (2017)

A. Hofmann, The Physics of Synchrotron Radiation, Cambridge University Press
(2007)

A.W. Chao, M. Tigner, H. Weise, F. Zimmermann, Handbook of Accelerator Physics
and Engineering, 3rd edition, World Scientific (2023)

J. Schwinger, On the Classical Radiation of Accelerated Electrons, Phys. Rev. 75,
1912 (1949)

A.A. Sokolov, I.M. Ternov, Radiation from Relativistic Electrons, AIP (1986)