

*Largely based on the lectures given by F. Tecker at the Graduate Accelerator Physics Course –
John Adams Institute for Accelerator Science (Oxford), March 2025.*

Linear Colliders

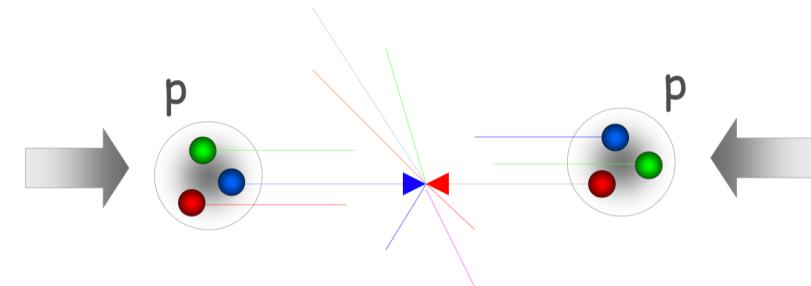
R. Corsini

Outline of the lectures

- Introduction:
 - The basics
 - A bit of history: from SLC to the present LC projects
- Luminosity & parameters optimization
- Introduction to linear collider proposals, ILC & CLIC
- Subsystems:
 - Sources
 - Damping Ring
 - Bunch Compressors
 - Main linac
 - Beam Dynamics, wake-fields and alignment
 - RF System
 - Beam Delivery System
- The superconducting solution: ILC
- The Two-Beam solution: CLIC

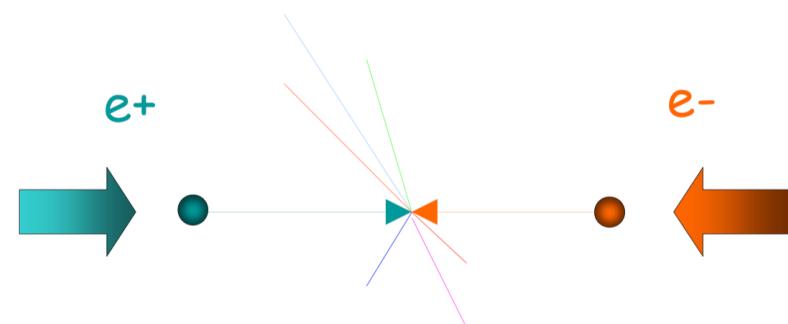
- Hadron collisions (p, ions):

- Compound particles (mix of quarks, anti-quarks and gluons)
- Parton energy spread, energy available $< E_{cm}$
- Can only use P_T conservation
- QCD processes produce large background

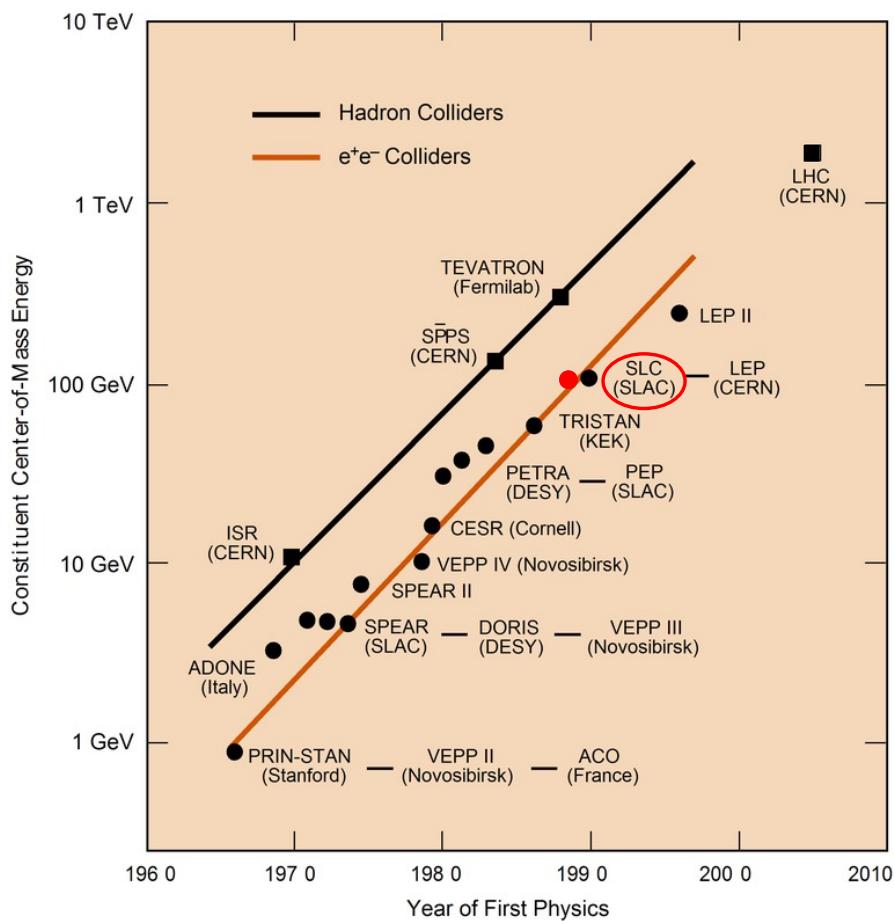


- Lepton collisions(e^- , e^+ , muons):

- Elementary particles \Rightarrow all energy available
- Well defined initial state
- Momentum conservation eases decay produce analysis
- Less background
- Polarization



- Photons also possible



- History:

- Energy constantly increasing with time
- Hadron Colliders at the [energy frontier](#)
- Lepton Colliders for [precision physics](#)
- LHC has found the Higgs with $m_H = 126 \text{ GeV}/c^2$
- A future Lepton Collider ([Higgs factory](#)) would complement LHC physics \Rightarrow precision measurements of the Higgs boson characteristics
- Recommended in the 2020 Update of the European Strategy for Particle Physics

Circular vs. Linear Lepton Collider

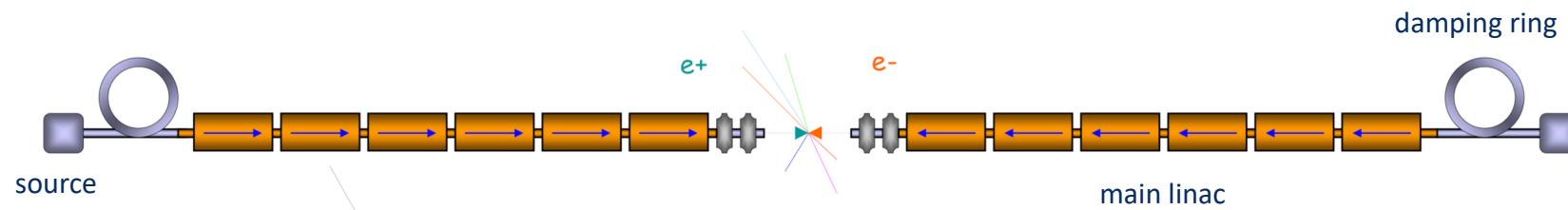
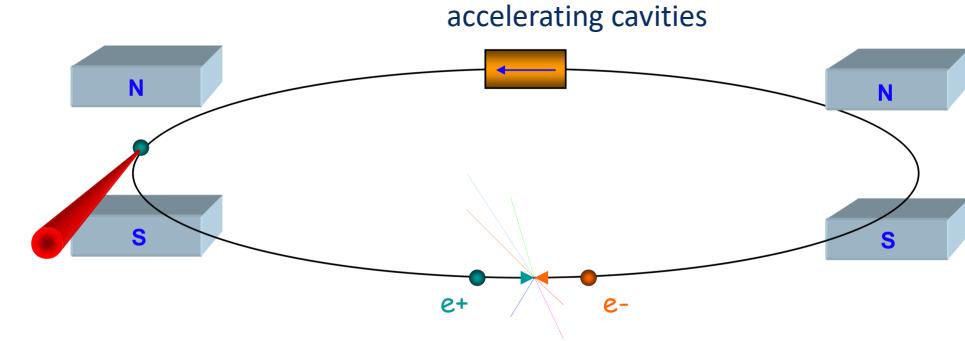
Accelerates beam over many turns
Can use beam many times in collision

However, charged particles emit **synchrotron radiation** in a magnetic field

$$\Delta E_{turn} = \frac{4}{3} \pi \frac{r_e}{(m_o c^2)^3} \frac{E^4}{\rho}$$

For light particles synchrotron radiation can be large

- At LEP lost 2.75GeV/turn for E = 105 GeV



Almost no radiation in a linac \Rightarrow No energy loss!

Beam has to achieve final **energy in single pass**
Must achieve **luminosity** with **single beam collision**

Need a **larger** lepton storage ring to **compensate for synchrotron radiation losses**.
LEP had already a $L = 27$ km, for $E_{cm} = 200$ GeV

- Synchrotron radiation:

- Emitted power

$$P = \frac{2}{3} \frac{r_e c}{(m_o c^2)^3} \frac{E^4}{\rho^2}$$

scales with E^4 !!

- Energy loss/turn

$$\Delta E_{turn} = \frac{4}{3} \pi \frac{r_e}{(m_o c^2)^3} \frac{E^4}{\rho}$$

must be replaced
by the RF system

- RF costs:

$$\epsilon_{RF} \propto \Delta E_{turn} \propto E^4/\rho$$

- Linear costs (magnets, tunnel, etc.)

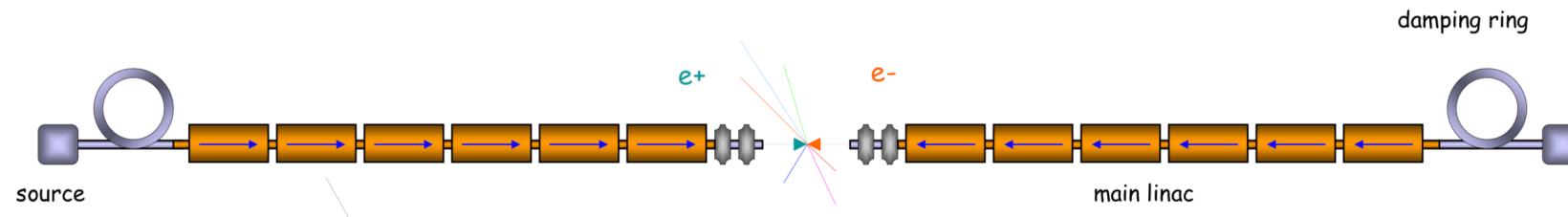
$$\epsilon_{lin} \propto \rho$$

⇒ Optimum when

$$\epsilon_{lin} \propto \epsilon_{RF} \Rightarrow \rho \propto E^2$$

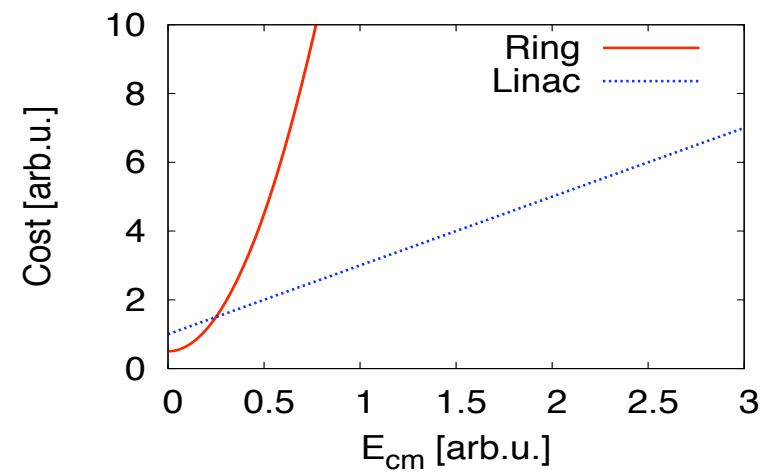
Must increase radius quadratically with energy

⇒ The **size** and the **optimized cost** scale as E^2 as well as the **energy loss per turn** (was already ~3% at LEP)

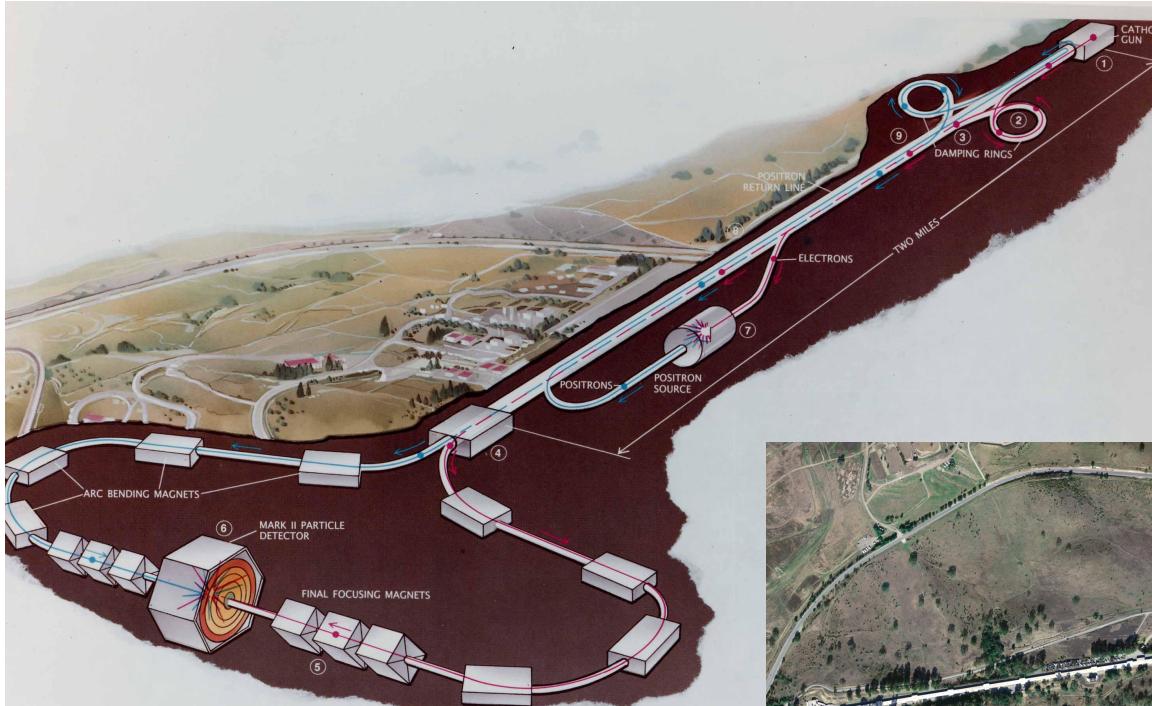


- NO bending magnets \Rightarrow NO synchrotron radiation
- Accelerating structures are used only once for each colliding beam
 \Rightarrow need lots of them !!!

\Rightarrow Cost and size scaling linearly with E



The first linear Collider, SLC



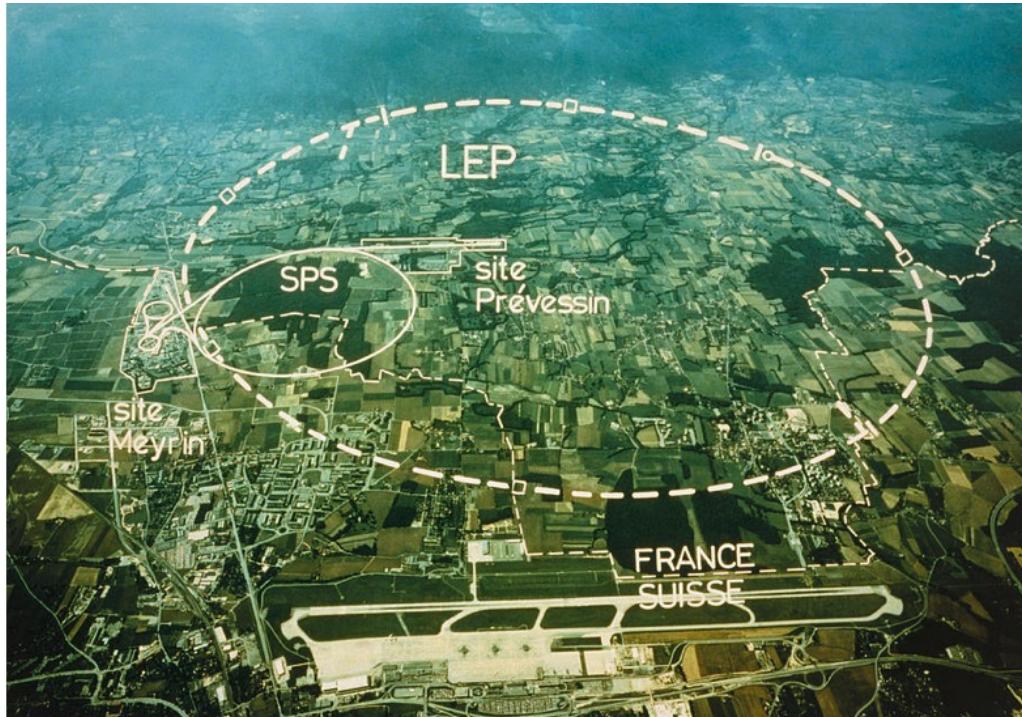
SLAC Linear Collider



Linac length = 3 km

Energy 92 GeV
Luminosity 1×10^{30}

SLAC NATIONAL
ACCELERATOR
LABORATORY



Ring length = 27 km

- Operated initially at the Z_0 energy, later upgraded with superconducting cavities to allow for W bosons production



Energy
Luminosity

$92 \text{ GeV} \Rightarrow 209 \text{ GeV}$
 1×10^{32}

LEP I copper
cavity



LEP II superconducting cavity



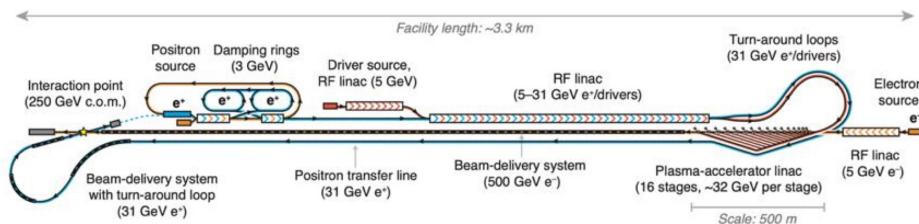
LEP – the largest electron-positron accelerator ever built – was dismantled in 2000
Its 27-kilometre tunnel now hosts the LHC

Linear Collider Projects, ~ 1990 to now

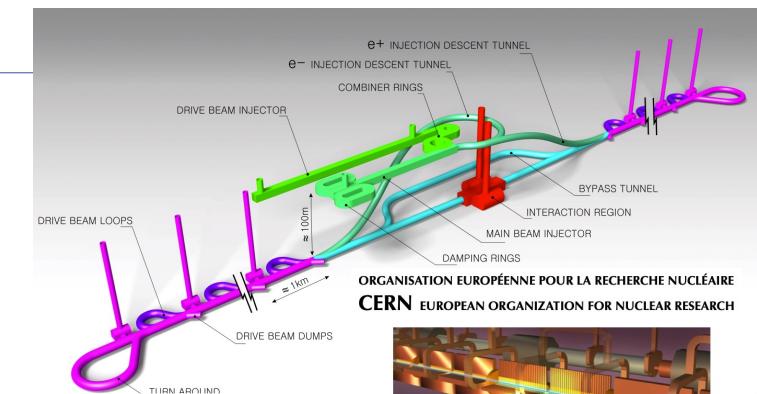
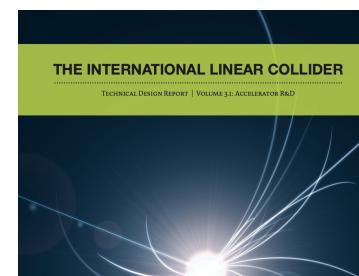
Linear Colliders: Overall and Final Focus Parameters – 500 GeV (c.m.)

	TESLA*	SBLC	JLC (S)	JLC (C)	JLC (X)	NLC	VLEPP	CLIC
Initial energy (c.o.f.m.) (GeV)	500	500	500	500	500	500	500	500
RF frequency of main linac (GHz) [†]	1.3	3	2.8	5.7	11.4	11.4	14	30
Nominal Luminosity ($10^{33} \text{ cm}^{-2} \text{s}^{-2}$) [†]	2.6	2.2	5.2	7.3	5.1	5.3	12.3	0.7-3.4
Actual luminosity ($10^{33} \text{ cm}^{-2} \text{s}^{-2}$) [†]	6.1	3.75	4.3	6.1	5.2	7.1	9.3	1.07-4.8
Linac repetition rate (Hz)	10	50	50	100	150	180	300	2530-1210
No. of particles/bunch at IP (10^{10})	5.15	2.9	1.44	1.0	.63	.65	20	.8
No. of bunches/pulse	800	125	50	72	85	90	1	1-10
Bunch separation (nsec)	1000	16.0	5.6	2.8	1.4	1.4	—	.67
Beam power/beam (MW)	16.5	7.26	1.3	2.9	3.2	4.2	2.4	8.3-9
Damping ring energy (GeV)	4.0	3.15	2.0	2.0	2.0	2.0	3.0	2.15
Main linac gradient, unloaded/loaded ^{††} (MV/m)	25/25	21/17	31/-	40/32	73/58	50/37	100/91	80/78
Total two-linac length (km)	29	33	22.1	18.8	10.4	15.6	7	8.8
Total beam delivery length (km)	3	3	3.6	3.6	4.4	3	2.4	
$\gamma \epsilon_x / \gamma \epsilon_y$ ($m\text{-rad} \times 10^{-8}$)	2000/100	1000/50	330/4.8	330/4.8	500/5	2000/7.5	300/15	
β_x^*/β_y^* (mm)	25/2	22/0.8	10/0.1	10/0.1	10/0.1	100/0.1	10/0.18	
σ_x^*/σ_y^* (nm) before pinch	1000/64	670/28	260/3.0	260/3.0	320/3.2	2000/4	247/7.4	
σ_z^* (μm)	1000	500	120	120	90	100	750	200
Crossing Angle at IP (mrad)	0	3	6.4	6.0	6.1	20	6	1
Disruptions D_x/D_y	0.56/8.7	.36/8.5	.29/25	.20/18	.096/8.3	.07/7.3	.4/215	0.29/9.8
H_D	2.3	1.8	1.6	1.4	1.4	1.34	2.0	1.42
Upsilon sub-zero	.02	.037	.20	.14	.12	.089	.059	0.07
Upsilon effective	.03	.042	.22	.144	.12	.090	.074	.075
δ_B (%)	3.3	3.2	12.7	6.5	3.5	2.4	13.3	3.6
n_γ (no. of γ 's per e)	2.7	1.9	2.2	1.5	.94	.8	5.0	1.35
N_{pairs} ($p_T^{\text{min}}=20 \text{ MeV}/c, \theta_{\text{min}}=0.15$)	19.0	8.8	31.6	10.3	2.9	2.0	1700	3.0
$N_{\text{hadrons}}/\text{crossing}$	0.17	0.10	0.98	0.23	0.05	0.03	45.9	0.05
$N_{\text{jets}} \times 10^{-2}$ ($p_T^{\text{min}}=3.2 \text{ GeV}/c$)	0.16	0.14	3.4	0.66	0.14	0.08	56.4	0.10

From the 1995 International Linear Collider (ILC) Technical Review Committee (TRC) Report (FERMILAB-PUB-95-438)



HALHF
Hybrid, Asymmetric,
Linear Higgs Factory



CLIC
Compact
Linear Collider

THE COMPACT LINEAR COLLIDER (CLIC)
READINESS REPORT



- The performance of particle colliders is usually quantified by the center of mass beam energy E_{cm} and the luminosity L
- The **luminosity** is the quantity that measures the **ability** of a particle accelerator to **produce the required number of useful interactions**. In particular, it is the proportionality factor between the number of events per second dR/dt and the cross section σ_p :

$$dR/dt = L \cdot \sigma_p$$

The unit of the luminosity is $\text{cm}^{-2}\text{s}^{-1}$

- A Collider luminosity is approximately given by

where:

n_b	= bunches / train
N	= particles per bunch
f_{rep}	= repetition frequency
$\sigma_{x,y}$	= transverse beam size at IP
H_D	= beam-beam enhancement factor (linear collider: typical value ~ 2)

$$L = \frac{n_b N^2 f_{rep}}{4\pi \sigma_x \sigma_y} H_D$$

$$L = \frac{n_b N^2 f_{rep}}{4\pi \sigma_x \sigma_y} H_D$$

- LHC ring $f_{rep} = 11 \text{ kHz}$

- LC $f_{rep} = \text{few-100 Hz}$ (power limited)

⇒ factor $\sim 100\text{-}1000$ in L already lost for the LC!

- Must push very hard on beam cross-section at collision:

factor of 10^6 gain! needed
to obtain high luminosity
of a few $10^{34} \text{ cm}^{-2}\text{s}^{-1}$

LEP: $\sigma_x \sigma_y \approx 130 \times 6 \text{ } \mu\text{m}^2$

LC: $\sigma_x \sigma_y \approx (60\text{-}550) \times (1\text{-}5) \text{ nm}^2$

Introduce centre-of-mass Energy E_{cm}

$$L = \frac{n_b N^2 f_{rep}}{4\pi \sigma_x \sigma_y} H_D \rightarrow L = \frac{(n_b N f_{rep} E_{cm}) N}{4\pi \sigma_x \sigma_y E_{cm}} H_D$$

$$\rightarrow L = \frac{\eta_{RF} P_{RF} N}{4\pi \sigma_x \sigma_y E_{cm}} H_D$$

Beam
power

- η_{RF} is the RF-to-beam power efficiency
- Luminosity L is proportional to the RF power P_{RF} and efficiency η_{RF} for a given E_{cm}

- Some numbers:

E_{cm}	= 500 GeV
N	= 10^{10}
n_b	= 100
f_{rep}	= 100 Hz

- Need to include efficiencies:

RF → beam	range 20-60%
Wall plug → RF	range 28-40%

AC power: a few hundred MW to accelerate beams for a high luminosity
 ⇒ This limits the practically achievable energy and luminosity

$$L = \frac{1}{4\pi E_{cm}} (\eta_{RF} P_{RF}) \left(\frac{N}{\sigma_x \sigma_y} H_D \right)$$

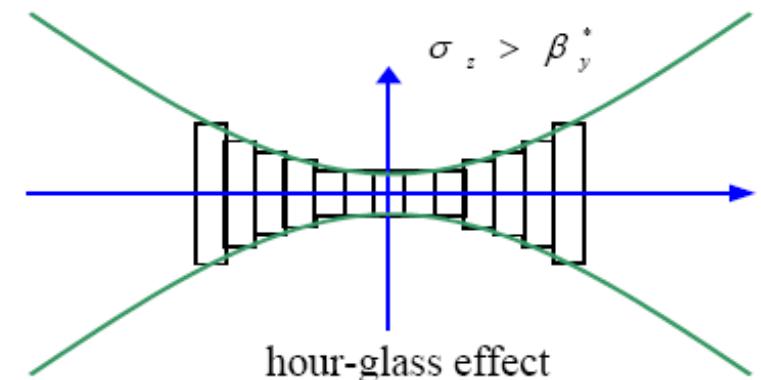
- Choice of acceleration technology (NC vs SC):
 - Efficiency
 - Available power
- Strong focusing needed for small beam size
 - Optical **aberrations**
 - Issues with **stability** and tolerances
- Beam-Beam effects:
 - Strong self focusing (**pinch effect**) ⇒
 - increases Luminosity
 - Photon emission (**Beamstrahlung**) ⇒
 - dilutes Luminosity spectrum
 - creates detector background

- β -function at the interaction point follows

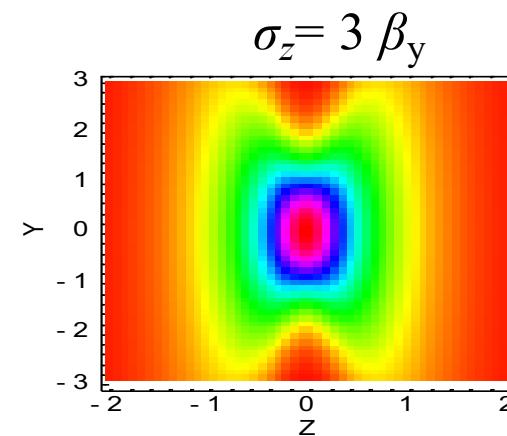
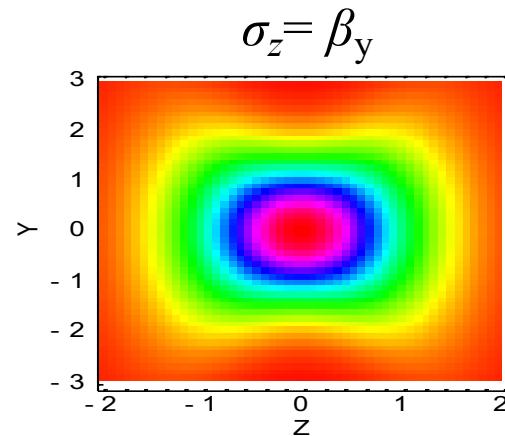
$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$

β^* beta function at the IP

N.B.: $\sigma_{x,y} = \sqrt{\frac{\beta_{x,y} \epsilon_{x,y}}{\gamma}}$

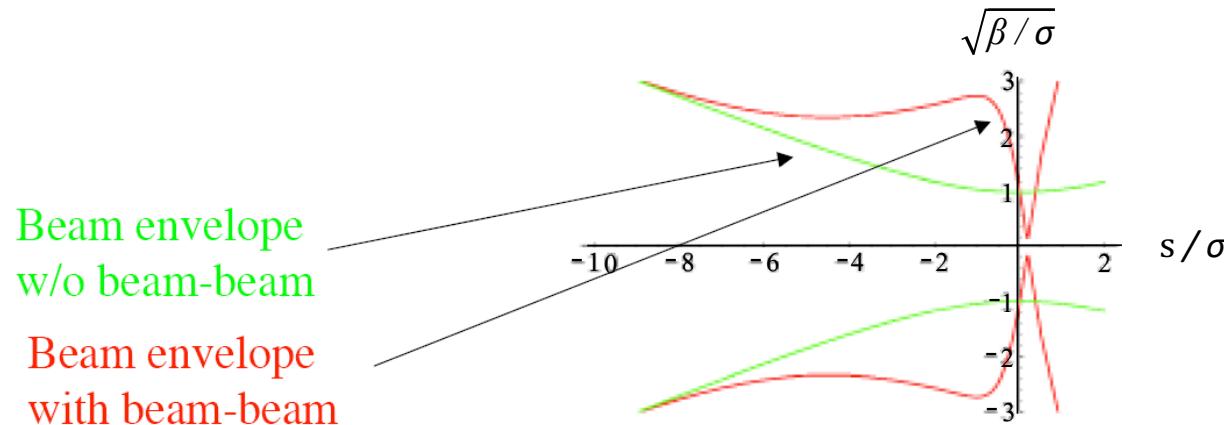
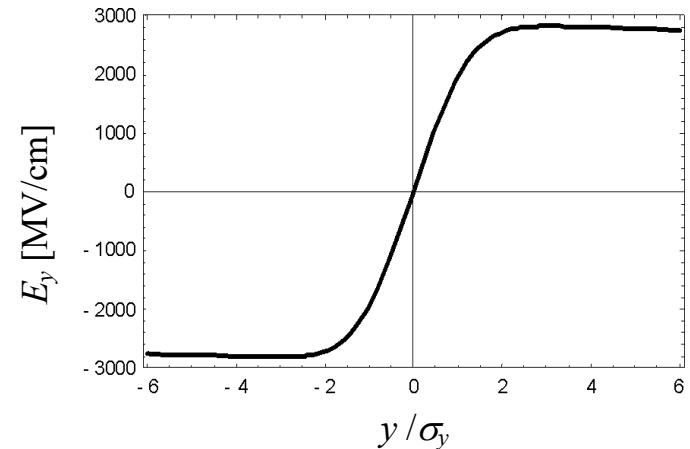


- Desirable to have $\sigma_z \leq \beta_y \Rightarrow$ **short bunch length** for high luminosity

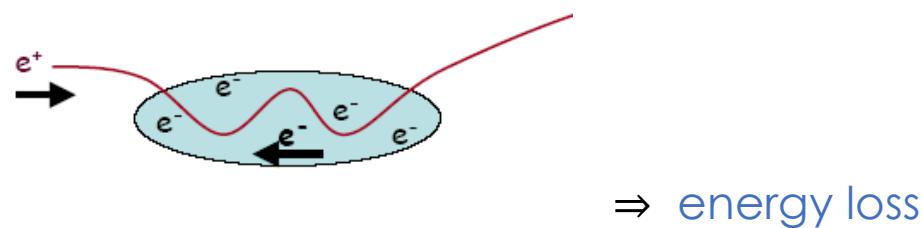


N.Walker

- Strong electromagnetic field of the opposing bunch:
 - Deflects the particles
“beam-beam kick”
 - Focuses the bunches
“pinch effect”
- Luminosity enhancement factor H_D



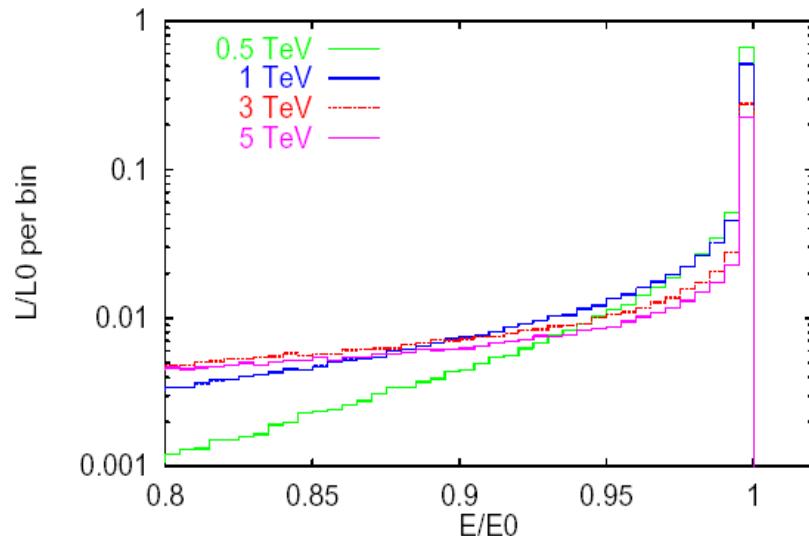
- “Synchrotron radiation” in the field of the opposing bunch (**beamstrahlung**)



- Smears out **luminosity spectrum**
- Creates e^+e^- pairs background in detector



- Quantified by **Disruption parameter**



$$D_{x,y} = \frac{2r_e N \sigma_z}{\gamma \sigma_{x,y} (\sigma_x + \sigma_y)}$$

- RMS relative energy loss by beamstrahlung

$$\delta_{BS} \approx 0.86 \frac{r_e^3}{2m_0c^2} \left(\frac{E_{cm}}{\sigma_z} \right) \frac{N^2}{(\sigma_x + \sigma_y)^2}$$

- We want

- σ_x and σ_y small for high luminosity
- $(\sigma_x + \sigma_y)$ large for small δ_{BS} \Rightarrow better luminosity spectrum

- Use flat beams with $\sigma_x \gg \sigma_y$

increase luminosity

by small σ_y

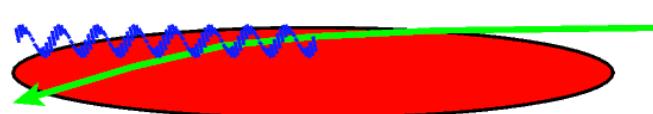
and minimise δ_{BS}

by (relatively) large σ_x

$$\delta_{BS} \propto \left(\frac{E_{cm}}{\sigma_z} \right) \frac{N^2}{\sigma_x^2}$$

Beam-beam Effect

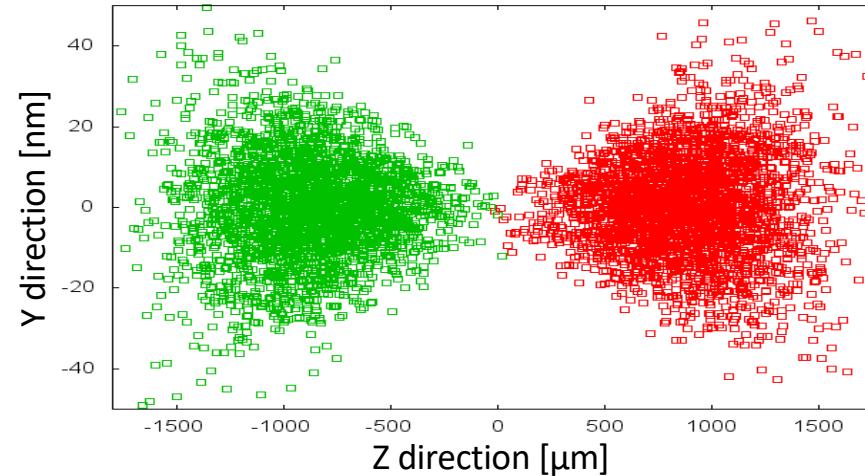
$$\mathcal{L} \propto H_D \left(\frac{N}{\sigma_x} \right) N n_b f_r \frac{1}{\sigma_y}$$



Intense beams to reach high luminosity

$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y}$$

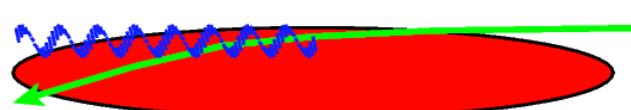
Beam-beam force switched off



D. Schulte

Beam-beam Effect

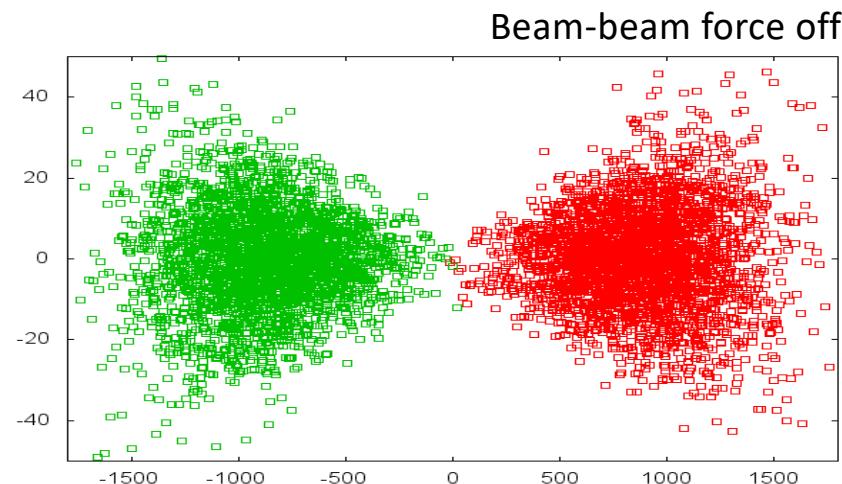
$$\mathcal{L} \propto H_D \left(\frac{N}{\sigma_x} \right) N n_b f_r \frac{1}{\sigma_y}$$



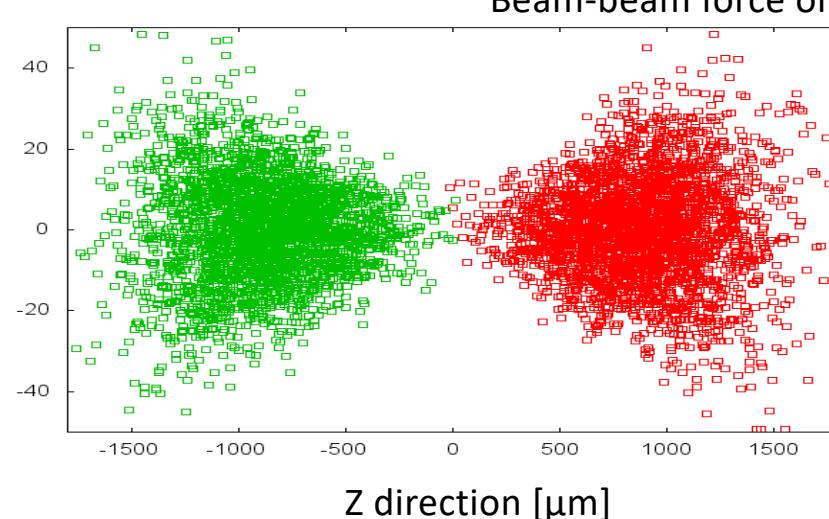
Beam focus each other

$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y}$$

Y direction [nm]

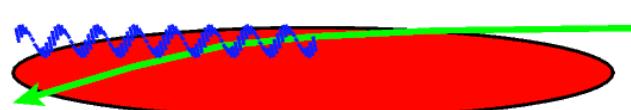


D. Schulte



Beam-beam Effect

$$\mathcal{L} \propto H_D \left(\frac{N}{\sigma_x} \right) N n_b f_r \frac{1}{\sigma_y}$$

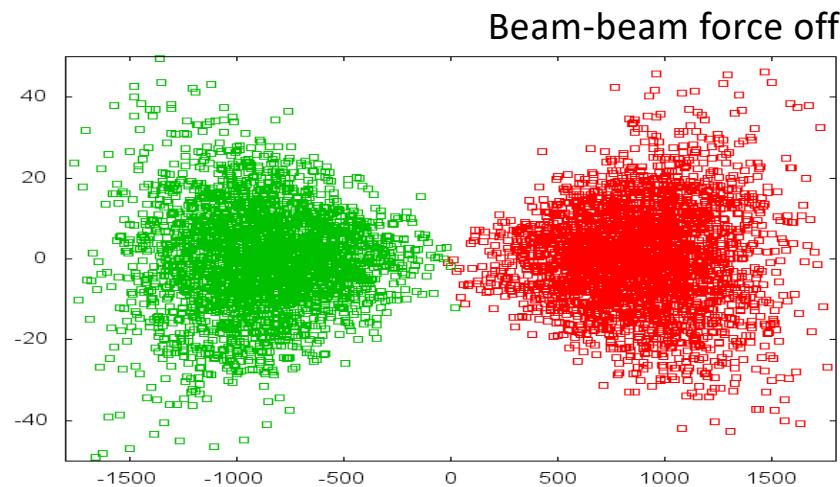


Emit beamstrahlung

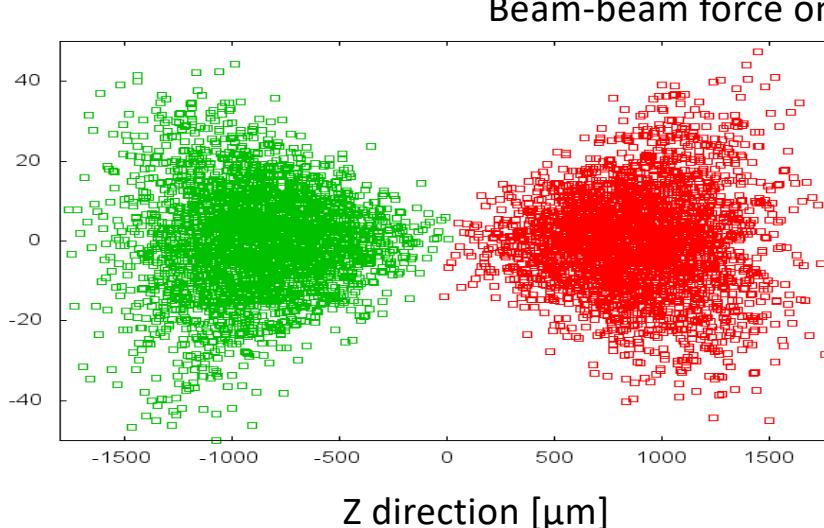
$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y}$$

$$n_\gamma \propto E_\gamma \propto \frac{N}{\sigma_x + \sigma_y}$$

Y direction [nm]

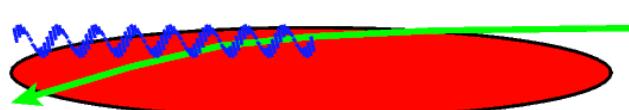


D. Schulte



Beam-beam Effect

$$\mathcal{L} \propto H_D \left(\frac{N}{\sigma_x} \right) N n_b f_r \frac{1}{\sigma_y}$$

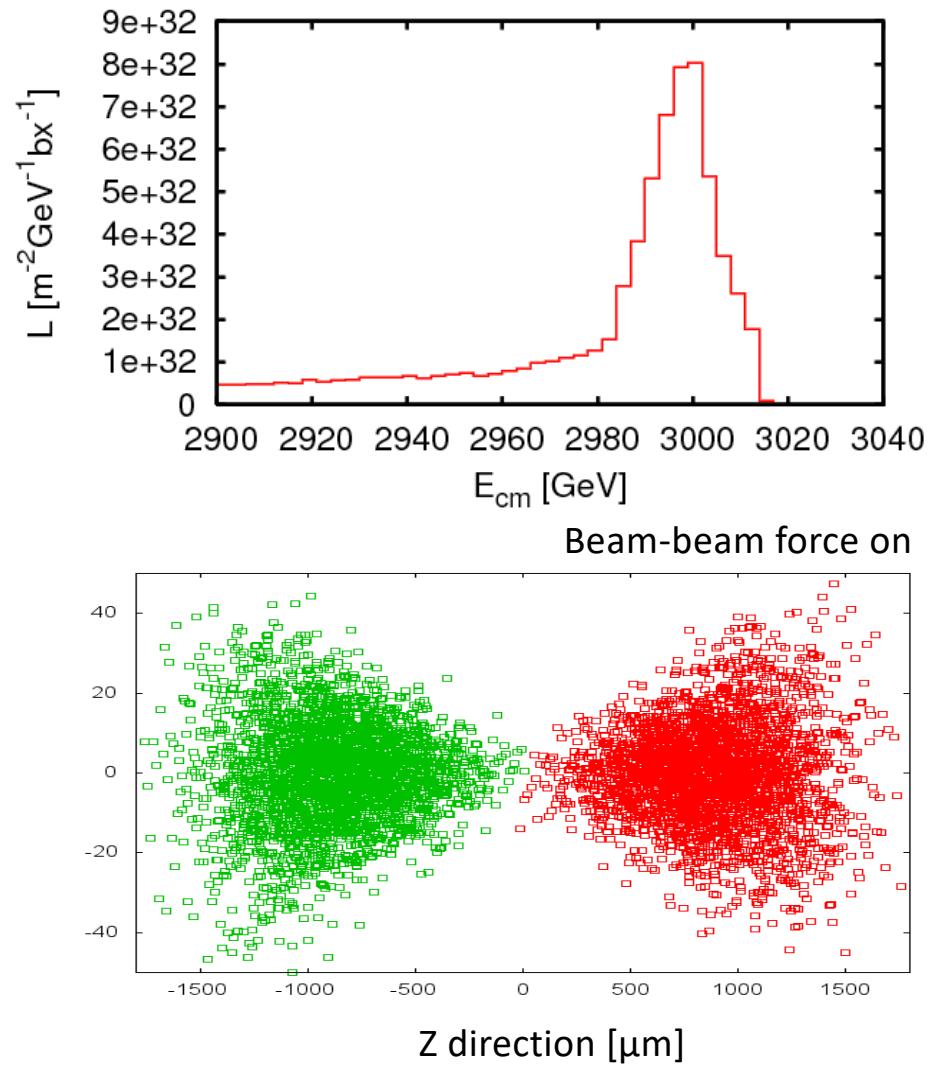


Develop luminosity spectrum

$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y}$$

$$n_\gamma \propto E_\gamma \propto \frac{N}{\sigma_x + \sigma_y}$$

$$\sigma_x \gg \sigma_y \quad \sigma_x + \sigma_y \approx \sigma_x$$



- Substitute $\delta_{BS} \propto \left(\frac{E_{cm}}{\sigma_z}\right) \frac{N^2}{\sigma_x^2}$ into $L = \frac{1}{4\pi E_{cm}} (\eta_{RF} P_{RF}) \left(\frac{N}{\sigma_x \sigma_y} H_D \right)$

- We get
$$L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}^{3/2}} \frac{\sqrt{\delta_{BS} \sigma_z}}{\sigma_y}$$

- Now use
$$\sigma_y = \sqrt{\frac{\beta_y \epsilon_{n,y}}{\gamma}}$$

- Then
$$L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}^{3/2}} \sqrt{\frac{\delta_{BS} \gamma}{\epsilon_{n,y}}} \sqrt{\frac{\sigma_z}{\beta_y}} \propto \frac{\eta_{RF} P_{RF}}{E_{cm}} \sqrt{\frac{\delta_{BS}}{\epsilon_{n,y}}} \sqrt{\frac{\sigma_z}{\beta_y}}$$

~ 1 (hourglass effect)

$$L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}} \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} H_D \quad \beta_y \approx \sigma_z$$

- We want **high** RF-beam conversion efficiency η_{RF}
- Need **high** RF power P_{RF}
- **Small** normalised vertical emittance $\varepsilon_{n,y}$
- Strong focusing at IP is implied (i.e., **small** β_y and hence **small** σ_z)
- Could also allow higher beamstrahlung δ_{BS} if willing to live with the consequences (luminosity spread and background)
 - Above result is for the low beamstrahlung regime where $\delta_{BS} \sim \text{few \%}$
 - Slightly different result for high beamstrahlung regime

Energy reach

$$E_{cm} \approx L_{linac} G_{acc}$$



High gradient

Luminosity

$$L = \frac{n_b N^2 f_{rep}}{4\pi \sigma_x^* \sigma_y^*} \times H_D \propto \frac{\eta_{beam}^{AC}}{\epsilon_y^{1/2}} \frac{P_{AC}}{E_{cm}} \frac{\delta_{BS}^{1/2}}{E_{cm}}$$

N.B.: $\sigma_{x,y} = \sqrt{\frac{\beta_{x,y} \epsilon_{x,y}}{\gamma}}$

- Acceleration efficiency
- Generation of small emittance
- Conservation of small emittance
- Extremely small beam spot at IP

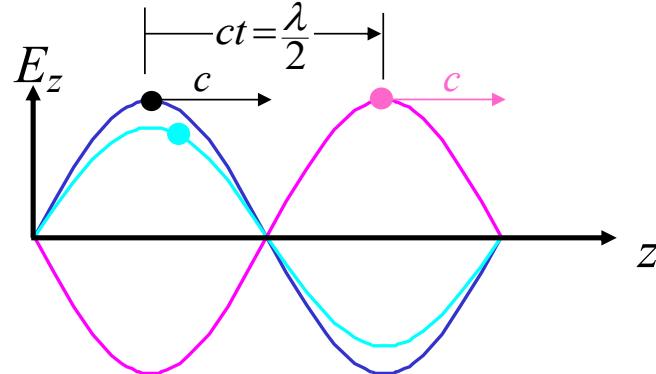


Superconducting RF (ILC) or Two-beam scheme (CLIC)

Damping rings

Wake-fields, alignment, stability

Beam delivery system, stability



Superconducting
(Niobium)

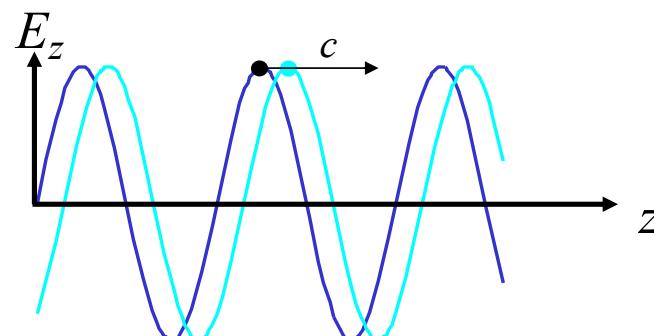


Normal
Conducting
(Copper)

Standing wave cavity:

bunch sees field:

$$\begin{aligned} E_z &= E_0 \sin(\omega t + \varphi) \sin(kz) \\ &= E_0 \sin(kz + \varphi) \sin(kz) \end{aligned}$$



Travelling wave structure:
need phase velocity = c
(disk-loaded structure)

bunch sees constant field:

$$E_z = E_0 \cos(\varphi)$$

ILC Cavities

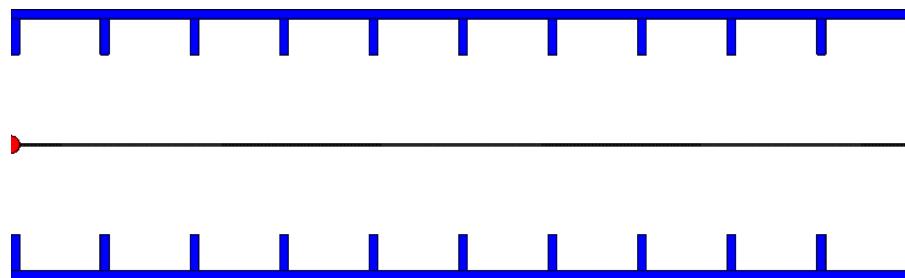
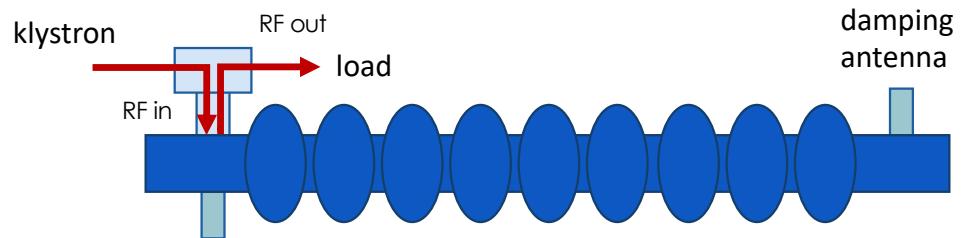


Standing wave structure

Superconducting cavity (Ni at 2 K)

RF frequency is **1.3 GHz**, 23 cm wavelength

Length is 9 cells = 4.5 wavelengths = **1 m**



Gradient is **31.5 MV/m**

Need about **16000 cavities**

Ring to Main Linac (RTML)
(including
bunch compressors)

Damping Rings

Polarised electron
source

e+ Main Linac

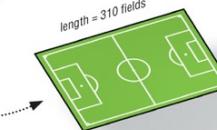
e+ source

Electrons

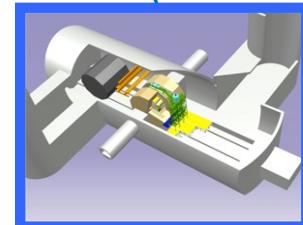
e- Main Linac

33 km

Interaction Point
Detector Cavern



length = 310 fields



ILC Scheme | © www.form-one.de

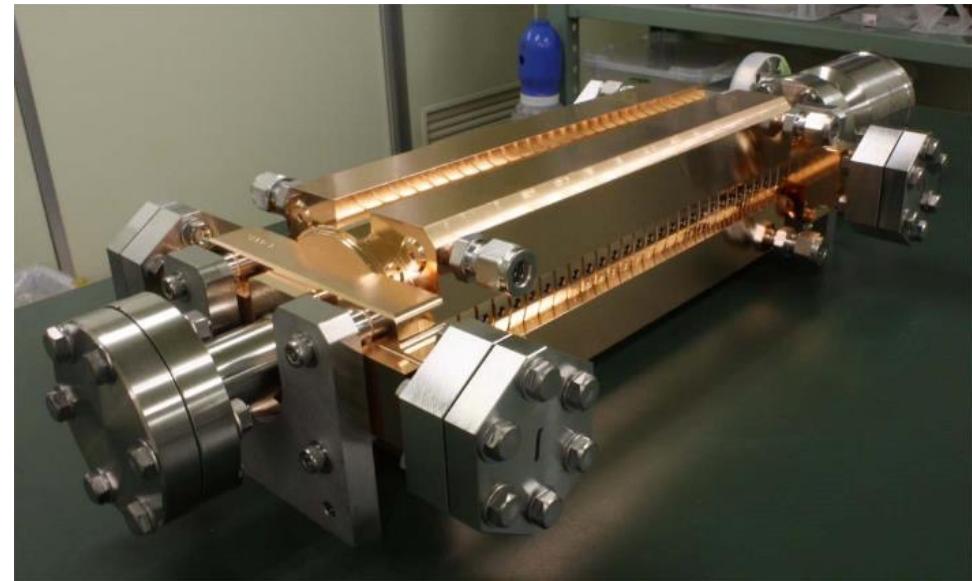
Parameters	Value
C.M. Energy	500 GeV
Peak luminosity	$1.8 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$
Beam power	10.5 MW
Beam Rep. rate	5 Hz
E gradient	31.5 MV/m +/-20%

12 GHz, 23 cm long, **normal conducting**
Loaded gradient 100MV/m

- ⇒ Allows to reach higher energies
- ⇒ 140,000 structures at 3TeV

losses in the walls and in the load

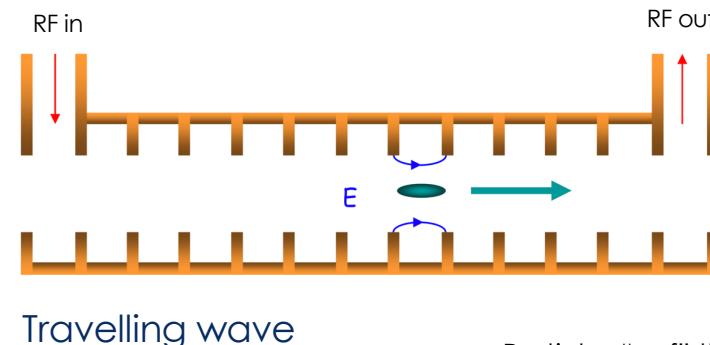
- ⇒ 50 RF bursts per second
- ⇒ 240 ns, 60 MW, 312 bunches
- ⇒ **Power during pulse 8.5×10^6 MW (3000 x ILC)**



Power flow

- 1/3 lost in cavity walls
- 1/3 in filling the structure and into load
- 1/3 into the beam

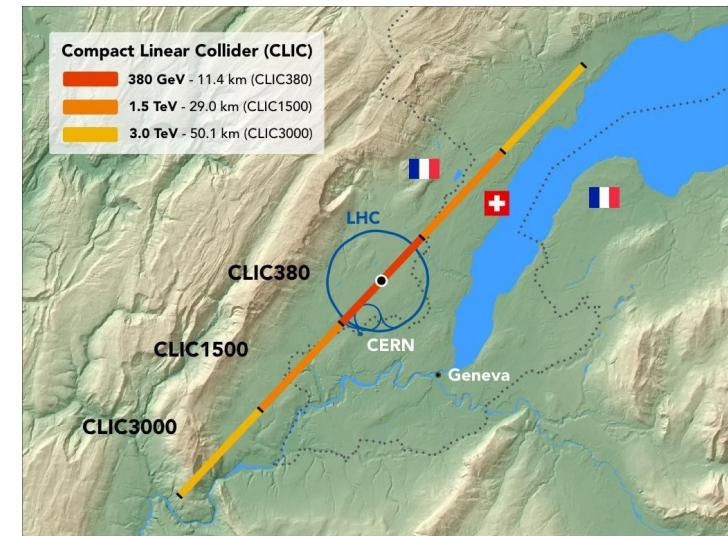
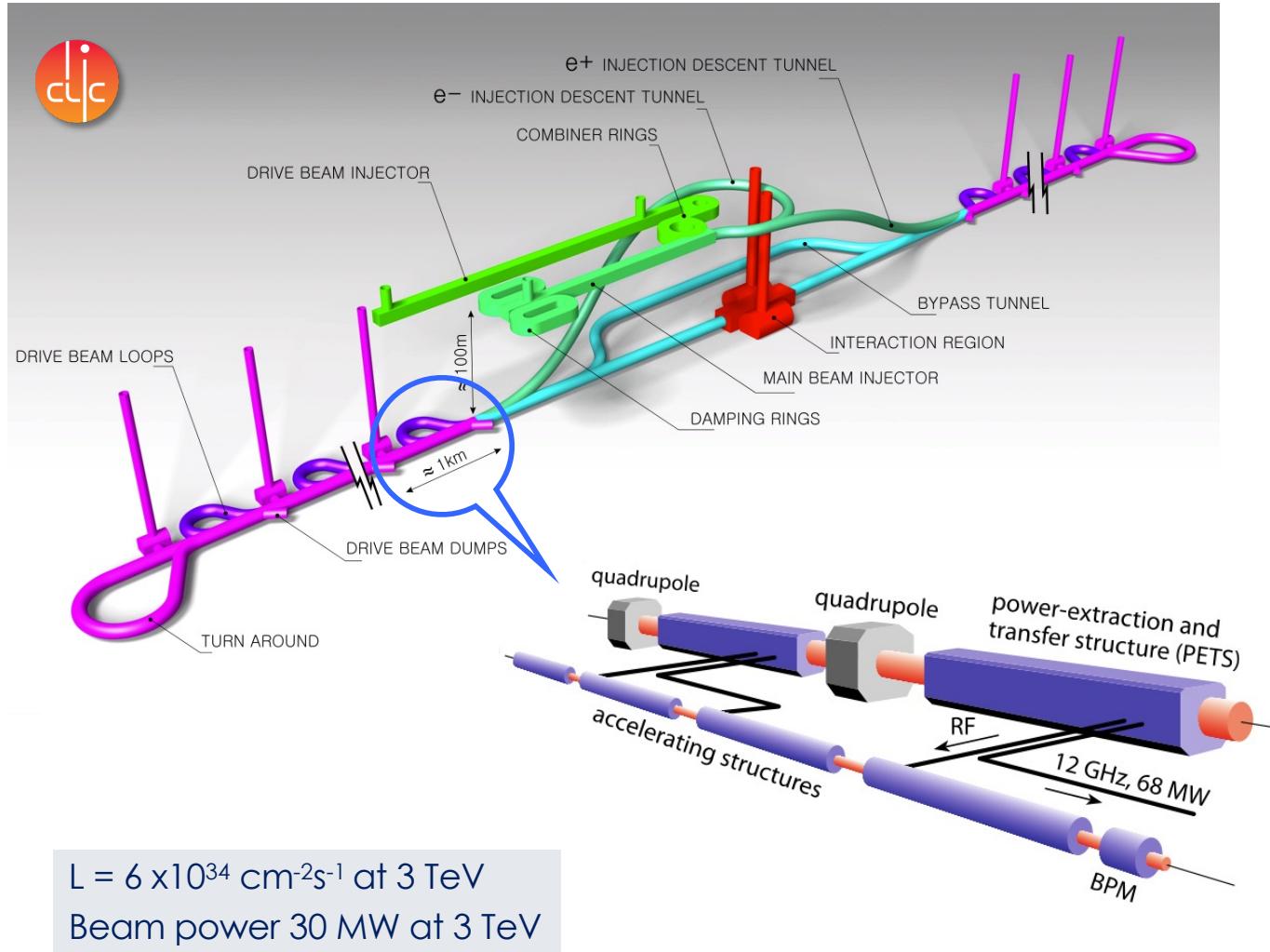
Average RF power about 3kW/m
About 1kW/m into beam



Particles “surf” the electromagnetic wave



The Compact Linear Collider - CLIC



CLIC can be built in **stages** of increasing collision energy: starting from 380 GeV, then \sim 1-2 TeV, and up to a final energy of 3 TeV.

To limit the collider length, the accelerating gradient must be very high - CLIC aims at 100 MV/m, 20 times higher than the LHC.

CLIC is based on a **two-beam acceleration scheme**, in which a high current e- beam (the drive beam) is decelerated in special structures (PETS), and the generated RF power is used to accelerate the main beam.

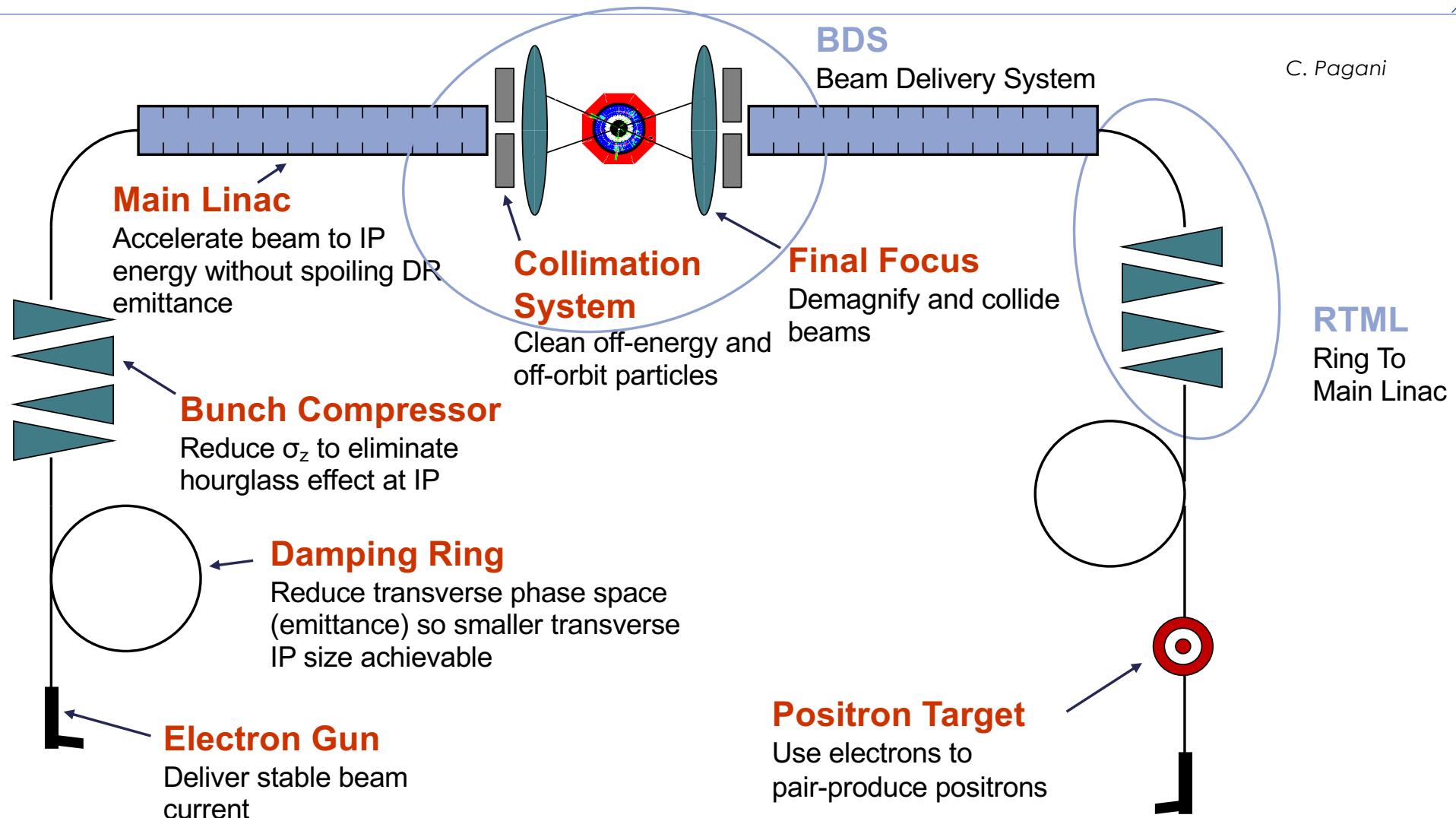
ILC and CLIC Main Parameters

Parameter	Symbol [unit]	SLC	ILC	CLIC	CLIC
Centre of mass energy	E_{cm} [GeV]	92	500	380	3000
Luminosity	L [$10^{34} \text{cm}^{-2}\text{s}^{-1}$]	0.0003	1.8	1.5	6
Luminosity in peak	$L_{0.01}$ [$10^{34} \text{cm}^{-2}\text{s}^{-1}$]	0.0003	1	0.9	2
Gradient	G [MV/m]	20	31.5	72	100
Particles per bunch	N [10^9]	37	20	5.2	3.72
Bunch length	σ_z [μm]	1000	300	70	44
Collision beam size	$\sigma_{x,y}$ [nm/nm]	1700/600	474/5.9	143/2.9	40/1
Vertical emittance	$\varepsilon_{x,y}$ [nm]	3000	35	30	20*
Bunches per pulse	n_b	1	1312	352	312
Bunch distance	Δz [mm]	-	554	0.5	0.5
Repetition rate	f_r [Hz]	120	5	50	50

End part I

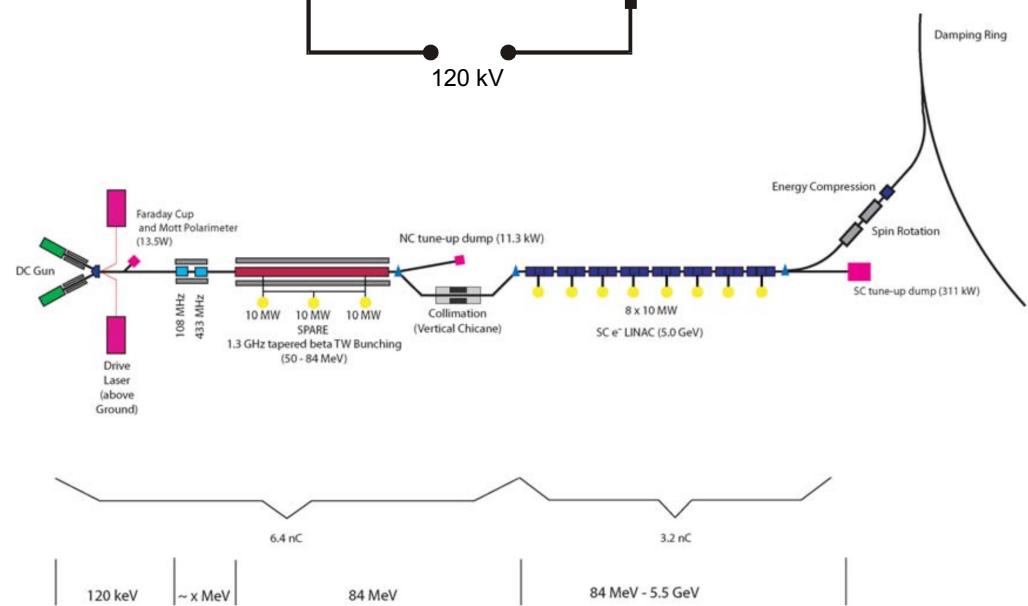
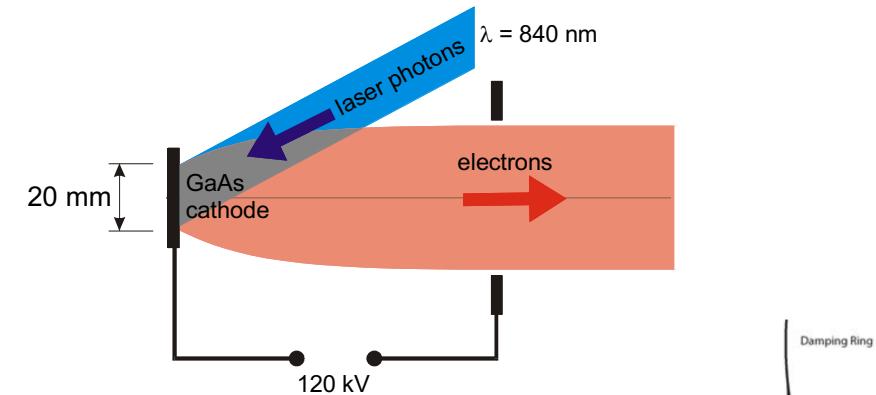
Generic Linear Collider

C. Pagani



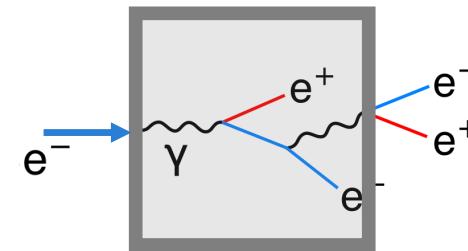
We need large number of bunches of (polarized) electrons – “standard” solution:

- laser-driven DC photo injector
 - circularly polarized photons on **GaAs cathode** (incompatible with RF gun)
 - $\epsilon_n \sim 50 \mu\text{m rad}$ too large by: factor ~ 10 in x plane factor ~ 500 in y plane
 - dominated by space charge
- ⇒ need a **damping ring**
- Laser + **RF bunching system** generate and capture the **bunch structure** adapted to the linac for further acceleration



Basic mechanism: pair production in target material

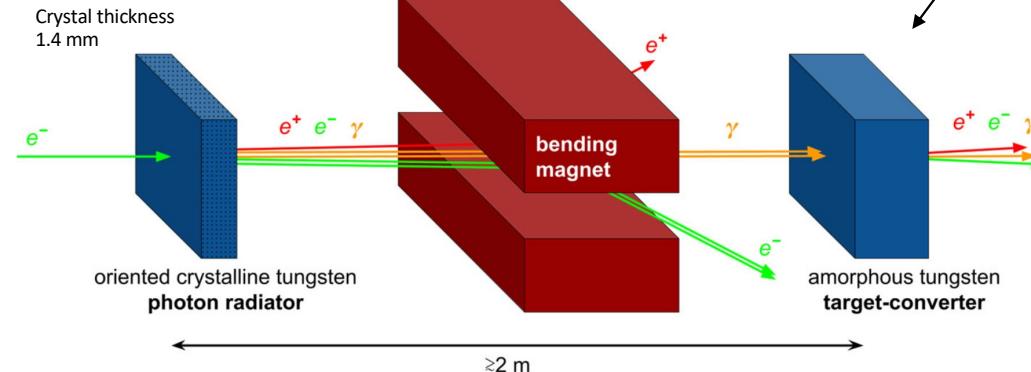
- Standard method: 'thick' target primary e⁻ generate photons these convert into pairs



Limitation: energy deposition in target

- Hybrid source:

- Crystal + Amorphous target
- Enhanced photon flux by channeling effect

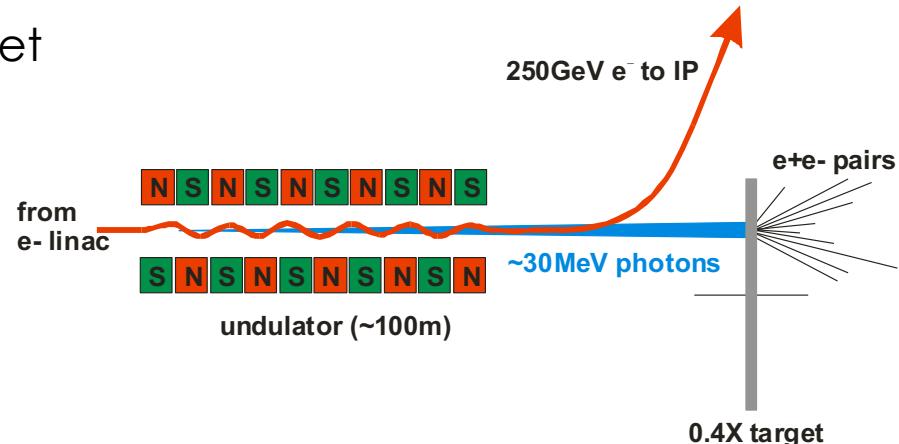


- Positrons are captured in accelerating structure inside solenoids and accelerated

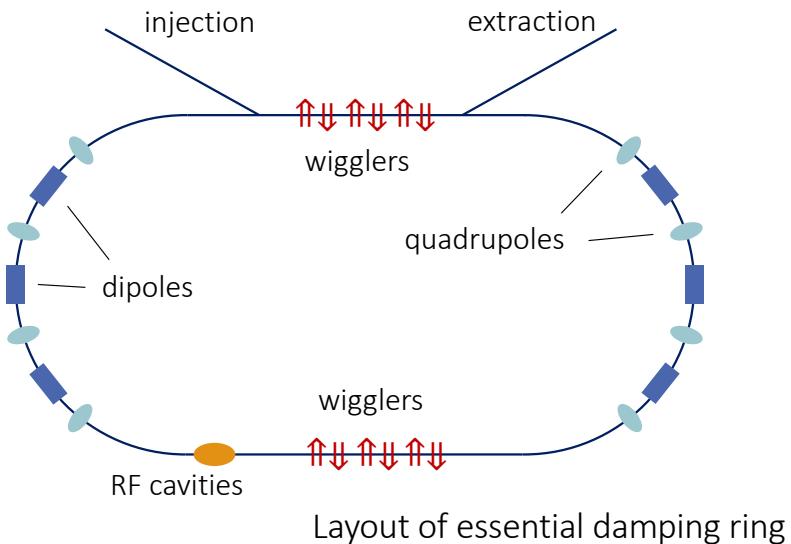
Undulator source:

high energy e⁻ produce photons in undulator magnet
+ thin conversion target

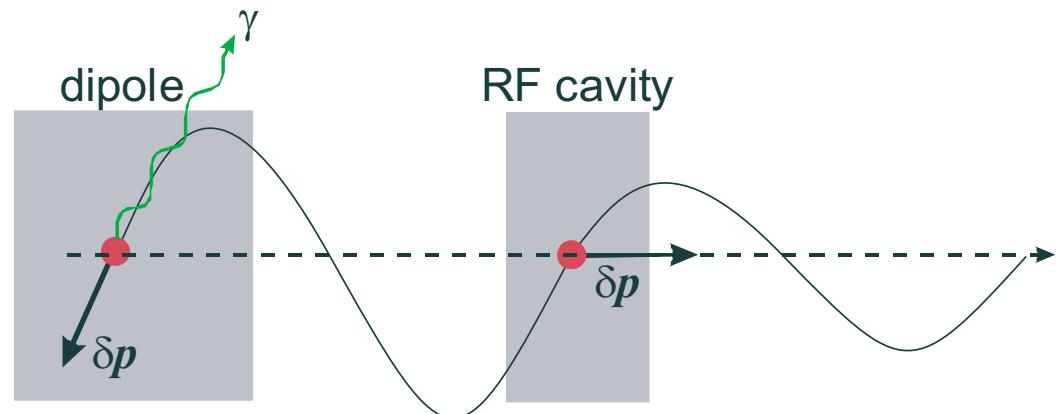
- ~0.4 rad. length \Rightarrow much less energy deposition in the target (5 kW compared to 20 kW)
 \Rightarrow no parallel targets needed
- Smaller emittance due to less coulomb scattering (factor ~2)
but still much bigger than needed $\varepsilon_n \sim 10.000 \mu\text{m rad}$
- Could produce polarised e⁺ by helical undulator
- But: need very high initial electron energy > 150 GeV !
- Use primary e⁻ beam
- Consequences for the commissioning and operation!



- e^- and particularly e^+ from the source have a much too high transverse ϵ_n
⇒ we have to reduce it (cooling)
- Solution: use synchrotron radiation in a damping ring



Layout of essential damping ring



Mechanism of radiation damping

- γ emission with transverse component
- Acceleration only in longitudinal direction

} **radiation damping!!!**

- Exponential damping to equilibrium emittance:

$$\mathcal{E}_f = \mathcal{E}_{eq} + (\mathcal{E}_i - \mathcal{E}_{eq}) e^{-2T/\tau_D}$$

initial emittance
 (~0.01 m rad for e^+)
 ↓
 final emittance
 equilibrium emittance
 damping time

- For e^+ we need emittance reduction by few 10^5

- ~ 7 - 8 damping times required

- Damping time:

P - emitted radiation power

$$\tau_D = \frac{2E}{P} \quad P = \frac{2}{3} \frac{r_e c}{(m_o c^2)^3} \frac{E^4}{\rho^2}$$

$$\tau_D \propto \frac{\rho^2}{E^3}$$

LEP: $E \sim 90$ GeV, $P \sim 15000$ GeV/s, $\tau_D \sim 12$ ms

$\tau_D \propto \frac{\rho^2}{E^3}$ suggests high-energy for a small ring. **But**

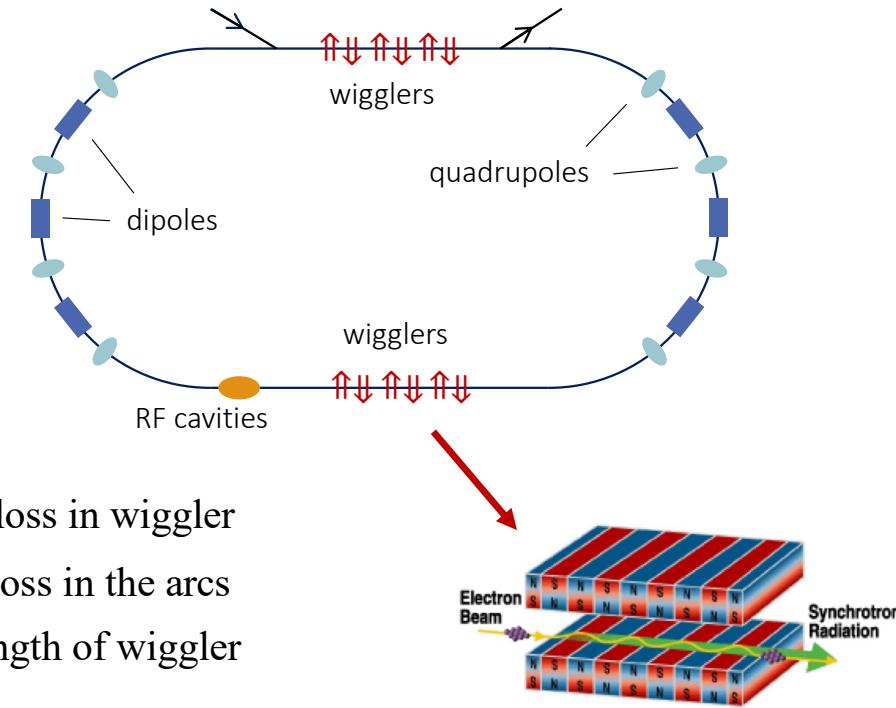
- required RF power: $P_{RF} \propto \frac{E^4}{\rho^2} \times n_b N$
- equilibrium emittance: $\mathcal{E}_{n,x} \propto \frac{E^2}{\rho}$ limit E and ρ in practice
- DR example:
 - Take $E \approx 2$ GeV
 - $\rho \approx 50$ m
 - $P_\gamma = 27$ GeV/s [28 kV/turn]
 - hence $\tau_D \approx 150$ ms - we need 7-8 τ_D !!! \Rightarrow store time too long !!!
 - Increase damping and P using wiggler magnets

Insert **wigglers** in straight sections in the damping ring

- Average **power radiated per electron** with wiggler straight sections

$$P = c \frac{\Delta E_{\text{wiggler}} + \Delta E_{\text{arcs}}}{L_{\text{wiggler}} + 2\pi\rho_{\text{arcs}}}$$

$\Delta E_{\text{wiggler}}$ energy loss in wiggler
 ΔE_{arcs} energy loss in the arcs
 L_{wiggler} total length of wiggler

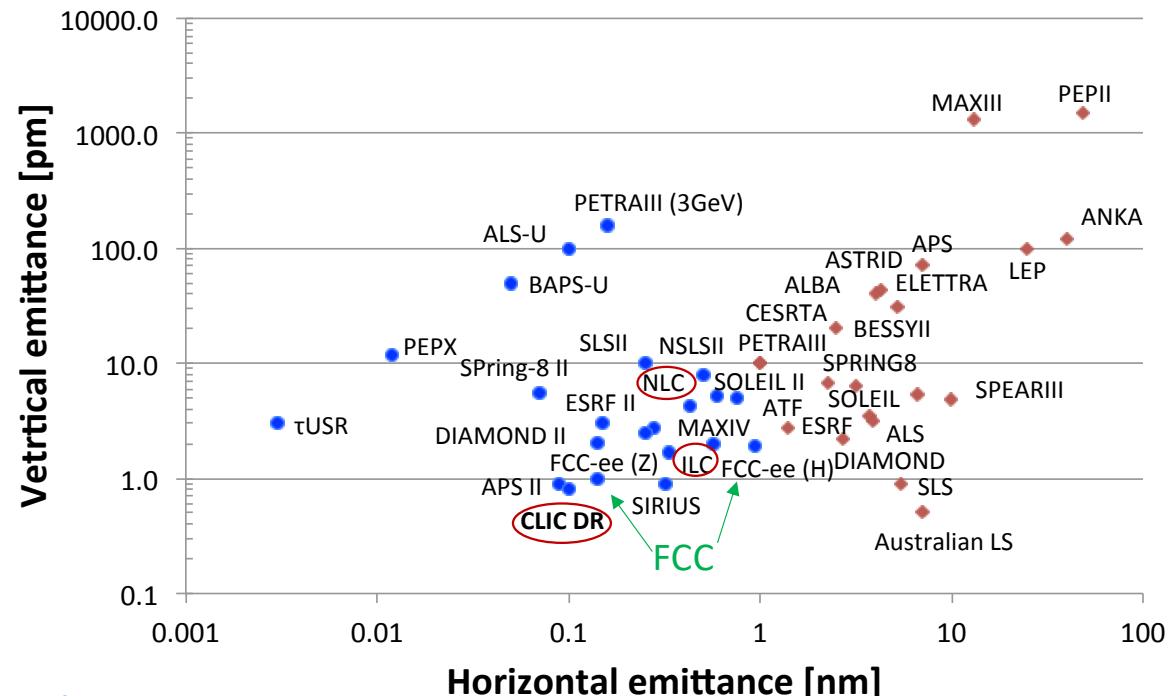


- Energy loss in wiggler:

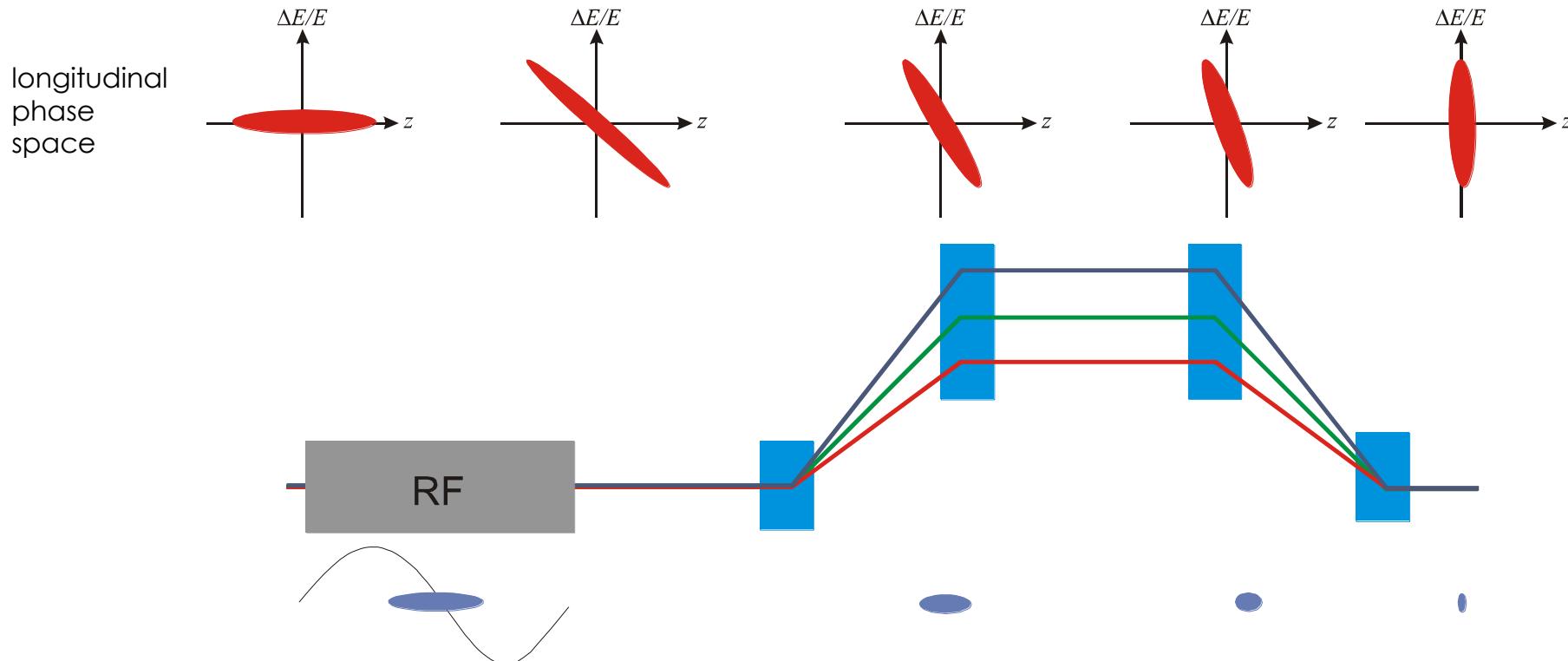
$$\Delta E_{\text{wiggler}} \approx \frac{K_\gamma}{2\pi} E^2 \langle B^2 \rangle L_{\text{wiggler}} \text{ with } K_\gamma \approx 8 \cdot 10^{-6} \text{ GeV}^{-1} \text{ Tesla}^{-2} \text{ m}^{-1}$$

$\langle B^2 \rangle$ is the field square averaged over the wiggler length

- Horizontal emittance ε_x defined by lattice
- Theoretical vertical emittance ε_y limited by
 - Space charge
 - Intra-beam scattering (IBS)
 - Photon emission opening angle
- In practice, ε_y limited by magnet alignment errors [cross plane coupling by tilted magnets]
- typical vertical alignment tolerance: $\Delta y \approx 30 \mu\text{m}$ \Rightarrow requires beam-based alignment techniques!
- DR emittance for LC in the range of existing/planned light sources

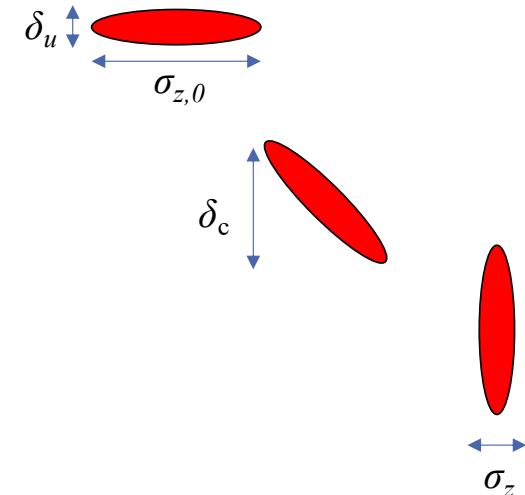


- Bunch length σ_z from damping ring: \sim few mm
- Required at IP: \sim few 100 μm or shorter
- Solution: introduce **energy/time correlation** and compress with magnetic chicane:



Initial (uncorrelated) momentum spread
 Initial bunch length
 Compression ratio
 Beam energy
 RF induced (correlated) momentum spread
 RF voltage
 RF wavelength
 Longitudinal dispersion (transfer matrix element)

δ_u
 $\sigma_{z,0}$
 $F_c = \sigma_{z,0} / \sigma_z$
 E
 δ_c
 V_{RF}
 $\lambda_{RF} = 2\pi / k_{RF}$
 R_{56}



conservation of longitudinal
 emittance ($\sigma_z \delta = \text{const}$)

$$\rightarrow F_c = \frac{\sqrt{\delta_c^2 + \delta_u^2}}{\delta_u} \Leftrightarrow \delta_c = \delta_u \sqrt{F_c^2 - 1}$$

fixed by DR

RF cavity

$$\delta_c \approx \frac{k_{RF} V_{RF} \sigma_{z,0}}{E} \Leftrightarrow V_{RF} = \frac{E \delta_c}{k_{RF} \sigma_{z,0}} = \frac{E}{k_{RF}} \left(\frac{\delta_u}{\sigma_{z,0}} \right) \sqrt{F_c^2 - 1}$$

compress at low energy

- Chicane (dispersive section) linear part $z_1 \approx z_0 + R_{56}\delta$

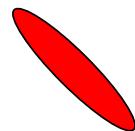
- Minimum bunch length for upright ellipse
⇒ correlation

$$\langle z\delta \rangle = 0 \quad \langle z\delta \rangle_f = \langle z\delta \rangle_i + R_{56}\delta^2 = 0$$



- Initial correlation

$$\langle z\delta \rangle_i = \frac{k_{RF}V_{RF}}{E} \sigma_{z,0}^2 = \delta_c \sigma_{z,0}$$



- With $\delta^2 = \delta_u^2 + \delta_c^2$ we get

$$R_{56} = -\frac{\delta_c \sigma_{z,0}}{\delta_c^2 + \delta_u^2}$$

- For high compression ratio ($\delta_c \gg \delta_u$)

$$R_{56} \approx -\frac{\sigma_{z,0}}{\delta_c}$$

$$\sigma_{z,0} = 2 \text{ mm}$$

$$\delta_u = 0.1\%$$

$$\sigma_z = 100 \mu\text{m} \Rightarrow F_c = 20$$

$$f_{RF} = 3 \text{ GHz} \Rightarrow k_{RF} = 62.8 \text{ m}^{-1}$$

$$E = 2 \text{ GeV}$$



$$\delta = 2\%$$

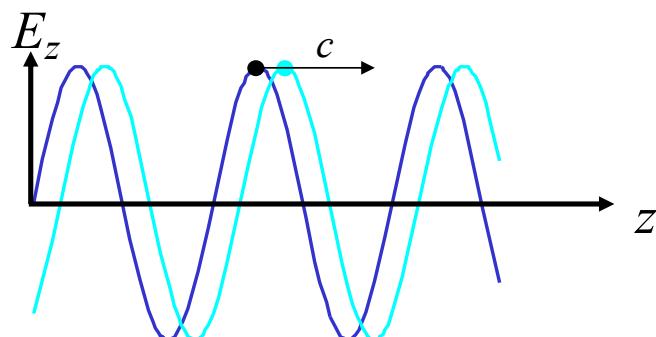
$$V_{RF} = 318 \text{ MV}$$

$$R_{56} = 0.1 \text{ m}$$

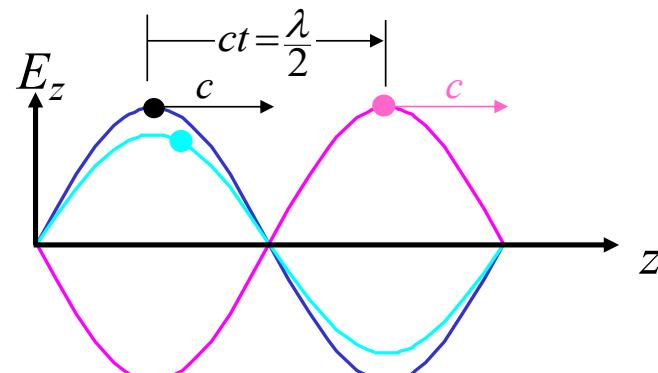
- Remark: we get a **large energy spread** after compression
⇒ large **chromatic effects** in the linac

⇒ use a **two-stage compression** with acceleration in between
to reduce relative energy spread along the line

- Now we got small, short bunches we "only" have to **accelerate** them to **collision energy**
- Reminder, **accelerating cavities**:



travelling wave structure:
need $phase\ velocity = c$
(disk-loaded structure)



bunch sees constant field:
 $E_z = E_0 \cos(\phi)$

standing wave cavity:

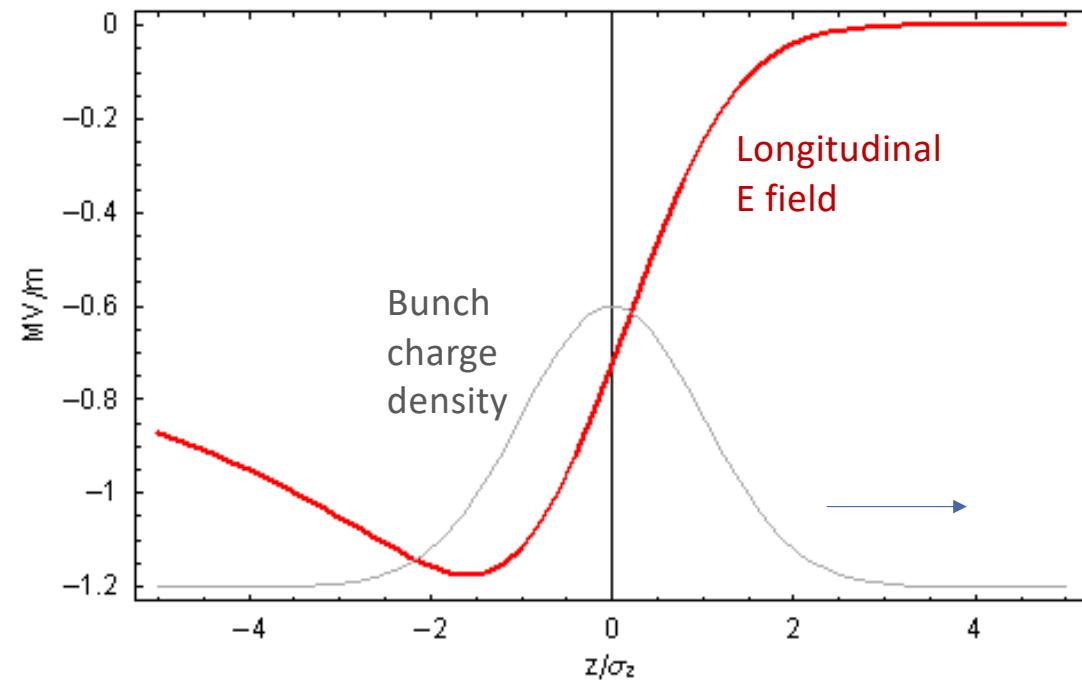
bunch sees field:

$$E_z = E_0 \sin(\omega t + \phi) \sin(kz)$$

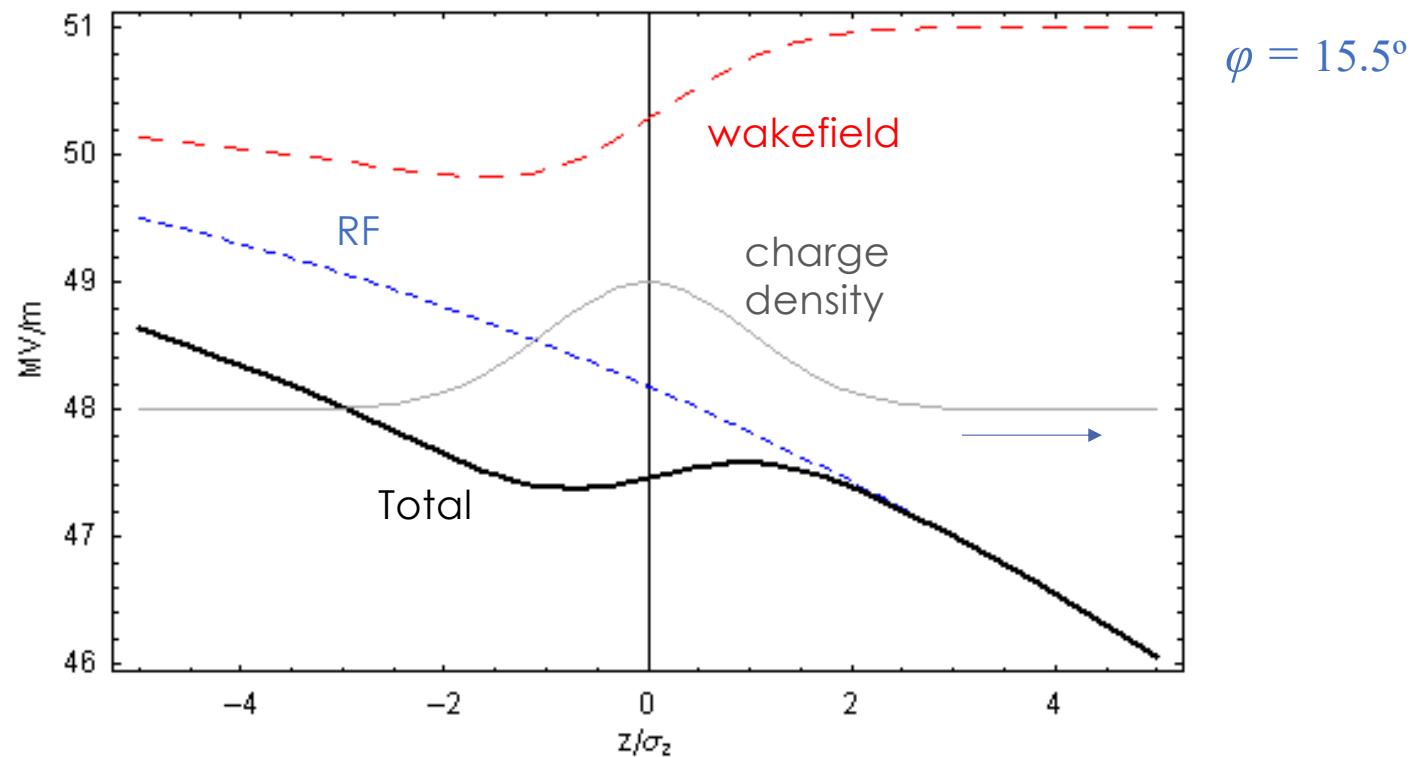
$$= E_0 \sin(kz + \phi) \sin(kz)$$

Beam **absorbs** RF power \Rightarrow **decreasing** RF field in cavities

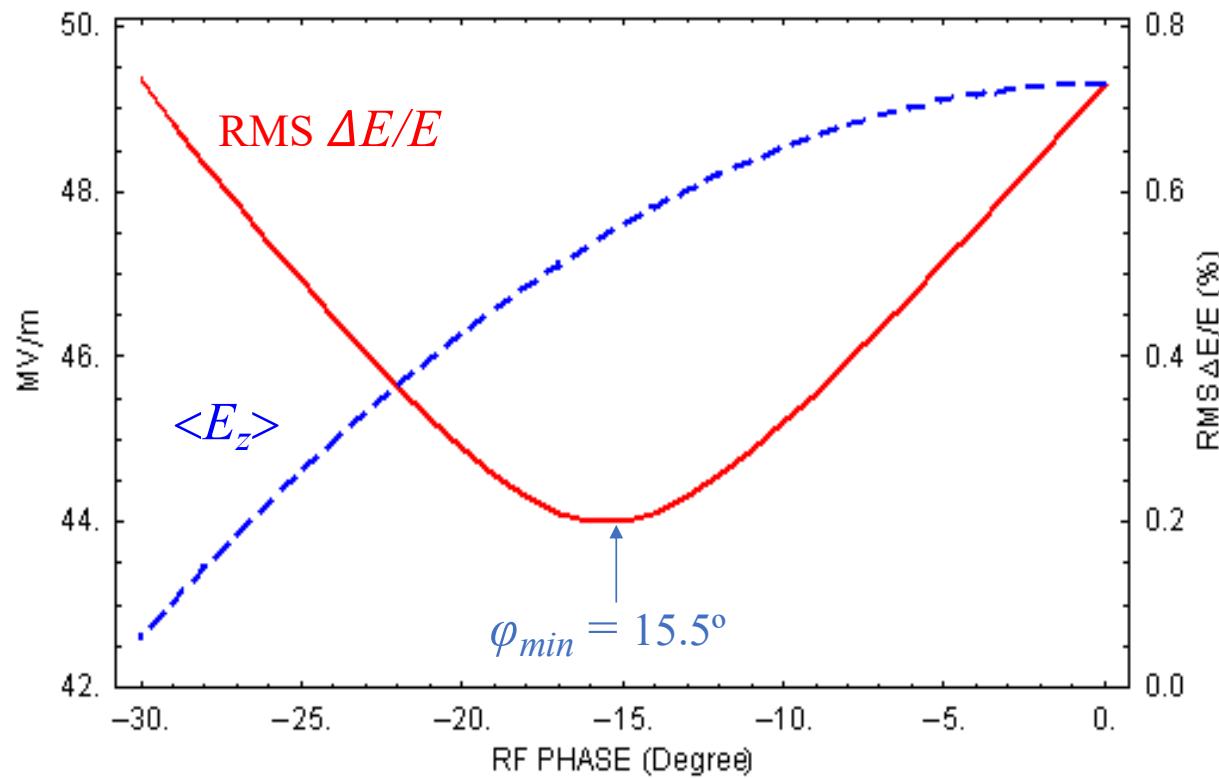
- Single bunch beam loading = single bunch longitudinal wake field
- Particles within a bunch see a decreasing field
- \Rightarrow energy gain **different** **within a bunch**



- Run **off-crest** and use **RF curvature** to compensate single bunch beam-loading
- Reduces the **effective gradient**



- Minimize momentum spread



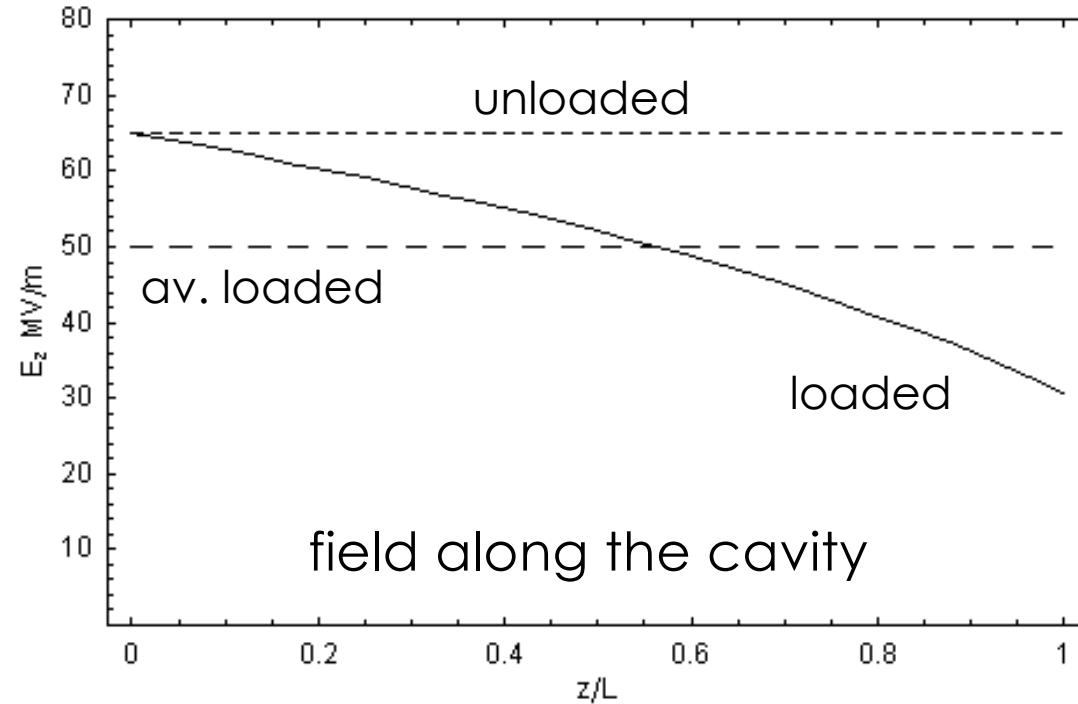
Beam absorbs RF power

⇒ gradient reduced along TW cavity for steady state

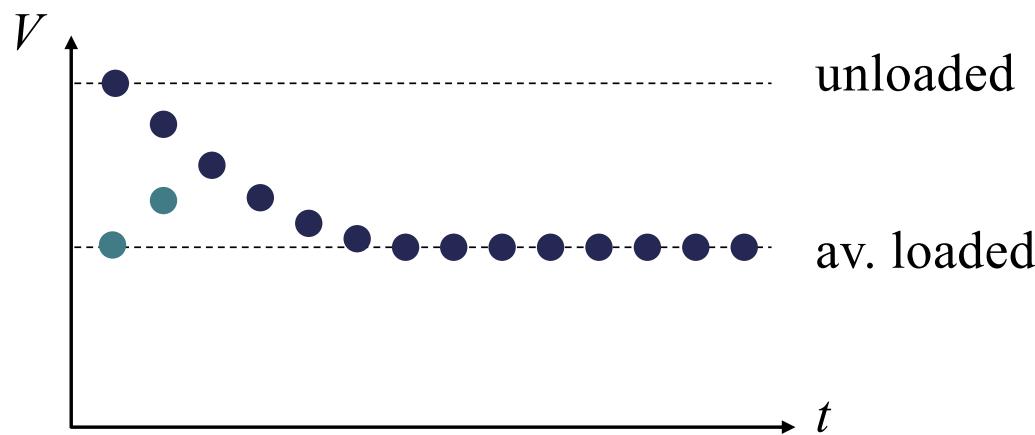
$$\frac{dP}{dz} = -\frac{E_z^2}{r_s} - I_b E_z$$

r_s shunt impedance

I_b peak beam current



- Transient beam loading (multi bunch effect):
 - First bunches see the **full unloaded field**, energy gain different from steady state
 - In the LC design, long bunch trains achieve steady state quickly, and previous results very good approximation.
 - However, transient over first bunches needs to be compensated
 - **'Delayed filling'** of the structure



With **superconducting** standing wave (SW) cavities:

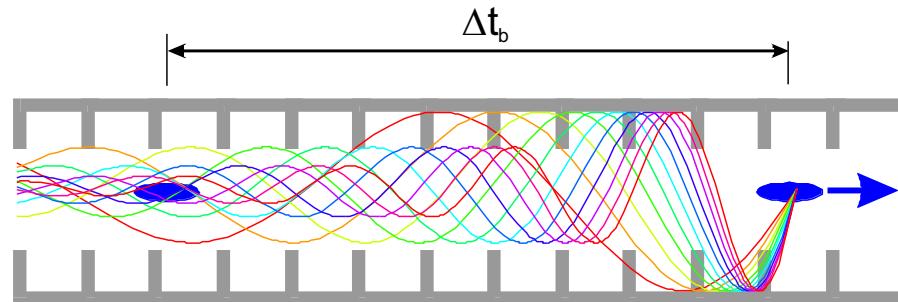
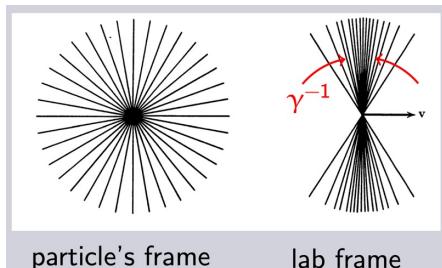
- Very **small losses** to cavity walls
- You can afford **long RF pulse** with
 - Many bunches
 - Large time between the bunches
- **RF feed-back** to compensate **beam-loading** before the next bunch arrives

⇒ **long bunch trains in SC linear collider design**

Linac must preserve the small beam sizes (and therefore the emittances), in particular in vertical plane y

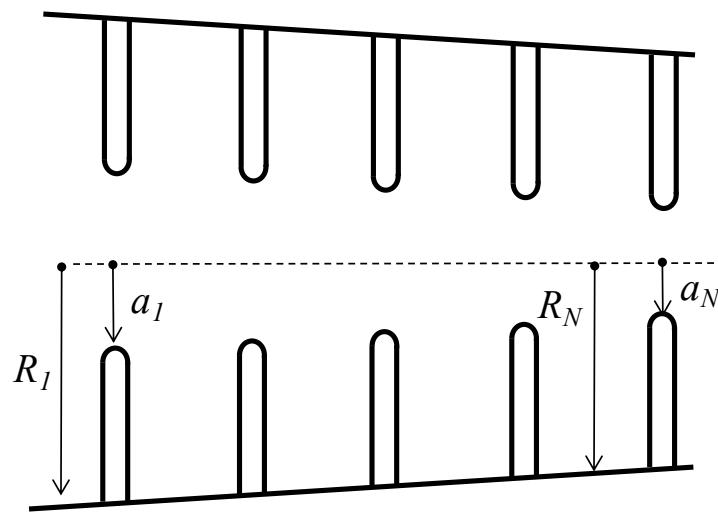
- Possible sources for emittance dilutions are:
 - Dispersive errors, chromaticity: $(\Delta E \rightarrow x, y)$
 - Transverse wakefields: $(z \rightarrow x, y)$
 - Betatron coupling: $(x \rightarrow y)$ and $(y \rightarrow x)$
 - Jitter: $(t \rightarrow x, y)$
- All these can increase the beam size at the IP
- Preserve beam size is fundamental to preserve the luminosity

Charged particles induce **electromagnetic fields** in the cavities
⇒ later particles are perturbed by these fields

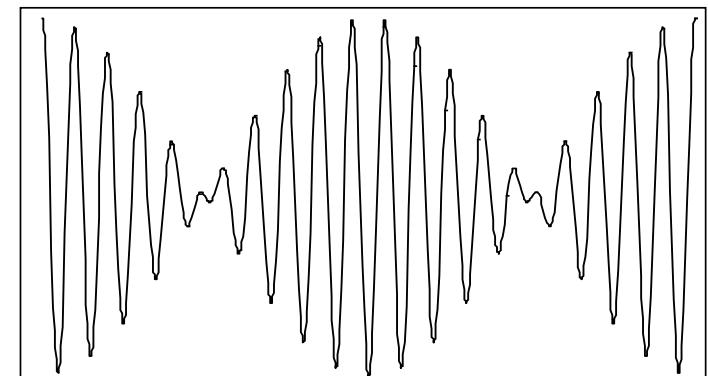


- Bunches passing **off-centre** excite **transverse higher order modes (HOM)**
- Fields can build up resonantly
- Later bunches are kicked transversely
 ⇒ multi-bunch and single-bunch **beam break-up (MBBU, SBBU)**
- Emittance growth!!!

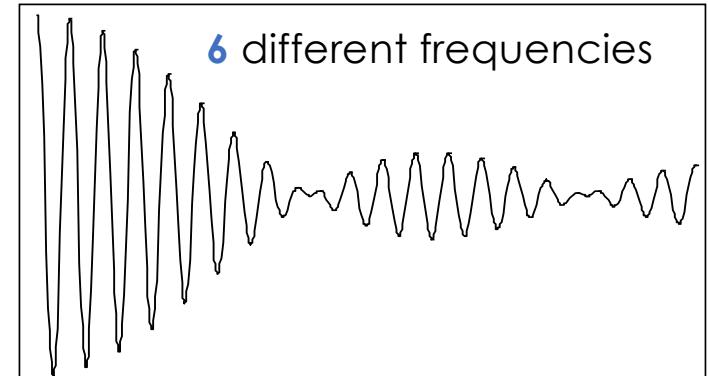
- Effect depends on a/λ (a iris aperture) and structure design details
- Transverse wakefields roughly scale as $W_{\perp} \propto f^3$
- **Less important for lower frequency:**
Superconducting (SW) cavities suffer less
- Long-range minimised by structure design
⇒ Dipole mode detuning



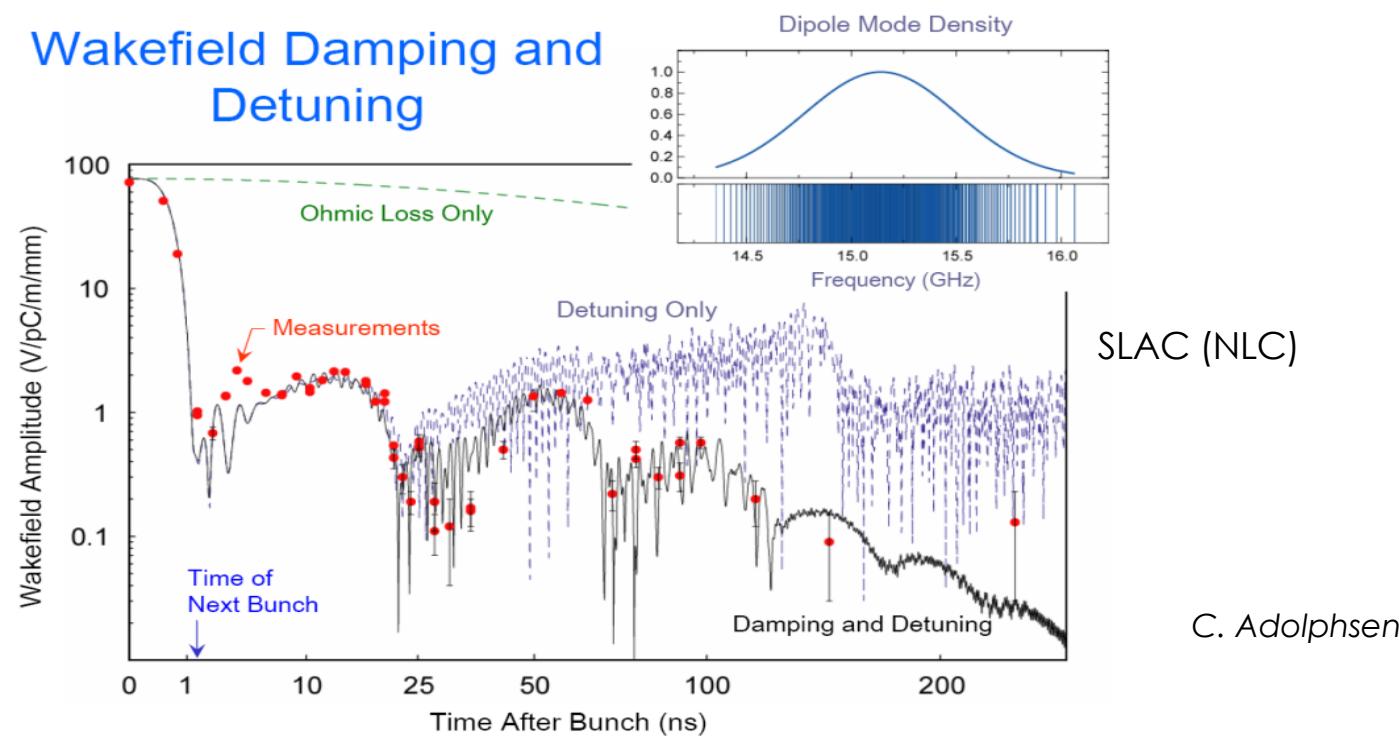
Long range wake of a dipole mode spread over **2** different frequencies



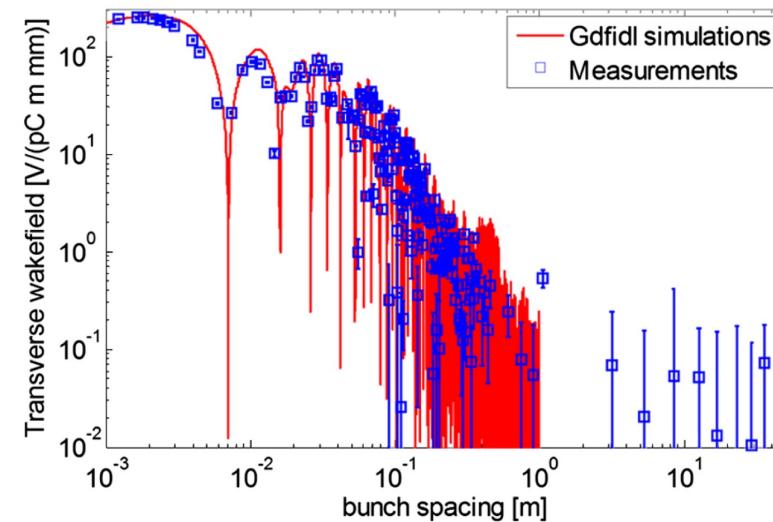
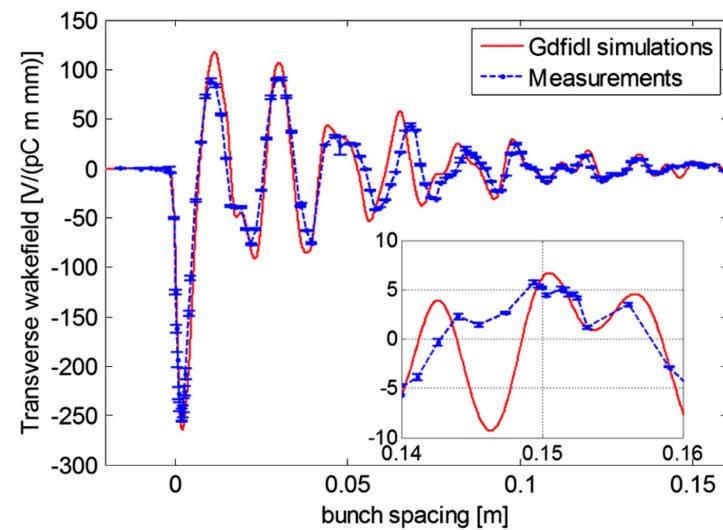
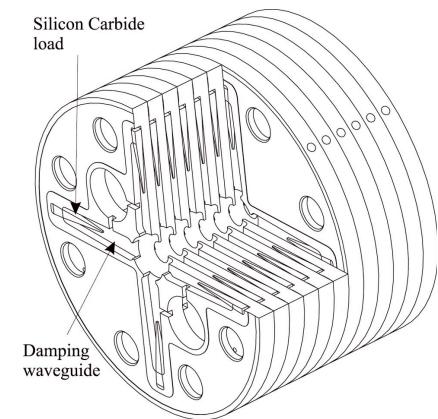
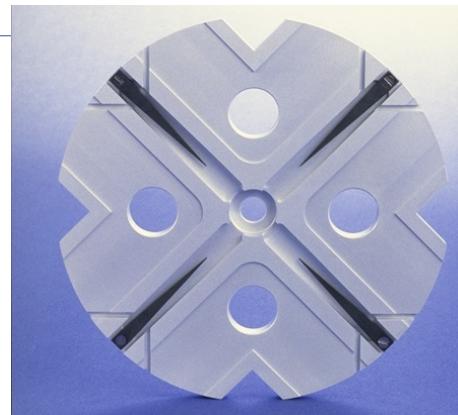
6 different frequencies



- Slight **detuning** between cells makes HOMs **decohere** quickly
- Will **recohere later**: for long trains need to be **damped** (HOM dampers)



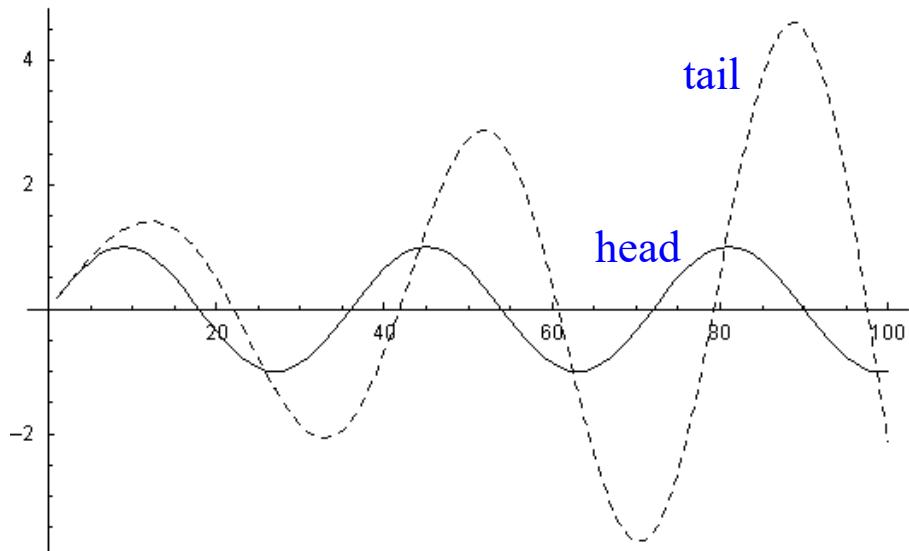
- Each cell damped by 4 radial WGs
- Terminated by SiC RF loads
- HOM enter WG
- Long-range wake efficiently damped



CERN (CLIC)
Measured at
FACET (SLAC)

A. Latina

- Head particle wakefields deflect tail particles
- Particle perform coherent betatron oscillations
- => head resonantly drives the tail



Tail particle
Equation of motion:

$$\frac{d^2 y_t}{ds^2} + k_1 y_t = f(W_\perp) y_h$$

Driven Oscillator !!

More explicit:

$$\frac{d^2 y(z)}{ds^2} + (1 - \delta) K_1 y(z) = \frac{Nr_0}{\gamma} \int_z^{\infty} dz' \rho(z') y(z') W_\perp(z' - z)$$

- Counteract effective defocusing of tail by wakefield by **increased focusing** on tail
BNS \Rightarrow Balakin, Novokhatski, and Smirnov
- Done by **decreasing tail energy** with respect to head
- Obtained by longitudinally correlated energy spread
(less off-crest than longitudinal wakefield compensation)
 \Rightarrow Transverse wakefields balanced by lattice chromaticity
- 2 particle model:

$$\Delta E = \frac{1}{8} \frac{W_{\perp} (2\sigma_z) Q L_{cell}^2}{\sin^2(\pi q_{\beta})}$$

q_{β} fractional β tune advance per cell

L_{cell} FODO cell length

- W_{\perp} non-linear
- Good **compensation achievable at the price of larger energy spread**

- BNS damping does not cure random cavity misalignment

- Emittance growth:

$$\Delta\epsilon \approx \delta Y_{RMS}^2 \left[\pi \epsilon_0 N r_e W_{\perp} (2\sigma_z) \right]^2 \frac{L_{acc} \bar{\beta}_i}{2\alpha G} \left[\left(\frac{E_f}{E_i} \right)^{\alpha} - 1 \right]$$

L_{acc} structure length

$\bar{\beta}_i$ initial average beta function

α scaling of the focusing lattice (~ 0.5)

G accelerating gradient

$E_{i,f}$ initial and final energy

- For given (maximum allowed) $\Delta\epsilon$, it scales as

$$\delta Y_{RMS} \propto \frac{1}{N W_{\perp}} \sqrt{\frac{G}{\beta}} \propto \frac{1}{N f^3} \sqrt{\frac{G}{\beta}}$$

- Higher frequency requires better structure alignment δY_{rms}
- Partially compensated by: higher G , lower β , lower N

End part II